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Test of an extended mathematical approach to calculate maize leaf area index and leaf angle distribution

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Abstract

Accurate estimates of leaf area index (LAI) and leaf angle distribution (LAD) are important for estimating the exchange of energy and gases in vegetative canopies. Direct estimates of LAI and LAD are laborious, time consuming and often times destructive. Stewart and Dwyer [Agric. For. Meteorol. 66 (1993) 247] introduced a mathematical method to calculate LAI and LAD of maize canopies based on descriptions of leaf width and leaf curvature. However, the method is limited in its application. The objectives of this research are to: (1) extend the method to account for leaves with maximum curvature height at the leaf tip (i.e., straight leaves, which commonly occurs with leaves at the top of maize canopies), so that the method provides a general application to maize canopies, and (2) test the performance of the extended model. LAI and LAD simulated using the mathematical method were tested against observed LAI and LAD values. Average leaf width coefficients, determined from all leaf positions, were used in the leaf area calculation. Leaf area simulations agreed overall within 2% (51 cm^2) of observed leaf area values. Estimated LAD agreed within 5% of the field observed LAD. The method offers a straightforward, reliable means of estimating LAI and LAD, when the appropriate coefficients are known, without requirements for special instrumentation or other conditions. The method also offers a means of checking the performance or reliability of other methods of characterizing canopy structure.

Keywords: Maize; Leaf curvature; Leaf area index; Leaf angle distribution

1. Introduction

The structure of vegetative canopies is central to plant–environment interactions such as light interception and scattering and the rate of mass and energy exchanges in plant canopies. The determination of leaf area index (LAI) and leaf angle distribution (LAD) is laborious and time consuming, requiring repeated characterization as the canopy develops (Daughtry, 1990). A large amount of effort can be devoted to characterizing these canopy characteristics, especially when characterizing them as functions of canopy height. Efforts to simplify the measurement has resulted in a variety of means, both direct (Daughtry, 1990) and indirect (Welles, 1990), including the use of canopy gap inversion (e.g., Welles and Norman, 1991). Stewart and Dwyer (1993) introduced a mathematical method to define LAI and LAD of maize canopies based on descriptions of leaf width and leaf curvature (a curve in the Cartesian coordinate

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system that describes the leaf position along its length). The method provides a means of obtaining LAI and LAD once certain input parameters for the particular canopy are measured. The method is non-destructive to the canopy, so that the same plant can be measured repeatedly. However, application of their method is limited to descriptions of leaf curvature where the leaf maximum height does not occur at the leaf tip; often newly emerged maize leaves at the top of the canopy can be straight, i.e., the maximum leaf height coincides with leaf tip. Therefore, the method was extended to include a solution for straight leaves.

2. Theory

In the mathematical expressions of Stewart and Dwyer (1993), leaf curvature and leaf width are solved for using five measured leaf parameters: (1) initial value of leaf width (at the ligule), W_0 (cm); (2) leaf length, L (cm); (3) coordinates of leaf maximum height (x_M, y_M) in cm from some predetermined reference height (e.g., at the ligule so that x_0 and y_0 are set to 0); (4) leaf tip coordinates (x_L, y_L) in cm; and (5) the initial value of leaf angle (at the ligule), θ_0.

Leaf curvature is derived from a quadratic equation where the ligule is defined at (0, 0):

$$Ax^2 + By^2 + Cxy + Dx + y = 0 \tag{1}$$

based upon horizontal and vertical leaf coordinates (x, y).

Coefficients A, B, C and D are defined through relations of the coordinates (x_M, y_M) and (x_L, y_L), and θ_0:

$$A = B\frac{y_M^2}{x_M^2} + \frac{y_M}{x_M^2}, \tag{2}$$

$$B = \frac{-y_M(x_L^2/x_M^2) - x_L y_L(\tan\theta_0/y_M) + 2x_L(y_L/x_M) + \tan\theta_0 x_L - y_L}{y_M^2(x_L^2/x_M^2) + y_L^2 - 2y_M x_L(y_L/x_M)}, \tag{3}$$

$$C = \frac{\tan\theta_0 - 2Ax_M}{y_M}, \tag{4}$$

$$D = -\tan\theta_0. \tag{5}$$

Substituting Eqs. (2)–(5) into Eq. (1) and solving for y, an equation is derived from which leaf curvature can be described. At any horizontal distance along the leaf (x_i) (Fig. 1) the corresponding vertical distance

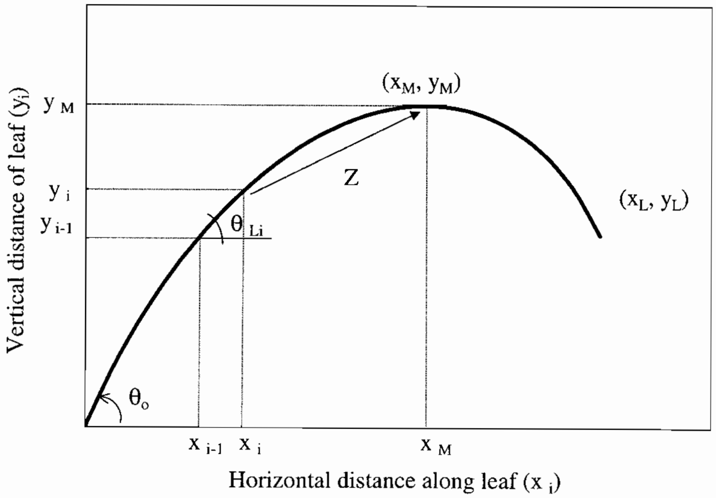

Fig. 1. Leaf curvature defined by the horizontal and vertical distances along the leaf (x_i and y_i) using Eq. (6) and input parameters: initial value of leaf angle (θ_0); leaf length, L (cm) (i.e., the length along z); coordinates of leaf maximum height (x_M, y_M) in cm from the ligule (x_0, y_0); and leaf tip coordinates (x_L, y_L). Once leaf curvature has been described, the solution for the average leaf angle θ_{L_i} along a curved leaf between x_{i-1} and x_i can be found.

y_i is

$$y_i = \frac{-(Cx_i + 1) + [(Cx_i + 1)^2 - 4Bx_i(Ax_i - \tan\theta_0)]^{0.5}}{2B}. \quad (6)$$

Once leaf curvature has been described the solution for the average leaf angle, θ_{L_i}, along a curved leaf between x_{i-1} and x_i can be found

$$\theta_{L_i} = \arctan\frac{y_{i-1} - y_i}{x_{i-1} - x_i}. \quad (7)$$

Leaf width at (x_i, y_i) along the leaf length, W_i, is described using a third order polynomial with defined shape coefficients, b_1 and b_2 as

$$W_i = W_0 + b_1 z_i + b_2 z_i^2 - (W_0 + b_1 L + b_2 L^2)\frac{z_i^3}{L_3}, \quad (8)$$

where W_0 is the initial leaf width, z_i the actual distance along the leaf from the ligule (Fig. 1) and L the leaf length. Coefficients b_1 and b_2 are determined by a non-linear least-squares regression of Eq. (8) using an independent data set of previously measured leaf width per leaf length characterization. Leaf area for a particular segment (S_i) in cm^2 is estimated by integrating the leaf width over the desired leaf segment (i.e., $S = \int W_i\,dz$). The LAI and LAD are estimated through solution of the leaf area and leaf angle, θ_{L_i}, of each leaf segment, S_i.

3. Revised method

The method was extended to account for straight leaves (i.e., when $x_M = x_L$ and $y_M = y_L$), where the coordinates of the tip and maximum height coincide. Under these conditions the coefficient B in Eq. (1) is nonexistent since the denominator in Eq. (3) reduces to zero. The leaf curvature (Eq. (1)) for straight leaves is then defined as

$$Ax^2 + Cxy + Dx + y = 0. \quad (9)$$

The coefficient D is unchanged (Eq. (5)) and coefficients A and C are recalculated as

$$A = \frac{y_L}{x_L^2} \quad (10)$$

$$C = \frac{\tan\theta_0 - (2y_L/x_L)}{y_L}. \quad (11)$$

Thus, straight leaves can be described given any horizontal distance along the leaf (x_i) with the vertical distance of the leaf (y_i) described as

$$y_i = \frac{-Ax_i^2 + x_i\tan\theta_0}{Cx_i + 1}. \quad (12)$$

To obtain smoother leaf curves than originally defined by Stewart and Dwyer (1993), leaf segments were defined based upon an increment equal to a 2 cm length along the leaf length (i.e., change in z of 2 cm) rather than the 10 cm segments along the x-axis. The leaf segment area (S_i) is estimated using the integral of the leaf width (Eq. (8)) from the initial (z_0) to the final leaf length (L) of the segment as follows:

$$S_i = \int_{z_{i-1}}^{z_i} W_i\,dz = W_0(z_i - z_{i-1}) + \frac{b_1(z_i^2 - z_{i-1}^2)}{2} + \frac{b_2(z_i^3 - z_{i-1}^3)}{3} - \frac{(W_0 + b_1 L + b_2 L^2)(z_i^4 - z_{i-1}^4)}{4L^3}. \quad (13)$$

LAI is estimated by adding all estimated leaf segment areas (S_i) for all leaves within a defined ground area for a particular plant, plot or observation date and dividing that sum by the defined ground area:

$$\mathrm{LAI} = \frac{\sum_{i=1}^{n} S_i}{\text{ground area}}. \quad (14)$$

Since the area of each leaf segment (S_i) has leaf angle θ_{L_i}, the leaf area can be associated within a particular leaf angle class [θ_{LAC}, a range of leaf normal angle values within the 0 to $\pi/2$ space (0–90°)]. The total leaf area associated with a particular leaf angle class is estimated as the sum of all leaf segment areas in that particular leaf angle class. The fraction of leaf area for each leaf angle class [$\mathrm{fLA}(\theta_{LAC})$] for a particular observation date is estimated as a normalized leaf area by dividing the total estimated leaf segment areas for each leaf angle class by the total estimated leaf area as

$$\mathrm{fLA}(\theta_{LAC}) = \frac{\sum_{j=1}^{n} S_i}{\sum_{i=1}^{n} S_i} \quad \text{for all } S_i \text{ of leaf angle class } \theta_{LAC}, \quad (15)$$

where θ_{LAC} indicates the leaf angle class defined within the 0 to $\pi/2$ interval and i indicates the leaf segment counter of n total leaf segments for the particular plant, plot or observation date. The LAD is estimated simply as the distribution of leaf area fractions [fLA(θ_{LAC})] for all defined leaf angle classes.

4. Materials and methods

Maize (*Zea mays* L.) was used in field experiments conducted during the summer of 1995 at the Kansas State University (KSU) Evapotranspiration Laboratory Research Site, 15 km south of Manhattan, KS (latitude 39°12′N, longitude 96°35′W, 385 m above sea level) and in 1996 at the University of Nebraska Agricultural Research and Development Center (ARDC), located near Mead, NE (latitude 41°09′N, longitude 96°30′W, 354 m above sea level). The maize (var. Pioneer 3162) field at the KSU site was planted in May 1995 in east–west row directions with a 0.76 m row spacing at a population of 75,500 plants/ha (Eudora silt loam). The maize (var. Pioneer 3225) at the ARDC was planted in May 1996 in north–south row directions with a 0.79 m row spacing at a population of 49,357 plants/ha (Sharpsburg silt clay loam).

The input parameters for the mathematical approach [initial slope (θ_0), initial width (W_0), leaf length (L) and coordinates of leaf maximum height (x_M, y_M) and leaf tip (x_L, y_L)] were measured on green leaves of two adjacent maize plants from the field on day of year (DOY) 187 and 193 (1995) from which leaf curvature, leaf area, LAI and LAD were simulated. Two plants from each measurement day constituted a plot in the analysis (i.e., two plots in 1995). The green leaves were assigned a leaf number at the time of measurement in relation to the position from the top of the maize plant among all leaves on the plant. A device to measure the input parameters was constructed from a compass, protractor, ruler and inclinometer (Rosenberg et al., 1983).

As a test of the performance of the approach, leaf area and LAD were determined independently but on the same leaves from which the input parameters were determined. Fifty leaves in total were measured in the two plots in 1995. In addition to the model input parameters, leaf segment angle and leaf width were measured at 15 cm intervals (in 1995) along the full leaf length for the same 50 leaves; leaf curvature and leaf area were determined from these additional measurements and considered as "observed" or "true" values to be compared with simulated values. The observed leaf area was determined for each of the 50 leaves as the area of the trapezoid defined by the measured leaf widths W_i and W_{i+1} along the leaf at each leaf segment interval. Observed LAI and LAD were determined on a plot basis.

In 1996 at the ARDC, model input values were measured on all leaves from nine plants, on average, selected within a 1.6 m × 1 m area of the maize field for each of three measurement periods. The plants from each measurement period constituted a plot in the analysis. The observed areas of individual leaves of all plots from the 1996 study were determined using a Model 3100 Area Meter (LI-COR, Lincoln, NE). [1]

Observed values of leaf area and LAI components from 1995 and 1996 were compared to simulated values as a means of evaluating the performance of the mathematical method. Leaf area and LAI were estimated using the integral of Eqs. (8), (13) and (14); LAD was estimated using Eq. (15). The comparisons between observed and simulated leaf areas were performed for leaves from maize plants of the five plots from the two field studies for a total of 352 leaves. As an additional means of comparison with the mathematical method, LAI was estimated with an LAI-2000 Plant Canopy Analyzer (LI-COR, Lincoln, NE) within the same five plots (from the 1995 and 1996 studies) from which the LAI was observed and simulated. The mean tilt angle (MTA) and the *G*-function [i.e., fraction of the leaf area projected in the direction of the beam from a particular zenith angle (Ross, 1981)] for the observed, simulated and LAI-2000 LADs were calculated for every leaf inclination angle from 0 to 90° as an additional way of evaluating the LAD.

Leaves from three plants (37 leaves) were measured on DOY 207 (1996) at the ARDC as an independent set of leaf width per leaf length measurements (at 10 cm interval) from which the two shape coefficients (b_1 and b_2) were derived. The shape coefficients were

[1] The use of company names and brand names is necessary to report factually on available data; however, the University of Nebraska and UFV neither guarantee nor warrant the standard of the product, and the use of the name by the University of Nebraska and UFV implies no approval of the product to the exclusion of others that also may be suitable.

determined using a nonlinear least-squares regression procedure (Marquadt, 1963) from which all simulated values were computed. Eq. (6) was used to define leaf coordinates (at a 2 cm interval) for leaves where the leaf tip was lower than the maximum height of the leaf and Eq. (12) was used to define leaf coordinates (at a 2 cm interval) for straight leaves. Simulated leaf area was calculated as the sum of the leaf segment areas of the leaf (i.e., sum of all i segments in Eq. (13)). LAI and LAD of each plot (a total of five plots) were simulated using Eq. (14) and the distribution of leaf area fractions from Eq. (15), respectively.

Observed and simulated leaf area, leaf curvature, LAI and LAD were compared using the determination coefficient (r^2) of the regression line, the mean relative error (MRE), the mean bias error (MBE) and the root mean square error (RMSE):

$$\mathrm{MBE} = \frac{\sum_{i=1}^{n}(S_i - O_i)}{n},$$

$$\mathrm{MRE} = \frac{\sum_{i=1}^{n}100(S_i - O_i)}{O_i},$$

$$\mathrm{RMSE} = \frac{[\sum_{i=1}^{n}(S_i - O_i)^2]^{0.5}}{n},$$

S_i and O_i are the simulated and observed values for each leaf or plot i.

5. Results and discussion

Shape coefficients b_1 varied in value from −0.10 to 0.38 in 1995 and from −0.10 to 0.33 in 1996. Coefficient b_2 varied from −0.0060 to −0.0011 in 1995 and from −0.0092 to 0.0122 in 1996. Coefficients for both years showed a somewhat systematic variation with ligule height (Fig. 2); for simplification purposes, the average values for these coefficients ($b_1 = 0.08$ and $b_2 = 0.0001$) from all leaves observed from DOY 207 (1996) were used as independent coefficients for leaf area, LAI and LAD computations. Leaf curvature was simulated for 44 of the 50 leaves measured in 1995 (i.e., those meeting model specifications where both x_M and $y_M \geq 0$); six leaves measured were broken or damaged such that the leaf coordinates did not match that of a normal maize leaf. The revised mathematical method includes using the originally published equations along with the newly derived

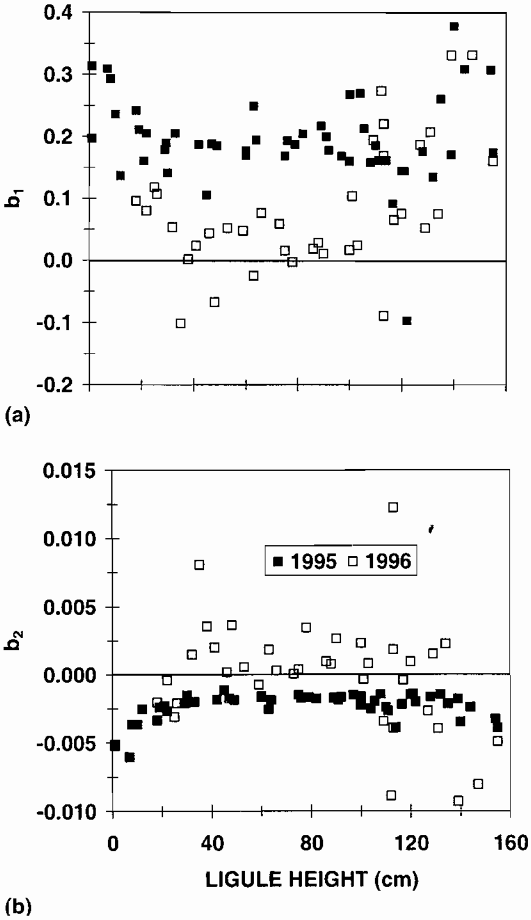

Fig. 2. Leaf shape factors versus ligule height as determined using 87 leaves from a total of seven plants. DOY 187 and 193 (1995) and DOY 207 (1996). (a) Shape factor b_1; (b) shape factor b_2.

equations where appropriate (eight of the 50 observed leaves were straight, i.e., $x_M = x_L$ and $y_M = y_L$, and were found near the top of the canopy). The accuracy of the revised mathematical method is indicated by the agreement between observed leaf curvature and simulated curvature from data collected in the 1995 study (Fig. 3). These results demonstrate that leaf profiles were represented within 0.7 cm (MBE) and −6.0% MRE (0.3 cm RMSE).

Typically, observed and simulated leaf areas increased from the lowest leaf to a peak value for mid-height leaves then decreased from mid-height leaves to leaves located at the top of the canopy (Fig. 4). Departures between observed and simulated leaf area

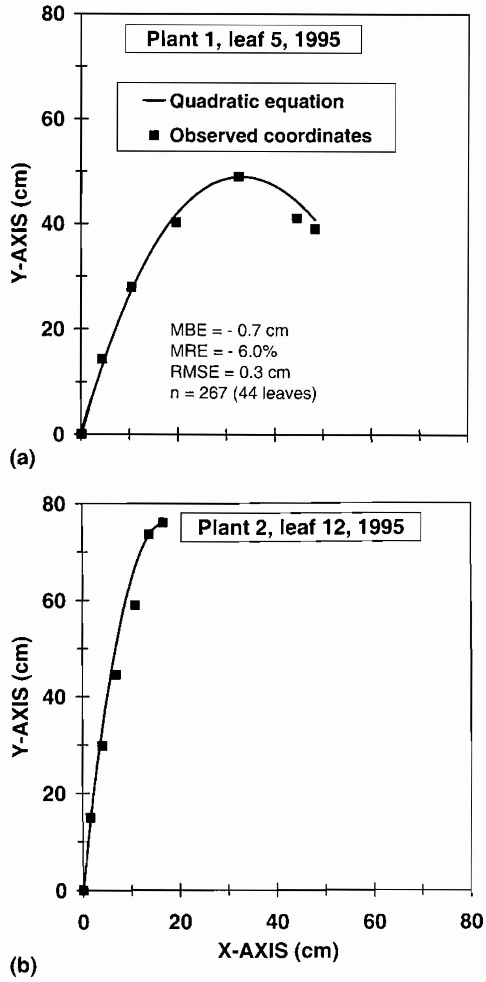

Fig. 3. A comparison of observed maize leaf vertical and horizontal coordinates with those simulated using the quadratic equation with field-measured values of initial leaf position, maximum leaf height and average leaf segment angles. (a) Leaf tip height less than the maximum leaf height (Eq. (6)). (b) Leaf tip height equal to maximum leaf height (Eq. (12)). MBE, MRE and RMSE were calculated (overall for 44 leaves) using y-axis length measurements at the simulated and observed (x, y) coordinates, excluding position (0, 0).

were the result of departures from the prescribed relationship with initial leaf width. For example, leaf number 6 in Fig. 4a was damaged along the leaf beyond the ligule resulting in a narrower leaf than is described in the mathematical approach given the actual leaf width at the ligule, thus resulting in an overestimate of leaf area for this particular leaf. Overall, observed and simulated leaf area agreed with a mean

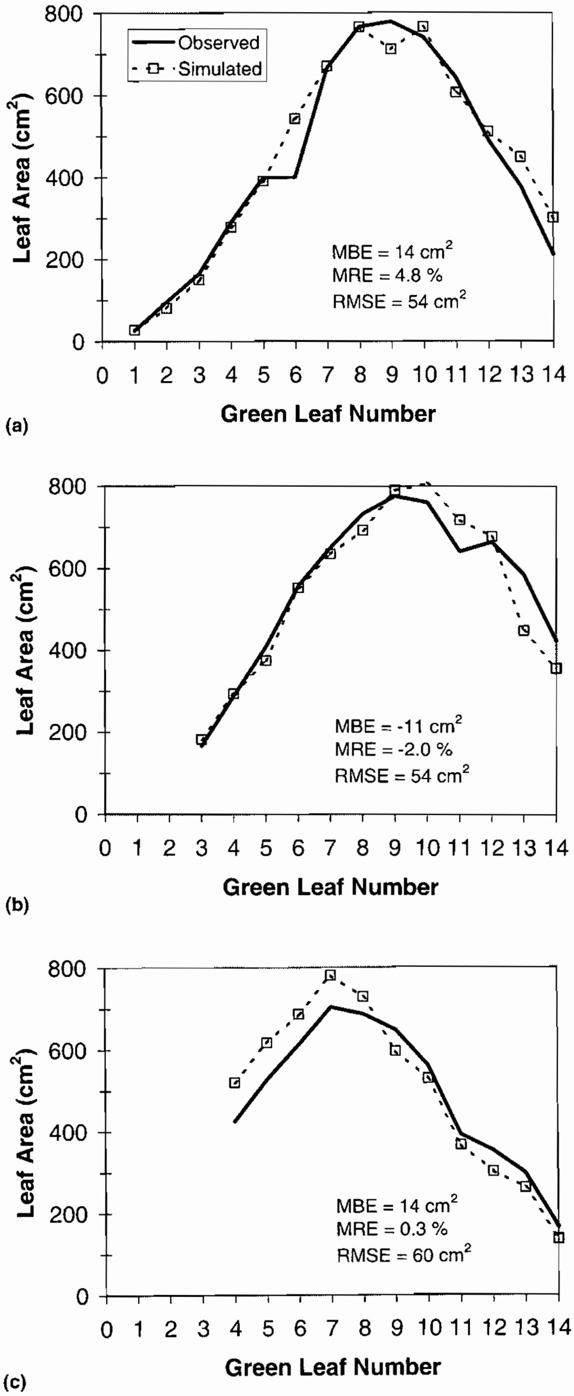

Fig. 4. Observed and simulated leaf area (in cm^2) versus leaf position from the bottom of the plant to the top of the plant (green leaf number = 14) for a particular plant on: (a) DOY 187; (b) DOY 193; (c) DOY 234.

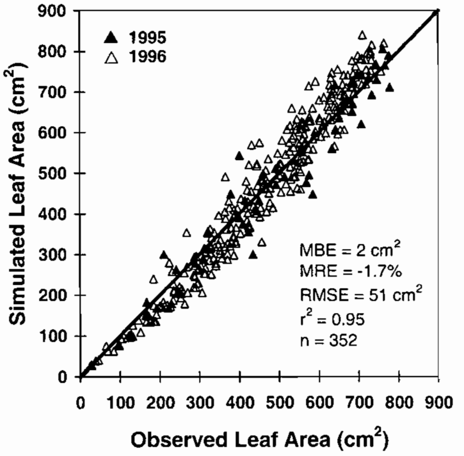

Fig. 5. Observed leaf area (in cm^2) (as determined using the measured leaf width in 1995 and the LI-COR 3000 leaf area meter in 1996) compared to simulated leaf area (in cm^2) (as determined using the modified method) for 352 leaves observed on DOY 187 and 193 (1995) and DOY 234 and 235 (1996).

bias of $2\,cm^2$ MBE (RMSE of $51\,cm^2$ and r^2 of 0.95) with a slight tendency to underestimate leaf area for small leaves (errors as large as $-100.8\,cm^2$) and to overestimate leaf area for large leaves (errors as large as $143.1\,cm^2$) (Fig. 5). LAI simulations resulted in an overall small underestimate of LAI values (Table 1) (−0.1 MBE, 0.1% MRE and 0.1 RMSE), despite the overall overestimate in leaf areas (attributed to the fact that the LAI underestimate from the two plots of 1995 overshadowed the LAI overestimate from the three plots of 1996). The LAI relative errors, on a plot basis, varied from −3.3 to 3.5%. LAI estimates using the LAI-2000 were within the 15% acceptance range (Welles and Norman, 1991) of observed LAI values (−0.5 MBE, −11.5% MRE and 0.8 RMSE) despite a slope of 0.4 and high intercept, the result of underestimating values in 1995 along with a limited number of data points. The revised method yielded estimates of 0.8–5% MRE (0.02 RMSE) of observed values on a plot basis; results closely approximated a spherical LAD (Fig. 6). The *G*-functions from observed and

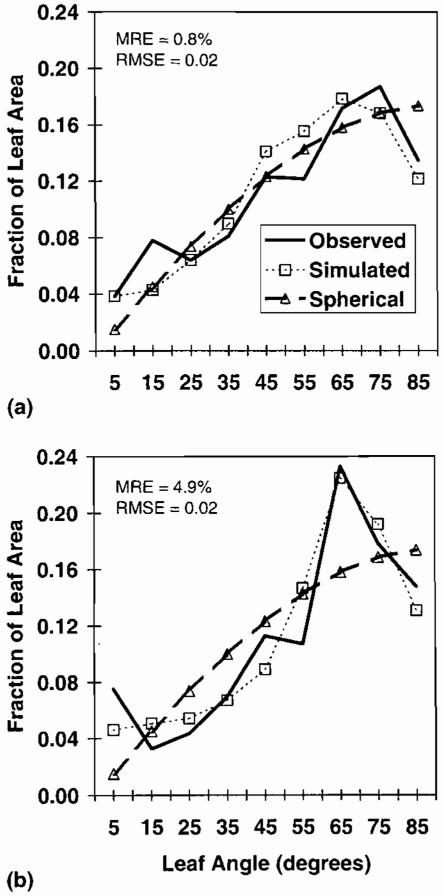

Fig. 6. Comparison between observed LAD, simulated LAD (as determined using the revised method for leaves from each plot) and the theoretical spherical LAD: (a) DOY 187; (b) DOY 193 (1995).

Table 1
Results of statistical comparisons between: (1) (true) observed (from detailed leaf segment measurements in 1995 and leaf area meter measurements in 1996) and simulated leaf areas (from Eq. (13)), and (2) observed (from detailed leaf segment measurements in 1995 and leaf area meter measurements in 1996) and simulated LAI_S by the model (Eq. (14)) and LAI estimated by the LAI-2000

	Observed values compared to simulated and/or estimated	n	MBE	MRE (%)	RMSE	Intercept	Slope	r^2
(1)	Leaf area (Eq. (13))	352	$2\,cm^2$	−1.7	$51\,cm^2$	−43.8	1.1	0.95
(2)	LAI_S (Eq. (14))	5	−0.1	0.1	0.1	0.2	0.9	0.99
	LAI (LAI-2000)	5	−0.5	−11.5	0.8	1.6	0.4	0.98

Table 2
MTA and *G*-function (Ross, 1981) statistics (minimum, maximum, average and standard deviation (S.D.)) calculated from 0 to 90° solar zenith angles at intervals of 1° (LAD assumed to be uniform in azimuth)

	DOY 187			DOY 193			Theoretical spherical LAD
	Observed	Simulated	LAI-2000	Observed	Simulated	LAI-2000	
MTA (°)	55	55	46	56	57	55	57
G-function							
Minimum	0.48	0.49	0.40	0.49	0.49	0.44	0.50
Maximum	0.53	0.53	0.67	0.51	0.50	0.55	0.50
Average	0.51	0.51	0.54	0.50	0.50	0.51	0.50
S.D.	0.01	0.01	0.09	0.01	0.01	0.04	0.01

simulated LADs confirmed this by approaching 0.5, the value of the *G*-function for a spherical canopy in any direction (Table 2). The average *G*-function for the LAD estimated with the LAI-2000 approached 0.5 although the range of values was larger than in the case of observed and simulated values. Having the spherical LAD allows the maize canopy to have half of its leaf area projected towards the sun regardless of sun position. The straight leaves on the top of the canopy gives additional advantage as a large amount of radiation reaches the middle of the canopy where most of the leaf area is located.

6. Conclusions

A method to calculate leaf curvature, leaf area, LAI, and LAD from five measured maize leaf parameters was extended and tested against observed values. Observed and simulated leaf areas agreed on average within 2% (RMSE of 51 cm^2). Observed and simulated LAD were on average within 5% of the field observed LAD. The method is straightforward (five input parameters for each leaf), compares favorably to estimates using canopy gap inversion, and can be used for accurate estimates of LAI and LAD. A single set of shape coefficients were used for the two commercial varieties used in this work, however, a different set may be required for leaves with other shapes. Observed and simulated LADs, as well as that from the LAI-2000, yielded average *G*-function (Ross, 1981) values of approximately 0.5 from 0 to $\pi/2$, confirming a spherical LAD for the canopy.

Although testing of the model was laborious, the method used in an operational mode is straightforward, once the shape factors are determined. The method has applications for radiative transfer and crop growth models, does not rely on sky conditions nor elaborate equipment needs, and offers the flexibility in selecting the leaf angle classes of choice. In addition, the revised method can be used as a tool to evaluate and/or improve the use of other methods which determine LAI and LAD and for any application that requires accurate field determination of LAI and LAD. The technique holds potential for other grass crops with leaves that have a defined curvature, such as sorghum and sugar cane, as long as the leaf shape coefficients are known.

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