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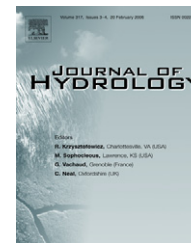
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New findings about the complementary relationship-based evaporation estimation methods

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Summary A novel approach has been found to estimate the equilibrium surface temperature (T_e) of wet environment evaporation (E_w) on a daily basis. Employing this temperature in the Priestley–Taylor equation as well as in the calculation of the slope of the saturation vapor pressure curve with pan measurements improved the accuracy of long-term mean evaporation (E) estimation of the Advection–Aridity (AA) model when validated by Morton’s approach. Complementarity of the potential evaporation (E_p) and E terms was considered both on a daily and a monthly basis with the involved terms always calculated daily from 30 yr of hourly meteorological measurements of the 1961–1990 period at 210 SAMSON stations across the contiguous US. The followings were found: (a) only the original Rome wind function of Penman yields a truly symmetric Complementary Relationship between E and E_p which makes the so-obtained E_p estimates true potential evaporation values; (b) the symmetric version of the modified AA model requires no additional parameters to be optimized; (c) for a long-term mean value of evaporation the modified AA model becomes on a par with Morton’s approach not only in practical applicability but also in its improved accuracy, especially in arid environments with possible strong convection; (d) the latter two models yielded long-term mean annual evaporation estimates with an R^2 of 0.95 for the 210 stations, which is all the more remarkable since they employ very different approaches for their E_p calculations; (e) with identical apparent E_p values the two models yielded practically identical long-term mean annual evaporation rates; (f) with the proper choice of the wind function to estimate apparent E_p the long-term mean annual E estimates of the modified AA model are still very close ($R^2 = 0.93$) to those of the Morton approach.

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Introduction

Recently there has been a renewed interest in Bouchet's (1963) complementary relationship (CR) of evaporation among hydrologists (Hobbins et al., 2001a,b; Crago and Crowley, 2005; Ramirez et al., 2005; Kahler and Brutsaert, 2006; Pettijohn and Salvucci, 2006; Szilagyi, 2001a,b, 2007; Szilagyi et al., 2001). It is understandable, since this is probably the only tool currently available to define areal evaporation based solely on widely available standard meteorological measurements, while other traditional methods, like the 'bucket model' approach, rely on water balance calculations, or need information about the water stress of the canopy, such as the Penman–Monteith (Monteith, 1973) equation. Note that here the word evaporation is used in a broad sense, including transpiration of the vegetation, since, as Brutsaert (1982) points out, the underlying physical process is the same, i.e., vaporization of water, independent of the source, be it soil, open water or the stomata of the vegetation.

The CR of Bouchet, as it is written recently, postulates an inverse relationship between actual (E) and potential evaporation (E_p) as $E + E_p = \text{constant}$. The underlying argument is that as an originally completely wet area of regional extent with an evaporation rate of E_w dries out under constant available energy (Q_n) for evaporation and sensible heat (H) transfer between land and the air, the increase in H (since a decreasing rate of evaporation will cool the surface less efficiently), ΔH , will be fully available to raise the corresponding level of E_p , thus their sum remains constant, i.e., $2E_w$. Note that with the drying of the area, the air flowing above it will dry out as well, therefore E_p will be affected not only by ΔH but simultaneously by an increase in the vapor pressure deficit (VPD) of the air. It is important to point out that both, a change in H and an accompanying change in VPD, are needed for the CR to operate.

Let us consider an open water surface, such as a shallow lake of a certain size surrounded by drying land under a constant Q_n term consumed by sensible and latent heat fluxes. Let us further assume that the flux transfer coefficients, f_E and f_H , respectively, for latent and sensible heat in a Dalton-type formulation of $E = -f_E \partial e / \partial z$ and $H = -f_H \partial T / \partial z$, where $\partial e / \partial z$ and $\partial T / \partial z$ denote the vertical gradients in vapor pressure and air temperature above the surface, would not change in time either. From a constant Q_n assumption, $\Delta H = -\Delta E$ must be true over the land as it dries, but not necessarily over the open water surface, since an additional heat transfer must be considered in its energy balance as wind blows from the drier and thus warmer land toward the water surface, unless this transported heat is fully consumed by a corresponding increase in open water evaporation, E_p , triggered by an increase in VPD of the drier air. When such a full conversion of ΔH into ΔE_p happens, one obtains a symmetrical CR, i.e., $\Delta H = -\Delta E = \Delta E_p$.

The interesting thing is that under a constant Q_n term, it probably is unlikely that only a certain portion of ΔH would raise the latent heat flux rate over the open water surface by letting the water surface become warmer instead in the expense of a reduced E_p increase. It is so because when this latter happened then an increased water surface temperature could further boost evaporation (since over a free water surface vapor pressure and temperature are related through the Clausius–Clapeyron equation), which then

would modify the sensible heat flux, eventually leading to a situation that ΔH is more or less fully consumed by a corresponding increase in open water evaporation. Therefore it is unlikely for the open water surface temperature to change significantly due to sensible heat exchange across its freely evaporating surface as long as the warmer air is sufficiently drier. This conclusion has been drawn earlier by Morton (1983) and Szilagyi (2001a, 2007).

Note that this way a near constant water temperature is fundamentally linked with a symmetric CR, meaning that one holds as long as the other, but only when the open water surface is of a certain extent. While in the open environment it may almost be impossible to verify this theoretical claim through direct measurements, simply because of the diurnal change in all the processes involved, it could, however, be performed in a fully controlled laboratory setting. The authors are anxious yet to see such an experiment.

As was mentioned, the open water body must have a certain size for the CR to become symmetric. Areal extent is important, because sensible heat transfer, due to differences of surface and air temperatures between the wet and drying surfaces, will take place not only across the free surface of the water body, but across its fixed boundary, let it be the bottom of a shallow lake or the side and bottom of an evaporation pan. When the size of the open water area is small, this additional heat transfer may be important enough to significantly alter the otherwise largely constant temperature (under a constant Q_n) of the E_p source. When this happens, the enhanced E_p rate from such an open water surface will no longer represent true potential evaporation, that is why Brutsaert (2005) named it apparent potential evaporation. Employing such values in the form of e.g., pan evaporation measurements in the CR will lead to a clear violation of its symmetric nature as was demonstrated by Kahler and Brutsaert (2006) and Szilagyi (2007). Another violation of the symmetric CR may occur when the E_p source is too large in size, since then the increased E_p -triggering effect of an enhanced VPD in the form of drier air being transported over the open water area weakens with distance along the wet surface, as the air becomes ever closer to saturation, leading to an overall diminished E_p response. For a summary of the different terms involved with the CR, see Fig. 1.

In this study Brutsaert and Stricker's (1979) Advection–Aridity (AA) model is investigated and subsequently modified based on a validation of long-term mean annual evaporation estimates by Morton's (1983) WREVP (also called CRAE) model. At a selected station, the evaporation estimates of the two versions (one with true potential evaporation from the Penman equation, and one with apparent potential evaporation values) of the modified AA model are compared with the WREVP model's estimates on a monthly and also on an annual basis. Finally, all the different annual evaporation estimates are compared to water balance estimates of evaporation of a nearby watershed.

Overview of the CR-based models for evaporation estimation

The original AA model (Brutsaert and Stricker, 1979) employs the Penman-equation (1948) for estimating E_p

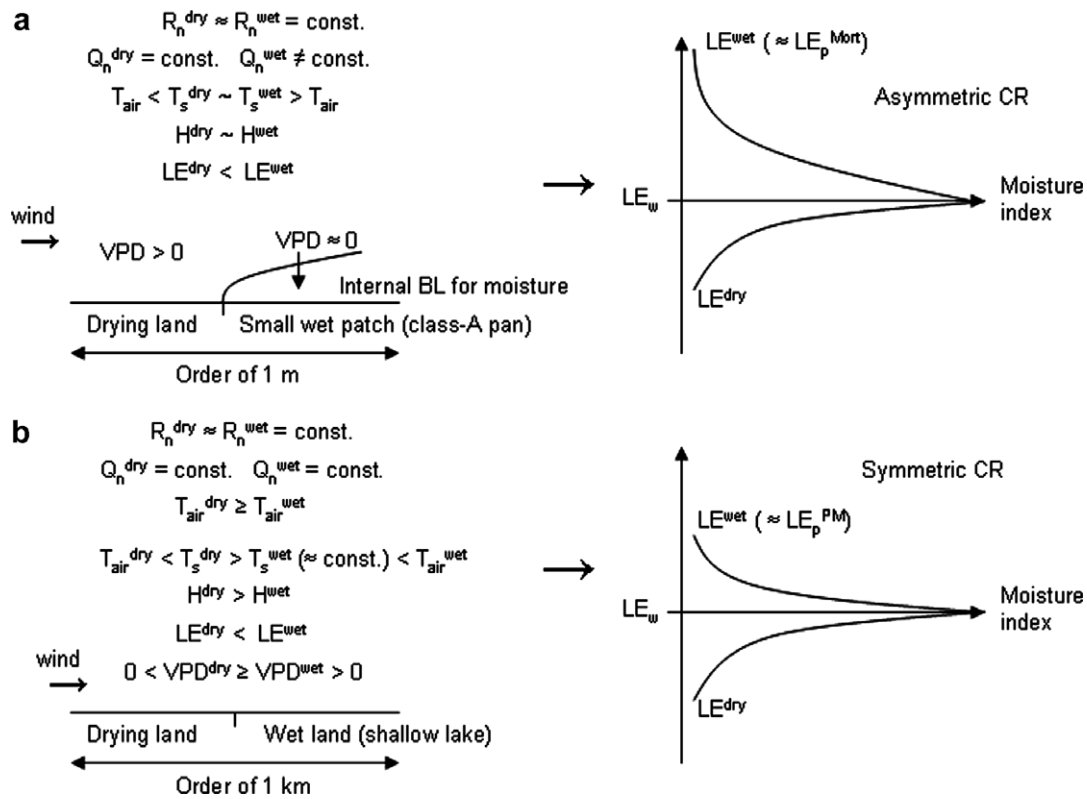


Figure 1 Schematic representation of the terms involved with the Complementary Relationship for a small wet patch (a), evaporating at an apparent potential level; as well as a plot-sized wet area (b), evaporating at the true potential evaporation rate. R_n : net radiation at the surface; Q_n : available energy at the surface for latent [LE] and sensible heat [H] fluxes; T_s : surface temperature; VPD: vapor pressure deficit; LE_w : wet environment latent heat flux; LE_p^{Mort} : WREVAP-estimated latent heat flux of potential evaporation employing the Rome wind function. BL: boundary layer; \sim : direct proportionality.

$$E_p = \frac{\delta}{\delta + \gamma} Q_n + \frac{\gamma}{\delta + \gamma} f(u)(e^* - e), \quad (1)$$

where δ is the slope of the saturation vapor pressure curve at the air temperature (and not at the required but typically unknown surface temperature), γ is the psychrometric constant, $f(u)$ is a wind function, e and e^* are the actual and saturation vapor pressures (in hPa) taken at the air temperature. Q_n now is expressed in water depth equivalent of mm d^{-1} . It also employs the Priestley–Taylor equation (Priestley and Taylor, 1972) for calculating the wet environment evaporation, E_w as

$$E_w = \alpha \frac{\delta}{\delta + \gamma} Q_n, \quad (2)$$

where α is the Priestley–Taylor parameter, with a value typically between 1.2 and 1.32. Actual evaporation then is estimated by the CR as

$$E = 2E_w - E_p. \quad (3)$$

Penman formulated his original Rome wind function for open water surfaces as $f(u) = 0.26(1 + 0.54u_2)$, where u_2 is the cup-anemometer-measured mean horizontal wind speed at 2 m height. Doorenbos and Pruitt (1975) modified the Rome wind function to be used for vegetated surfaces of agricultural areas as $f(u) = 0.26(1 + 0.86u_2)$. In both equations u_2 must be in ms^{-1} , to obtain evaporation in mm d^{-1} in (1).

Following Kahler and Brutsaert (2006), Szilagyi (2007) modified the symmetric structure of the AA model for use with apparent potential evaporation values. His version of the AA model is based on the assumption that when the freely evaporating water/wet surface is small so that its temperature would always be close to the drying land's surface temperature (Fig. 2) due to the aforementioned heat transfer across its fixed boundary, then the Bowen ratio, Bo , when written for the small wet area, would be

$$\begin{aligned} Bo = \frac{H}{E_p} &= \gamma \frac{T_s - T_a}{e_s^* - e_a} = \gamma \frac{T_s - T_a}{e_s^* - e_a^*} \left(1 - \frac{e_a^* - e_a}{e_s^* - e_a} \right) \\ &= \frac{\gamma}{\delta(T_s)} \left(1 - \frac{e_a^* - e_a}{e_s^* - e_a} \right) \approx \frac{\gamma}{\delta(T_s)}, \end{aligned} \quad (4)$$

where e_a^* and e_s^* are the saturation vapor pressure values at the air and surface temperatures, respectively. Due to the small size of the wet patch, T_a and e_a now must be obtained very close to the water/wet surface in order to have a proper fetch. The right-hand-side of (4) follows as long as the existence of a thin, saturated air layer above the water surface can be surmised. The potential existence of such an air layer in general may be supported the larger the temperature difference between the warmer water surface and cooler air. Note that δ now is in fact the slope of the saturation vapor pressure curve at the water/wet surface (since the flux transfer is written for the surface) and not the air

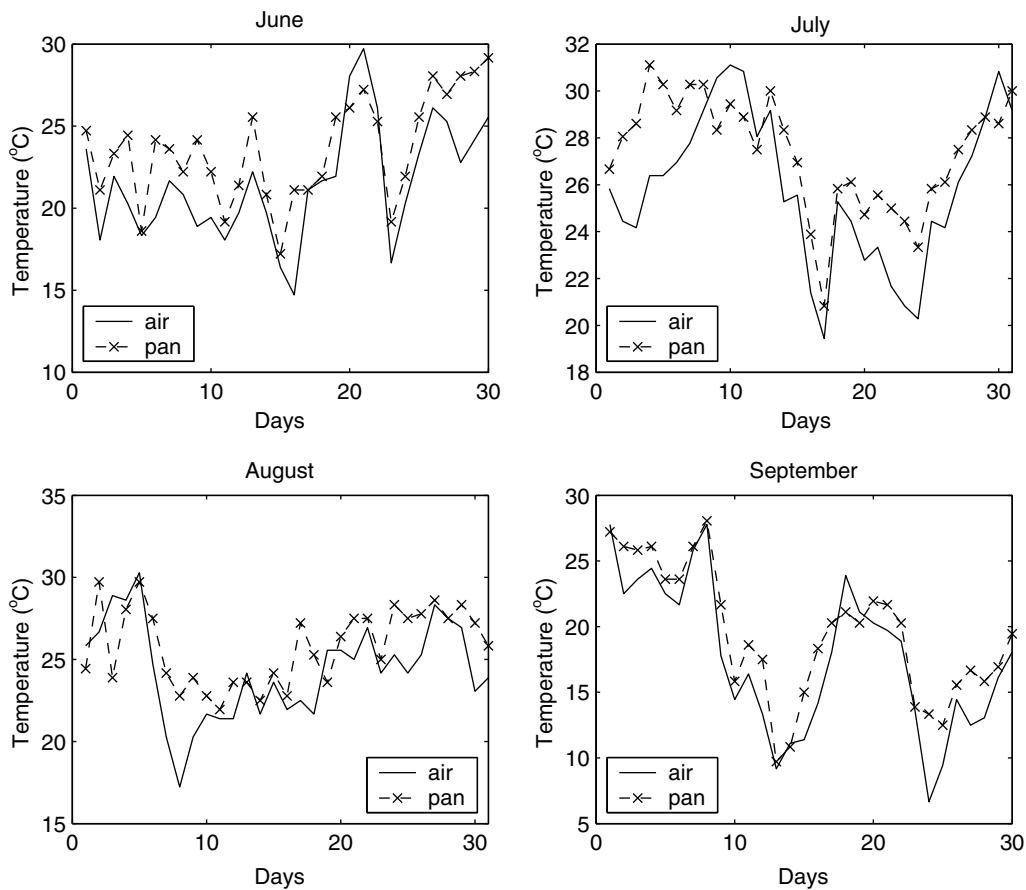


Figure 2 Mean daily air and class-A pan water surface temperatures for the summer of 1989 near the Tuttle Creek dam, Kansas, USA.

temperature, as in (1). While Szilagyi (2007) assumed that the small wet patch surface temperature is practically the same as that of the surrounding drying land (T_{dl}), it is probably more appropriate to assume a more general linear relationship between the two in place of an identity, which thus yields a similar relationship for the two sensible heat transfer terms as $H = aH_{dl} + b$, which, when inserted into (4) and after rearrangement, yields

$$\begin{aligned} \frac{dH}{dt} &= \frac{d(aH_{dl} + b)}{dt} = \frac{d[a(Q_n - E) + b]}{dt} = \frac{-a dE}{dt} \\ &\approx \frac{d}{dt} \left(\frac{\gamma}{\delta(T_s)} E_p \right) = E_p \frac{d}{dt} \left(\frac{\gamma}{\delta(T_s)} \right) + \frac{\gamma}{\delta(T_s)} \frac{dE_p}{dt}. \end{aligned} \quad (5)$$

By neglecting the temporal change in the γ/δ function, and employing finite differences in (5) one obtains

$$\Delta E \approx -\frac{c\gamma}{\delta(T_s)} \Delta E_p, \quad (6)$$

where $c = a^{-1}$. The CR for apparent potential evaporation values results by specifying the initial value in (6) as the wet environmental evaporation rate, E_w , i.e.,

$$E = E_w + \frac{c\gamma}{\delta(T_s)} (E_w - E_p). \quad (7)$$

In practice surface temperatures are rarely known, so in (7) δ may be evaluated at the known air temperature (Szilagyi, 2007), as in (1). This way a properly calibrated value of

c must correct for this switch in temperatures plus for the omission of the temporal change in the γ/δ function in (6). As an alternative, rather than evaluating δ at the actual air temperature in (7), one may evaluate it at an intermediate temperature between actual air and, yet unknown, initial wet environment surface temperatures. This choice will be discussed later.

Morton's (1983) CR-based model employs a term different from the Penman equation for the E_p calculation. He introduces the equilibrium surface temperature, T_e , for the open water source, which is obtained by iteration so that at T_e the E_p rate calculated by a Dalton-type approach and separately by an energy balance would be the same. Then the E_w term also is calculated at T_e , using the Priestley–Taylor equation which is subsequently transformed by a constant multiplier and an additive constant so that these transformed E_w values would result in a symmetric CR around them. Moreover, the flux transfer coefficient in the Dalton-type equation for E_p is formulated in a way that would not require wind measurements.

As later we will see, Morton's long-term mean evaporation estimates can be almost identically recreated by a modification of the original AA model. Morton has very carefully optimized the parameters in his model using an extensive array of pan, lake and catchment evaporation data. He maintained that in fact his approach is the only evaporation estimation method that had been universally calibrated and

would not need any additional optimization. The comprehensive study of Hobbins et al. (2001a) seems to justify Morton's claim. Over 120 basins within the US, minimally impacted by human activity, Morton's model gave a practically unbiased estimate of long-term mean annual evaporation, verified by water balances over the watersheds. In about 90% of the watersheds his evaporation estimates were within 5% of water balance closures. Hobbins et al. (2001a) also concluded that Morton's model at the same time slightly under- and over-estimates water-balance-obtained watershed-scale evaporation rates in humid and arid environments, respectively. However, for its undeniably good overall performance the Morton model was chosen as a practical tool of verification in the present CR-based evaporation estimation investigation.

Comparison of the long-term means of the CR-based models' evaporation estimates

Comparison of long-term mean annual evaporation estimates of the WREVAP and classical AA models

Both the Morton and AA models were applied for evaporation estimation using hourly measurements over the 1961–1990 period, aggregated for daily values at the 210 stations of the Solar and Meteorological Surface Observation Network (SAMSON) within the contiguous United States (Fig. 3). 19 out of these stations additionally had pan evaporation data for the growing-season (May–September). Both models require air pressure, temperature, and humidity, as well as global incident radiation data that were available at these stations. By employing true potential evaporation estimates the AA model additionally needs wind measurements at 2 m height. Since the wind measurements, u_r , at the SAMSON stations were taken at different heights, z_r , (always higher than 2 m) along the 30-yr period, they had to be

converted to values representative at 2 m above the ground. This was achieved by a power function of $u_2 = u_r(2/z_r)^{1/7}$ (Brutsaert, 2005). With the algorithm of Morton's WREVAP model, the daily global incident radiation values were converted into net radiation values, expressed in mm d^{-1} that were considered equal to Q_n for the day. This ensured that the same variables were entered into the two models (with the exception of the wind velocity), so differences in their output could not be blamed for differences in data inputs.

Both evaporation terms, E_p and E_w , were first calculated on a daily basis. Comparison of the WREVAP models' E_p values from daily and monthly data respectively, revealed problems with its daily estimates. In fact, Morton has always cautioned not to use his model on a daily basis. Therefore, his model subsequently has been applied on a monthly basis with monthly input values in this study, as was similarly done by Hobbins et al. (2001a,b), and as Morton himself recommended. With the AA model in its E_p and E_w estimates such a problem has not been detected, so these variables were always calculated with daily data and subsequently aggregated for monthly values as the default case before the CR was invoked. Brutsaert and Stricker (1979) and Morton (1983) recommended that the shortest time-period the CR should be applied over is 3–5 days, simply, because on a shorter time interval a passing weather front can significantly upset any dynamic equilibrium in the fluxes between the land and atmosphere. Interestingly, whether the CR is invoked on a monthly, using daily estimates of E_p and E_w subsequently aggregated for months, or directly on a daily basis does not change the AA-model results, only the optimized value of the Priestley–Taylor parameter may change slightly as seen below.

Fig. 4 displays the long-term mean annual evaporation estimates by the WREVAP and the original AA models for all 210 stations. Here a value of 1.28 was employed for α in (2). As can be seen, the correlation is strong ($R^2 = 0.93$)



Figure 3 Distribution of the SAMSON stations. The triangles denote stations with growing-season (May–September) class-A pan evaporation data.

between the two types of estimates, except for arid climates, where the AA model consistently overshoots the WREVAP estimates which are certainly much closer to real-

ity even though they themselves overshoot true evaporation rates in such regions (Hobbins et al., 2001a). Nonetheless, the AA model has practically the same station-averaged

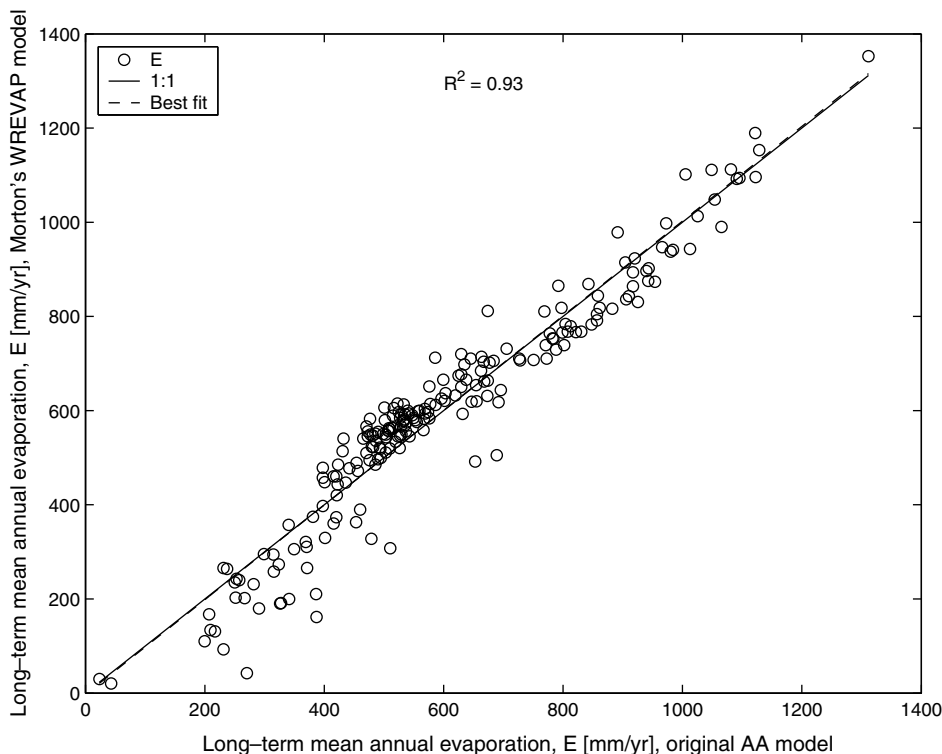


Figure 4 Regression plot of the original AA and WREVAP model estimates of long-term mean annual evaporation for the 210 SAMSON stations.

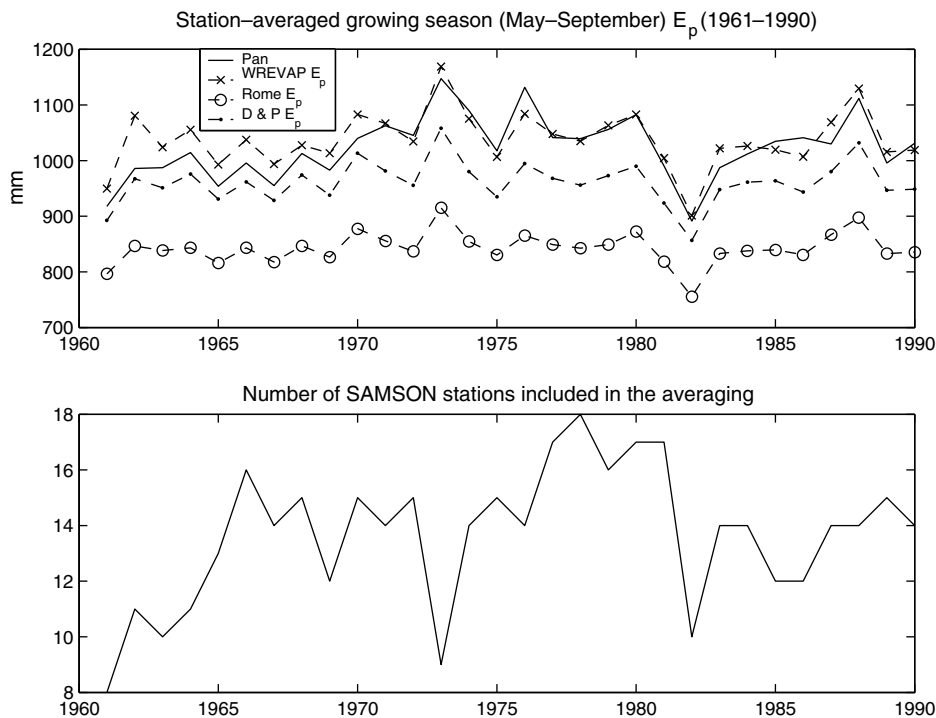


Figure 5 Station-averaged growing-season class-A pan measurements as well as different potential evaporation estimates by year. 'Rome' and 'D & P' E_p denote the Penman-equation E_p by the Rome and the Doorenbos–Pruitt wind functions, respectively. The number of stations fluctuates due to availability of data.

mean (595 mm yr^{-1}) as the WREVAP model (596 mm yr^{-1}), and the slope of the regression curve is an almost perfect unity (1.005) with an intercept value of a mere -2.4 mm yr^{-1} . (When the same is performed with the CR being invoked on a daily basis the optimized value of α changes to 1.29 to obtain practically the same E estimates).

The success of the AA model (except in arid regions as evidenced by Fig. 4 and also reported by Hobbins et al., 2001a), with its symmetric CR tells us an important property about its E_p estimates. Namely, the Rome wind function in the Penman equation was optimized by Penman so that it would describe the evaporation rate of a wet area just about the right size, i.e., the size of a plot with a characteristic length of a hundred meters, as was discussed above. Indeed, for the optimization of the Rome wind function he used sunken pans and subsequently validated his evaporation estimates with small reservoir evaporation measurements (Penman, 1948) as well. While for a class-A pan, daily mean water surface temperatures (Fig. 2) can be expected to be higher than those of the surrounding air (Jacobs et al., 1998), for sunken or insulated pans they cannot (Oroud, 1998; Martinez et al., 2005). This is so because hot air close to the surface cannot heat the side (and bottom) of such pans, neither can direct sunshine. Also, with depth the soil temperature changes quickly, so the deeper the sunken pan the less energy it can receive through heat conduction from the soil surface. That is why a sunken pan, such as employed by Penman, may approximate the evaporation rate of a large enough open water surface. This way the evaporation rate, E_p , of the Penman equation employing the Rome wind function is not an apparent potential evaporation rate, but rather one that is closer to a true potential evaporation rate, meaning that it describes the evaporation of a large enough (but not too large so that it would significantly influence the over passing air) open water source where any heat transfer to it across its fixed boundary (as opposed to its free open water surface) is already negligible. Note, however, that this is not the case with either the Morton-calculated E_p values or the wind function recommended by Doorenbos and Pruitt (1975), as evidenced by Fig. 5.

Comparison of long-term mean annual class-A pan measurements and E_p estimates of the Penman equation as well as the WREVAP model

Let us consider the consequences of Fig. 5. First, it is clear that Morton's E_p values are very close to the actual class-A pan evaporation rates. He assumes that class-A pan E_p would always take place at an equilibrium surface temperature, T_e , i.e., the temperature that would be more-or-less conserved at the E_p source as the wet environment dries and warms around it. As Fig. 2 demonstrates, it is not so as the daily mean water temperature of a class-A pan in the summer is typically higher than the ambient drying environment air temperature. Whether WREVAP calculates T_e correctly or not, would not affect the WREVAP model's evaporation estimates much eventually, because of (a) the freely adjustable stability parameter Morton employs in his Dalton-type equation for E_p in place of the wind measurements and; (b) other adjusting factors in his E_w esti-

mates. So, with the proper tuning of these parameters it can be achieved that the WREVAP E_p values estimate pan evaporation correctly without employing the correct water surface temperature.

Fig. 5 also demonstrates that the wind function recommended by Doorenbos and Pruitt (1975) for the E_p calculation of a vegetated surface of a crop field (as opposed to the Rome wind function that was worked out for an open water surface) causes the E_p estimates to be intermediate between the class-A pan values, an apparent potential evaporation measure, and the 'true' E_p values represented by the Penman equation with the Rome wind function. Here the vegetated area over which the calibration of the wind function by Doorenbos and Pruitt (1975) took place had to be of a typical plot size in order to be a useful prediction tool of water requirements of agricultural plants. So if the area of the E_p source is more-or-less the same as implied by the Rome wind function, why would then a vegetated surface typical of agricultural crops and with unimpeded access to water behave more like a class-A pan and not like an open water surface with the same plot size? The answer most likely is twofold: (a) a crop field has a larger roughness height than an open water surface, making the airflow over it more turbulent; (b) the advected warm air can warm up the plants, since they are not covered completely by water that would consume this heat through evaporation, consequently, as the stomata open, the water vapor is already at this elevated plant temperature, leading to enhanced evaporation, yet not being able to keep the plant temperature as steady as an open water surface achieves with respect to its temperature, simply because stomata cover only a fraction of the plant surface.

There remains to discuss the most important consequence of Fig. 5, at least for our goal of evaporation estimation. It is seen that the Penman equation with the Rome wind function predicts the evaporation rate of not an apparent potential evaporation source, such as a class-A pan, but rather that of a true potential evaporation source. As was postulated above, such a source would more-or-less conserve its equilibrium (T_e) or wet environment surface temperature. This is of importance because the original AA model calculates the wet environment evaporation, E_w , at the actual, measured air temperature and not the equilibrium temperature. The equilibrium temperature, T_e , can, however, be easily calculated (different from the WREVAP model) as follows.

Calculation of the equilibrium surface temperature, T_e , and comparison of the long-term mean annual evaporation estimates of the WREVAP and modified AA models

Writing the Bowen ratio for the open water surface representing a true potential evaporation source

$$Bo = \frac{H}{E_p} = \frac{Q_n - E_p}{E_p} = \gamma \frac{T_s - T_a}{e_s^* - e_a} = \gamma \frac{T_e - T_a}{e^*(T_e) - e_a}, \quad (8)$$

the unknown T_e can be expressed iteratively, since all the other terms are known (the E_p from the Penman equation employing the Rome wind function). Note that H can only be expressed as $Q_n - E_p$ in (8) for the wet surface because,

unlike the small wet patch case, local energy transfer across the wet area's fixed boundary is negligible due to its relatively large size. This T_e value now can be applied in (2) to calculate the wet environment evaporation, E_w , at the equilibrium temperature. Note that in (2) δ is customarily taken at the (only) available air temperature, but correctly it should be taken at the wet environment surface temperature, T_e . This way the modified AA model employing true potential evaporation values becomes

$$E = 2E_w(T_e) - E_p. \quad (9)$$

Fig. 6 displays the evaporation estimates by the so-modified AA model with $\alpha = 1.31$ (which was kept throughout with the CR being invoked on a monthly basis) in (2), and plotted against the WREVAP values. The R^2 value increased to 0.95 between the two model estimates, indicative of a very strong linear correlation ($r = 0.974$). As seen, the modified AA model improves dramatically in arid environments – where the difference between actual air and equilibrium surface temperatures is expected to be the largest – no longer overshooting the WREVAP values. The station-averaged long-term mean annual evaporation value of 595 mm yr^{-1} again is almost identical to the WREVAP model's. While a jump from 0.93 (Fig. 4) to 0.95 (Fig. 6) in the R^2 value between the original and modified AA models may not seem significant, the improvement is indeed in arid regions (compare the lower parts of the two Figures). Also, it can be claimed for the first time that the long-term mean annual evaporation estimation of the AA model has probably improved over that of the WREVAP model because of the latter's slight undershoot of the evaporation rates in humid and a similar overshoot in arid regions. The modified AA

model corrects this problem via its smaller than unit slope of the best-fit line in Fig. 6. (The same results can be obtained with the CR invoked on a daily basis employing $\alpha = 1.29$).

Formulation of the modified AA model for apparent potential evaporation values and comparison its long-term mean annual evaporation estimates with those of the WREVAP model

As it is obvious from Fig. 5 that Morton's E_p estimates are apparent potential evaporation values, with such values Szilagyi's (2007) modification of the AA model, (7), making use now of the known T_e , can be written as

$$E = E_w(T_e) + \frac{\gamma}{\delta(T_e)} [E_w(T_e) - E_p], \quad (10)$$

where a value of unity was assigned for c . Note that the choice of $c = 1$ and the application of the equilibrium temperature rather than a temperature of, e.g., $0.5(T_a + T_e)$ – since the surface temperature changes from T_e to even beyond T_a over the apparent E_p source as the environment dries – is arbitrary, because now the aim is not to calibrate (10) from location to location, but rather, to evaluate how the modified AA model employing apparent potential evaporation values fare in general with the WREVAP model's long-term mean evaporation estimates. Such a comprehensive and objective calibration, lacking water balance data for each station, could not be performed here.

Fig. 7 displays the so-derived E estimates against those by the WREVAP model employing the same Morton-derived E_p estimates in both models. As can be seen, this version of

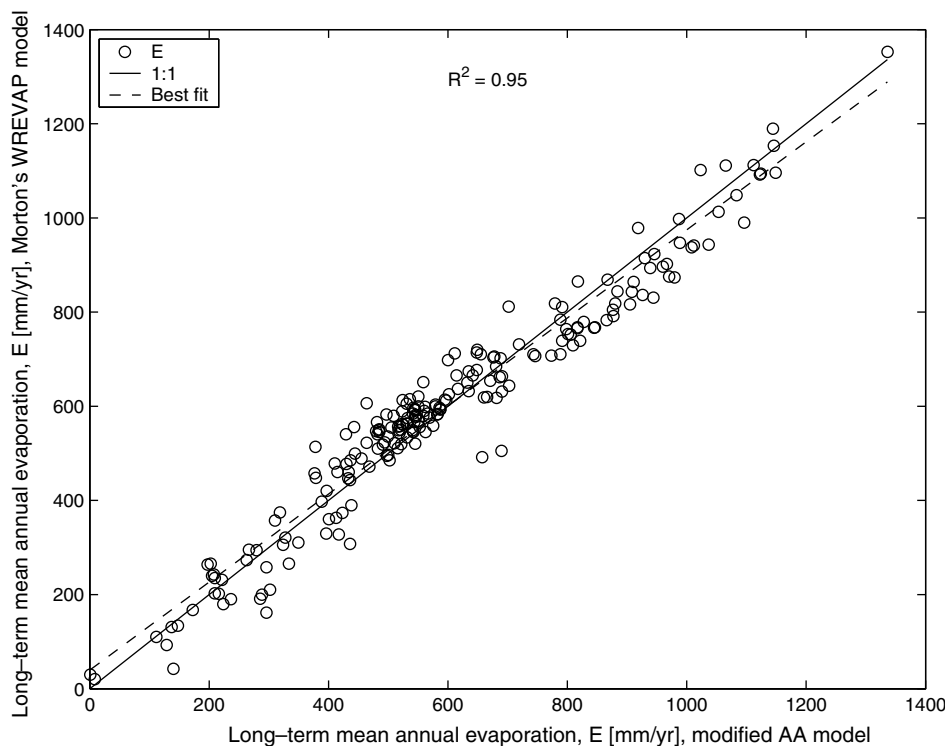


Figure 6 Regression plot of the modified AA, employing the Penman equation with the Rome wind function, and WREVAP model estimates of long-term mean annual evaporation for the 210 SAMSON stations.

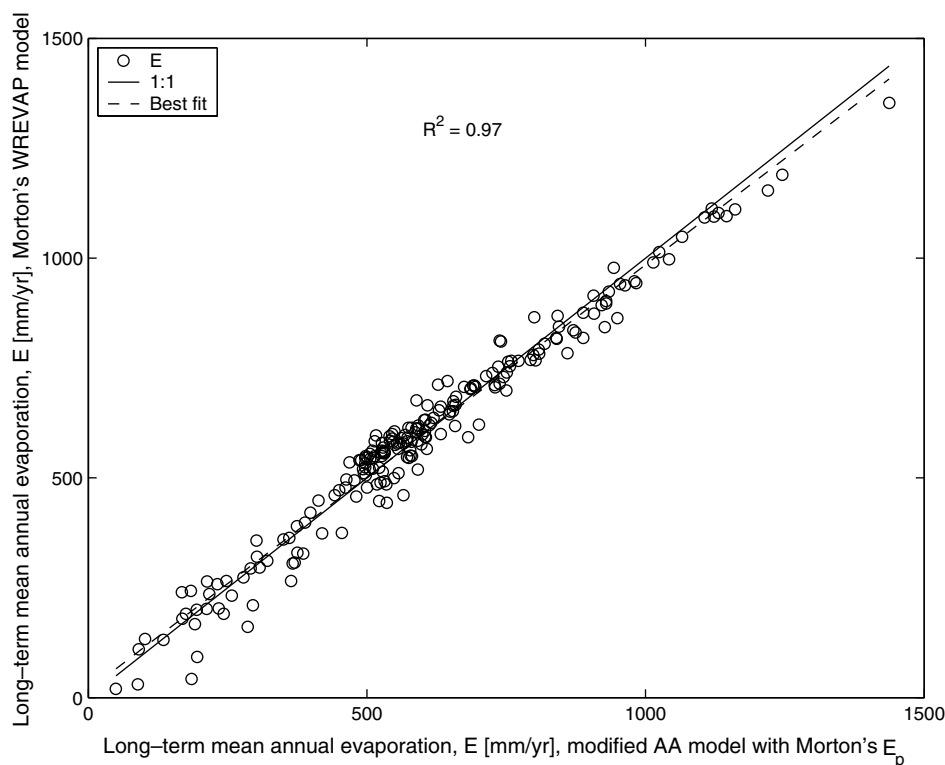


Figure 7 Regression plot of the modified AA and WREVP model estimates of long-term mean annual evaporation for the 210 SAMSON stations. The modified AA model, with the $\gamma/\delta(T_e)$ term, now employs the Morton E_p estimates.

the modified AA model yields very similar long-term mean E estimates (station-averaged long-term mean is 598 mm yr^{-1}) to the WREVP model ($R^2 = 0.97$), even correcting the slight over- and undershoot of the latter in arid and humid regions, respectively, similarly to the previous case (Fig. 6) of employing true E_p values in the modified AA model. Here the application of T_e instead of T_a both in the E_w and the γ/δ terms makes a significant difference in the E estimates (a jump from 0.87 to 0.97 in the R^2 value, not shown). The relationship however improves even further ($R^2 = 0.99$, and station-averaged long-term mean is 599 mm yr^{-1}) when replacing the $c\gamma/\delta$ term in (7) with a constant value of 0.55 (which, interestingly, is the γ/δ value at about 13°C , very close to the station-averaged mean annual temperature of 12.2°C). The result is displayed in Fig. 8. Note, it does not mean that this last version of the AA model would result in better evaporation estimates than the one that employs the $c\gamma/\delta$ multiplier, it simply means that it yields long-term mean E estimates closer to those of the Morton model.

The very high correlations ($r > 0.98$ in Figs. 7 and 8) between the E estimates of the last two versions of the modified AA and the WREVP models demonstrate that the two (i.e., the modified AA and the WREVP model) are almost identical for long-term mean annual evaporation rates when the same inputs are applied. At the same time the modified AA model is expected to yield slightly improved E estimates over the WREVP model in Fig. 7 since it corrects for the slight under and overshoot of the WREVP model in humid and arid regions, respectively. Almost identical outputs between the modified AA and WREVP models, at least on a long-term basis, seem to confirm what Granger already sta-

ted in 1989, i.e., albeit the WREVP model's E and E_p rates are perfectly symmetric about its wet environment evaporation, E_w^* values, the WREVP model is inherently based on an asymmetric CR. This is so because the E_w^* values were obtained by Morton via transforming the Priestley–Taylor E_w values in order to obtain such an, in this case, 'artificial' perfect symmetry.

Another conclusion to be drawn here is that the symmetric or asymmetric nature of the AA model is not an important issue. Simply, the AA model is symmetric when in its E_p estimates the Rome wind function is employed (reflecting that via the application of the Rome wind function the Penman E_p estimates are not apparent potential but rather true potential evaporation measures) and becomes asymmetric for an apparent potential evaporation source such as a class-A evaporation pan. Here it should be noted that via the application of the Rome wind function in the Penman E_p estimates, a symmetric relationship between E and such E_p is not always guaranteed, as was observed by Szilagyi (2007) with data from the FIFE experiment in the Konza prairie. His observed asymmetry could be the result of a combination of the following two factors: (a) the Rome wind function's ability to describe true E_p may, to a certain degree, depend on the actual environmental conditions; (b) the meteorological measurements required by the Penman equation were obtained in the town of Manhattan, Kansas, while the measured evaporation fluxes were taken in a prairie location, several kilometers away. (The same holds for Morton's apparent potential evaporation estimates: at certain stations out of the 19 included in Fig. 5, his E_p estimates were very close to actual pan measurements, but at others

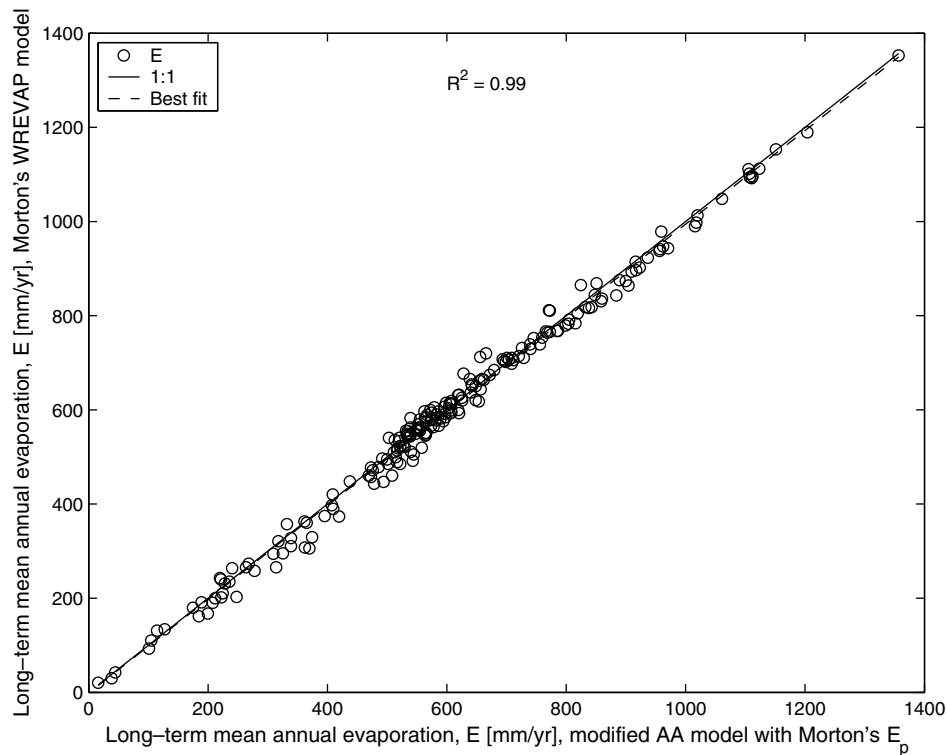


Figure 8 Regression plot of the modified AA and WREVAP model estimates of long-term mean annual evaporation for the 210 SAMSON stations. The modified AA model now contains a constant (0.55) for the γ/δ term and employs the Morton E_p estimates.

not so at all). All this may suggest a possible need for local calibration of the Rome wind function's parameters to describe true E_p properly.

Estimating apparent potential evaporation by modifying the wind function in the Penman equation

Finally, by tweaking the wind function's parameters in the Penman equation, as was demonstrated by Doorenbos and Pruitt (1975), it is possible to replicate the E_p of an apparent potential evaporation source, such as a vegetated surface or even a class-A pan. The latter is demonstrated in Fig. 9 where a good correlation ($R^2 = 0.92$) could be achieved between the Penman and Morton long-term E_p values by a wind function $f(u) = 0.49(1 + 0.35u_2)$, which is traditionally written this way rather than a simple linear relationship of $y = ax + b$. The resulting E estimates are displayed in Figs. 10 and 11. (The same results were obtained when invoking the CR on a daily basis with $\alpha = 1.29$). Here the AA is in the same form as for Figs. 7 and 8, i.e., $E = E_w(T_e) + [\gamma/\delta(T_e)][E_w(T_e) - E_p]$, and $E = E_w(T_e) + 0.55[E_w(T_e) - E_p]$, respectively. The so-derived E estimates, however, are not the same as in Figs. 7 or 8 due to differences in the Penman- (employing the above tweaked wind function) and Morton-estimated E_p values in Fig. 9. With this new wind function the modified AA model overshoots the evaporation rate in arid climates. Naturally, the evaporation estimates would improve by local calibration of c and the chosen temperature δ is evaluated at. In order to avoid such additional parameter calibrations for the wind function and the $c\gamma/\delta$

term (which cannot be performed when independent measures of the sought for E are totally lacking) the symmetric version of the modified AA model with the original Rome wind function should always be preferred for practical evaporation estimation. Finally, Fig. 12 demonstrates how these new E_p estimates generally stack up with pan evaporation measurements.

Comparison of the CR-based models' evaporation estimates on a monthly basis: a case study

Among the SAMSON stations, Omaha, Nebraska not only has a record of growing-season (April–September) class-A pan evaporation but it is located at the base of the Elkhorn watershed (drainage area of about 20,000 km², Fig. 13) for which precipitation and flow-rate measurements have also been available for the SAMSON data period of 1961–1990. This way annual evaporation could also be obtained as the difference of spatially-distributed precipitation and flow volume as described in detail by Hobbins et al. (2001a). Watershed-representative mean annual precipitation for the period is 763 mm, while evaporation is 583 mm.

The annual course of the different monthly evaporation terms is demonstrated in Fig. 14 for the 1981–1985 period, the rest of the data having very similar attributes. As has been mentioned, WREVAP E_p is typically close to class-A pan measurements, while the Penman E_p is banded by the pan and wet environment evaporation rates. A general feature of the actual monthly evaporation estimates is that the modified AA-model values tend to be somewhat larger than

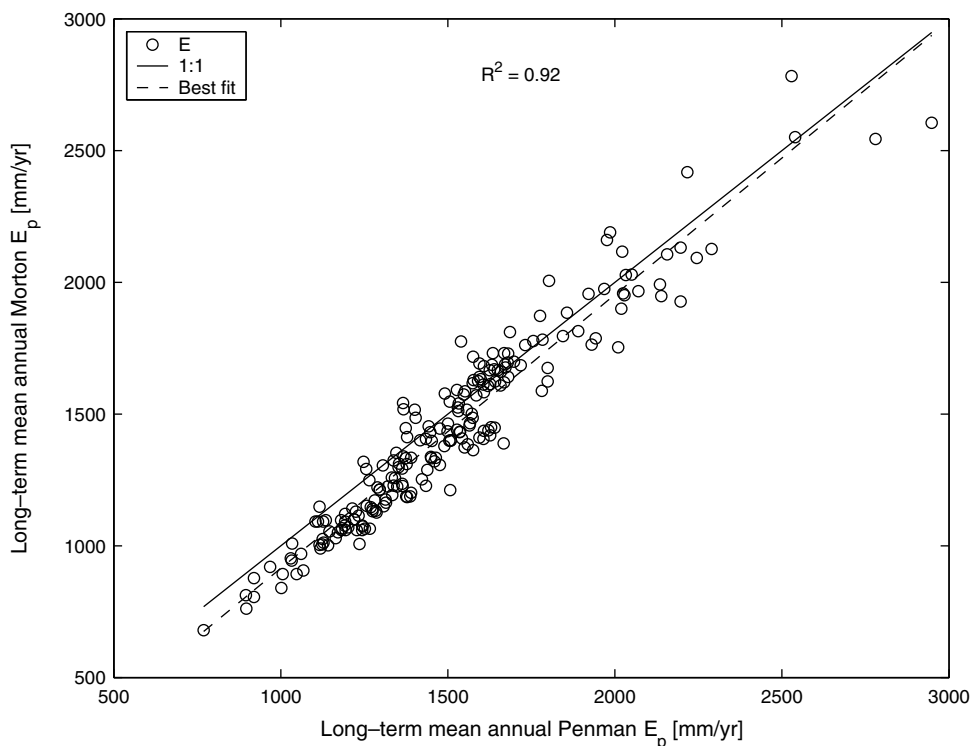


Figure 9 Regression plot of the Penman equation, employing $f(u) = 0.49(1 + 0.35u_2)$, and WREVAP estimates of long-term mean annual E_p for the 210 SAMSON stations.

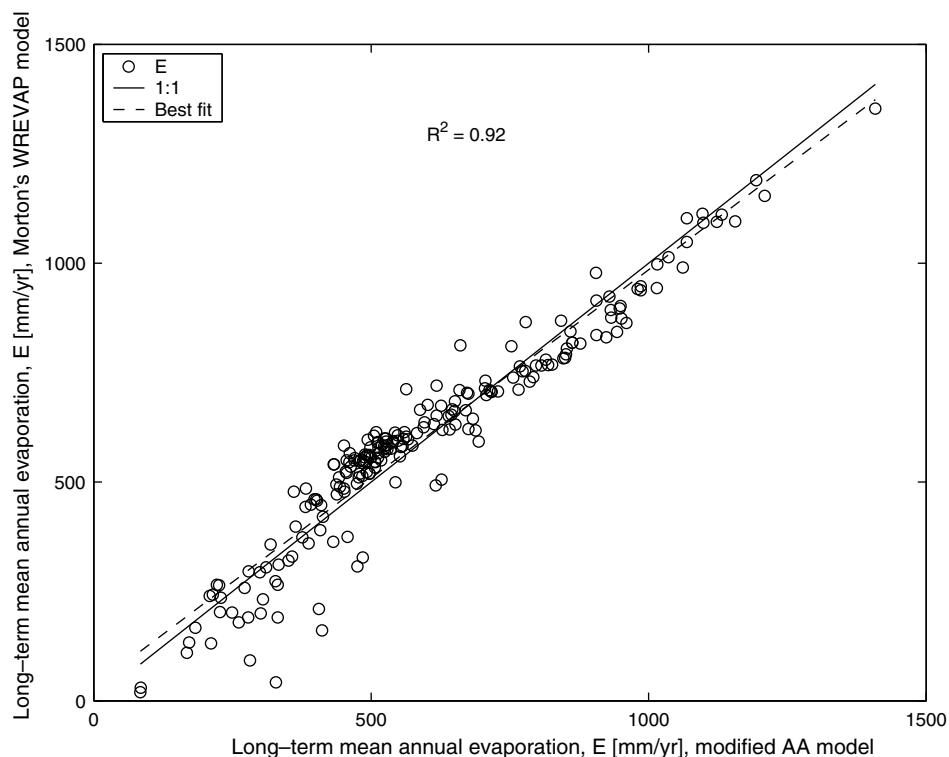


Figure 10 Regression plot of the modified AA and WREVAP model estimates of long-term mean annual evaporation for the 210 SAMSON stations. The modified AA model employs the Penman E_p values with the same wind function as in Fig. 9.

the WREVAP ones in the summer months, and become practically zero in the winter months, while the latter do not. This winter behavior of the modified AA-model estimates

certainly warrants further research, since evaporation can hardly be expected to be zero even in that season, especially not in November typically being a month with plenty

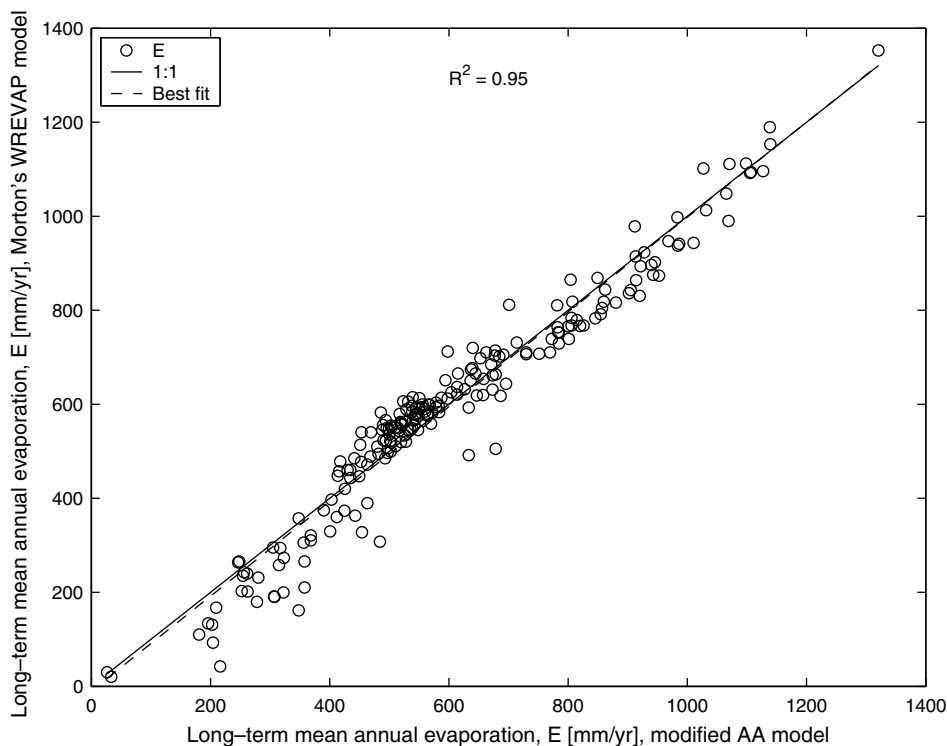


Figure 11 Regression plot of the modified AA and WREVP model estimates of long-term mean annual evaporation for the 210 SAMSON stations. In the modified AA model now the γ/δ term is replaced by a constant (0.55) and the wind function is the same as in Fig. 9.

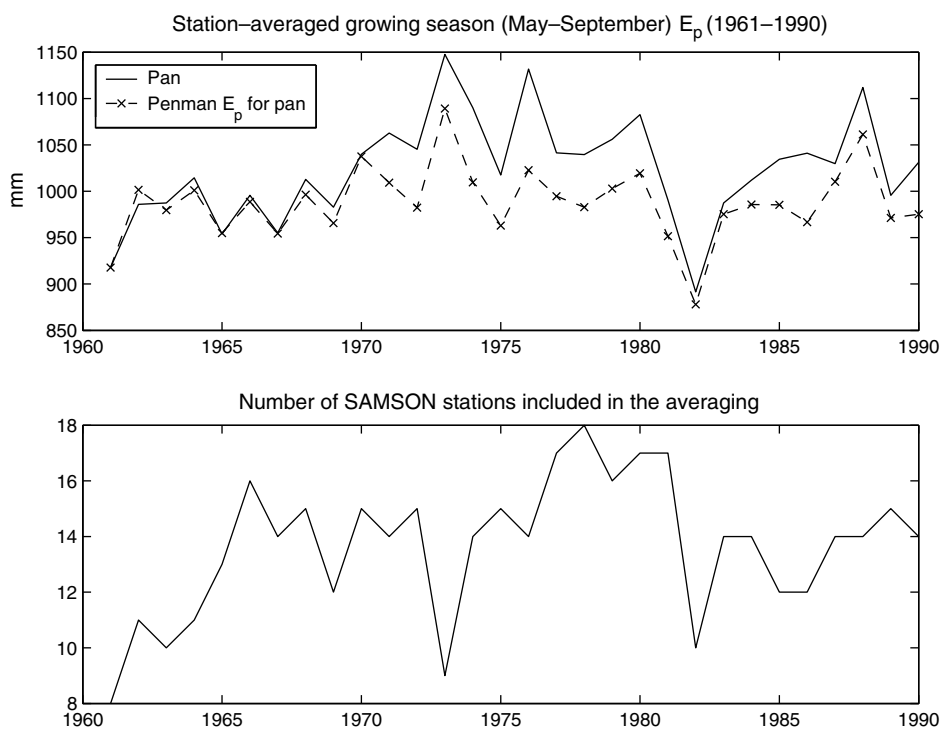


Figure 12 Station-averaged growing-season class-A pan measurements and potential evaporation estimates of the Penman equation by year. The wind function in the Penman equation is the same as in Fig. 9. The number of stations fluctuates due to availability of data.

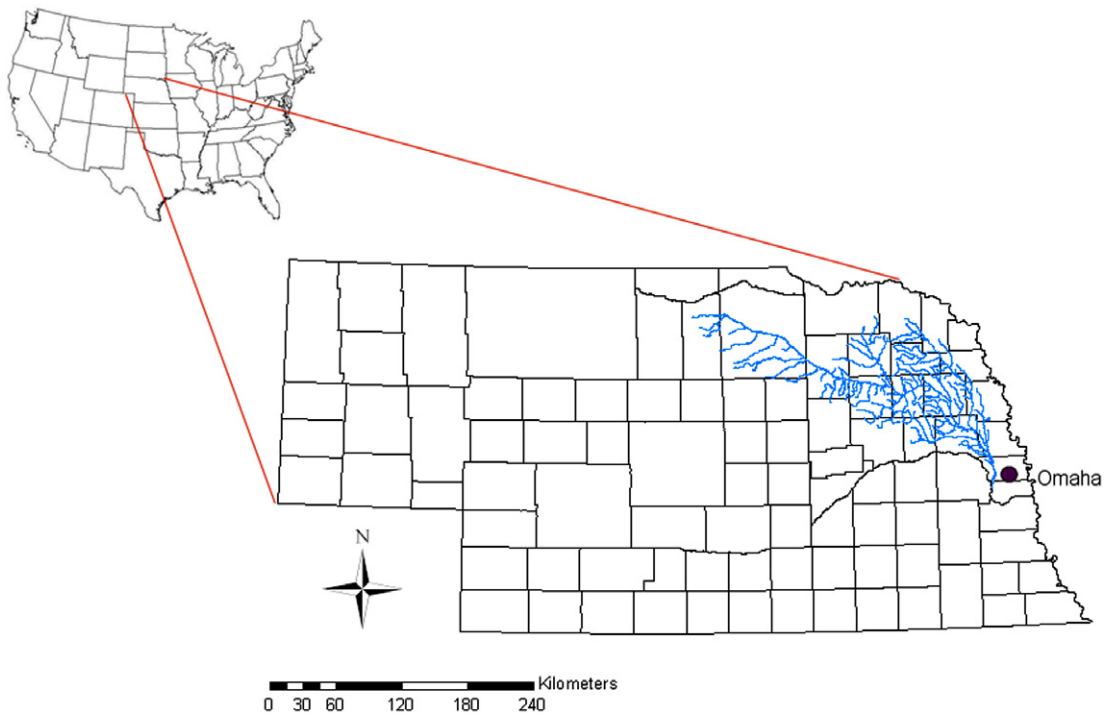


Figure 13 Location of the Elkhorn watershed in Nebraska.

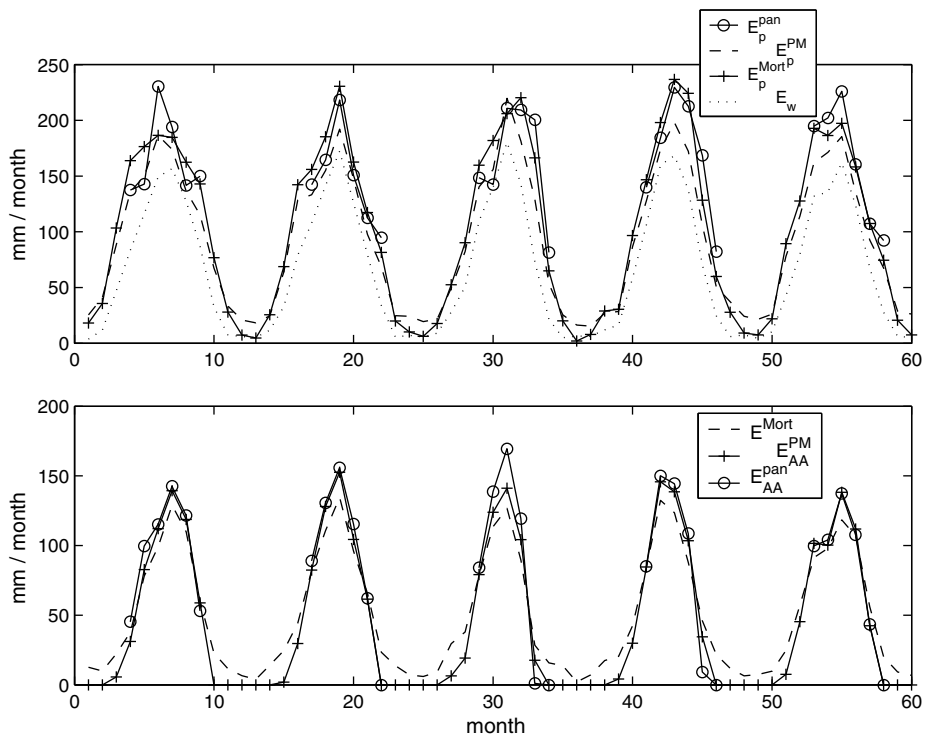


Figure 14 Different evaporation terms by month for 1981–1985, Omaha, Nebraska. E_w : wet environment evaporation by the Priestley–Taylor equation evaluated at T_e ; E_p^{Mort} : E_p by the WREVAP model; E_p^{PM} : E_p by the Penman equation employing the Rome wind function; E_p^{pan} : measured class-A pan evaporation; E_{AA}^{pan} : evaporation estimate by the modified AA model with pan data; E_{AA}^{PM} : evaporation estimate by the modified AA model with the Penman equation values; E_p^{Mort} : evaporation estimate of Morton’s WREVAP model.

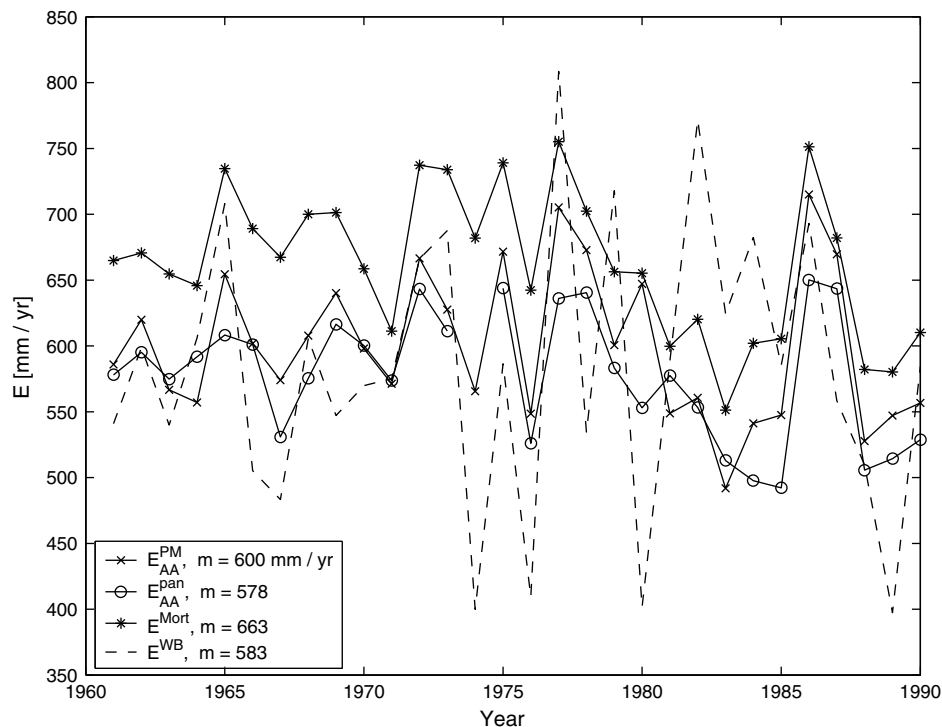


Figure 15 Annual evaporation estimates for the Elkhorn watershed in Nebraska. E_{AA}^{pan} : evaporation estimate by the modified AA model with pan data; E_{AA}^{PM} : evaporation estimate by the modified AA model with the Penman equation values; E^{Mort} : evaporation estimate of Morton's WREVAP model; E^{WB} : water-balance-derived evaporation; m : 30-yr mean value.

of sunshine and mild temperatures (although dry) in Nebraska, yet characterized by practically zero evaporation rates, according to the AA model. Characteristic also is that the two versions of the modified AA model give largely similar E estimates even though the one employing the pan values relies on (10), i.e., c being unity and δ evaluated at T_e , and not on (7) with c and T having been calibrated.

Fig. 15 displays the annually aggregated E estimates. While the water-balance-derived E values fluctuate more (possibly because inter-annual soil moisture change is not zero) than any of the model estimates, the long-term mean annual E of 583 mm is well preserved by the modified AA model, and overestimated by the WREVAP model for this station. The two versions of the modified AA model yield similar values on an annual basis yet again.

Summary and conclusions

Two popular evaporation estimation models that employ the complementary relationship were discussed. It was found that the WREVAP and the modified AA models produce practically identical long-term mean annual evaporation estimates as long as the same inputs are used. The proposed modification of the AA model involves a novel estimation of the equilibrium surface temperature, T_e , (entirely different from Morton's (1983) derivation) to be used in the Priestley–Taylor equation for obtaining the wet environment evaporation, E_w , as well as in the γ/δ term, the latter when the modified AA model is run with apparent potential evaporation values, such as class-A pan evaporation measurements or a good proxy of them.

While the WREVAP model may have been thought to be built on a symmetric CR, it is not, as Figs. 7 and 8 demonstrate, experimentally confirming the claim of Granger (1989). Apparent symmetry between potential and actual evaporation rates in the WREVAP model is achieved by a linear transformation of the Priestley–Taylor wet environment evaporation rates, E_w . The application of E_w as the reference level of evaporation is supported by the observation that this is the rate both E_p (independent whether it is true or an apparent potential evaporation rate) and E converge to in energy limited (i.e., wet) conditions.

The AA model is more flexible in its structure than the WREVAP model, because the former can work with any type of E_p , be it apparent or true potential evaporation value, as well as their estimates, while the WREVAP model can work (due to its fixed optimized parameters and its complex FORTRAN code) only with its own estimate of apparent potential evaporation, approximating class-A pan evaporation data. While the modified AA model requires wind measurements in the Penman equation, the latter being employed for calculating T_e even when the E estimation is based on apparent potential evaporation rates, it may be of certain advantage if one is interested in factoring out the relative contributions of the different variables that may have led to the observed overall declining trends in long-term pan evaporation rates (e.g., Brutsaert and Parlange, 1998; Roderick and Farquhar, 2002; Roderick et al., 2007) and their effect on actual evaporation around the globe.

When the modified AA model is applied with true E_p rates estimated by the Rome wind function in the Penman equation, the CR becomes symmetric: $E = 2E_w(T_e) - E_p$. It then

contains only one free parameter, the Priestley–Taylor coefficient, α , which in the present study was calibrated to be 1.31 when working with daily values to obtain E_w and E_p on a daily basis, but applying the CR on a monthly basis to obtain monthly E values. When the CR was invoked on a daily basis in the modified AA model, the optimized value of the Priestley–Taylor coefficient changed slightly, i.e., to $\alpha = 1.29$.

When employing the modified AA model with apparent potential evaporation values, an additional term, g , has to be considered in the, thus, asymmetric CR, i.e., $E = E_w(T_e) + g[E_w(T_e) - E_p]$. Szilagyi (2007) derived g to be c_7/δ that may work well with class-A pan evaporation measurements as was demonstrated here employing the pan evaporation estimates of Morton (Fig. 7). Otherwise g can be taken as a simple constant, c_0 , as has been suggested by Kahler and Brutsaert (2006). In either case, the value of c or c_0 must probably be calibrated for the specific location and type of apparent potential evaporation available, and they may further express seasonal dependence. The δ term may be evaluated at the equilibrium surface temperature, T_e , as was performed in this study, or at an intermediate temperature between actual air temperature, T_a , and T_e .

With the optimization of the wind function in the Penman equation, it is possible to estimate apparent potential evaporation rates such as represented by pan evaporation measurements or by measured fluxes of a vegetated surface with unimpeded access to soil moisture. However, there is no practical gain in doing so in terms of evaporation estimations, since the modified AA model with true E_p estimates based on the Rome wind function (or with pan measurements instead) can always be employed without the need of extra calibration of the wind function's parameters.

Finally a word of caution for potential users of the AA or WREVAP models, yet unfamiliar with the CR concept. The largest evaporation rate the CR can predict is the wet environment evaporation rate, specified by the Priestley–Taylor equation (2). It predicts the evaporation rate of a large, more-or-less homogeneous land area typically under water-limited conditions. Therefore, it can not be used directly to predict, for example, the evaporation rate of an irrigated plot even when the actual evaporation rate is below the potential level (but still over the wet environment evaporation rate). One may ask then how it is possible that the WREVAP model has a lake evaporation module, lake evaporation rate, depending on lake-area, being characteristically larger than the wet environment evaporation rate. It does so because lake evaporation, especially that of shallow lakes and on a monthly basis, is typically banded by apparent potential (from above) and wet environment evaporation (from below) rates, so by a proper weighting of these limits – in accordance with the size of the lake – shallow-lake evaporation can be estimated. In the WREVAP model lake area is implicitly assumed to be large enough so that this dependence on areal extent is negligible, thus, lake evaporation is uniformly performed by the application of the linearly transformed wet environment evaporation estimates, E_w^* , of the Priestley–Taylor equation. For deeper lakes an additional heat storage term is introduced. Working out a similar relationship within the AA framework is yet to be done.

Acknowledgments

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