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Reply to “Comment on two Papers About the Generalized Complementary Evaporation Relationships by Crago et al.”

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REPLY
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This article is a companion to Liu et al. (2020), https://doi.org/10.1029/2019WR026292.

Key Points:
- Four concerns raised by the comment are addressed, along with misconceptions of the role of x_{min}
- In the rescaled Complementary Relationship, the minimum value of x is computed for each data point
- The computed minimum value of x is a firm lower limit

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Reply to "Comment on 'Two Papers About the Generalized Complementary Evaporation Relationships by Crago et al.'"

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Abstract Liu et al. (2020, https://doi.org/10.1029/2019WR026292) provide comments on two papers introducing the rescaled Complementary Relationship (CR) of evaporation. Four concerns regarding the rescaled CR were raised and are addressed herein. Specifically, it is shown here that (1) the rescaled CR is not simply an asymmetric advection-aridity model; (2) a discontinuity in the slope of the rescaled CR at the point where evaporation is zero is not problematic because that point represents a true minimum value; (3) a particular formulation of the rescaled CR presented in an earlier paper was not recommended by the present authors, but was simply intended to illustrate that a range of boundary conditions can be accommodated; and (4) while behavior of the model advocated by Liu et al. 2020) can be modified to match the lower limit introduced by the rescaled CR, there is little theoretical justification or practical utility for doing so. Inherent in several of the concerns seems to be a misconception held by the authors of the Comment regarding the nature of the lower limit in the rescaled CR. Once this misconception is addressed, none of the concerns represent a weakness in the rescaled CR concept.

1. Introduction

The present authors welcome a discussion with Liu et al. (2020, hereinafter "L19") regarding the rescaled Complementary Relationship (CR) for evaporation and transpiration introduced by Crago et al. (2016; hereinafter "CSQH16") and further developed by Szilagyi et al. (2016), Crago & Qualls (2018; hereinafter "CQ18"), and Ma and Szilagyi (2019). Hopefully, this dialog will bring clarity to the issues involved.

L19 outlined the Generalized Nonlinear Advection-Aridity (GNAA) model developed by Brutsaert (2015; hereinafter "B15"), in which $y = E/E_{pa}$ is found to be a polynomial function of $x = E_{p0}/E_{pa}$, where E_{p0} is the potential evaporation rate given by the Priestley and Taylor (1972) equation, E_{pa} is the apparent potential evaporation rate given by the Penman (1948) equation, and E is the actual regional evaporation rate. The coefficients of the polynomial were determined such that the four boundary conditions introduced by B15 were satisfied, namely: (i) y = 1 at x = 1, (ii) y = 0 at x = 0, (iii) dy/dx = 1 at x = 1, and (iv) dy/dx = 0 at x = 0. The present authors agree with L19 that the GNAA developed by B15 is a major development within CR theory, and that it works well with a wide range of data. However, the present authors are convinced that it can be improved by rescaling the x axis.

L19 describes the reasoning behind the rescaling proposed by CSQH16. It starts with the observation that y can hypothetically go to zero if there is no moisture available to evaporate in the region, but under these conditions, x is still greater than zero. The value of x at which y = 0 is denoted $x_{min} = E_{p0}/E_{max}$, where E_{max} is the hypothetical value of E_{pa} in a desiccated region with regional E = 0. This maximum, E_{max}, can be thought of as the hypothetical evaporation from a small wet patch in a region with a desiccated lower atmosphere, or as the value of E_{pa} one gets when the humidity of the air is zero. With this definition, E_{max} cannot physically go to infinity, so x will not go to zero unless E_{p0} is zero (in which case the value of x is completely disconnected from the availability of moisture in the region and the CR should not be applied with that data point). CSQH16 suggested a new "rescaled" CR as $y = X$, where $X = (x - x_{min})/(1 - x_{min})$.

L19 expressed several reasonable and understandable concerns regarding the rescaled CR project. They are directed specifically at the formulation of the rescaled CR presented in CSQH16 and CQ18. However, starting with CSQH16 (see also Szilagyi et al., 2016; CQ18; Ma & Szilagyi, 2019), the present authors have noted that the rescaled CR concept, in which E_{max} (or x_{min}) is a fundamental variable, can accommodate formulae other than y = X. For example, in CSQH16, a third-order polynomial equation that could meet four boundary conditions was presented.

Some of the concerns raised in L19 clearly apply to any of the rescaled CR formulations, but some of them only apply to the formulation y = X of CSQH16 and CQ18. The concerns are addressed below in the order in which they appear in L19. When a concern only applies to y = X, rather than to all rescaled CR formulations, it will be noted in the discussion. The goal of this reply is to clarify and address these concerns.

2. First Concern: $y = X$ Is Simply an Asymmetric Advection-Aridity Model

The claim here seems to be that the rescaled CR looks like, and does not improve upon, the asymmetric Advection-Aridity model. The similarity between y = X and the "traditional asymmetric AA [Advection-Aridity] model" (L19) was pointed out already by CSQH16. That is, for a given value of x_{min}, y = X can be drawn on a graph of x versus y as a straight line from the point (1, 1) down to the point (x_{min},0). This graph looks the same as the traditional AA model, and as CSQH16 and L19 both noted, it can be written in the same form.

However, since x_{min} is found independently for each data point, the points on a graph of X versus y are rearranged from their positions on a graph of x versus y when the rescaled CR is employed. In other words, unlike the traditional AA model or the GNAA model, knowledge of the (x, y) position of a data point does not by itself indicate the (X, y) position, because X depends also on the additional variable, x_{min}. Whereas each of the asymmetric AA and GNAA models provides a single line or curve in (x, y) space, the rescaled CR provides a continuous family of lines. This is a substantive distinction (and improvement), as demonstrated by CQ18 (e.g., see their Figure 7). This concern applies specifically to the y = X formulation.

3. Second Concern: dy/dx Is Discontinuous at $x = x_{min}$

L19 suggest that there are problems with the rescaled CR when $x < x_{min}$ or when x_{min} goes to zero. In principle, these two conditions are impossible, because E_{max} is defined as the largest evaporation rate possible (i.e., with a saturated patch at the surface and with no humidity in the air at measurement height). Thus, E_{max} should, in principle, always be greater than E_{pa}, so that $x > x_{min}$ and $x_{min} > 0$. Mathematically, the original formulation of E_{pa} and E_{max} by CSQH16 could conceivably result in a value of $x < x_{min}$, because E_{pa} used the Penman equation while E_{max} used a mass transfer equation; later versions (e.g., Szilagyi et al., 2016, CQ18) have based both E_{pa} and E_{max} on the same equation. With E_{max} thus correctly formulated, the physically valid range of x is in fact (x_{min}, 1), and x_{min} must be greater than 0 (unless $E_{p0} = 0$, in which case the CR should not be used, as noted in section 1 of this reply).

Since x_{min} is truly the lower limit of x, the continuity of the slope dy/dx for $x < x_{min}$ is irrelevant—that range of x cannot be occupied in the real world. CSQH16 did point out that in a sense B15's boundary condition (iv) was satisfied with y = X because, in a counterfactual world in which values of $x < x_{min}$ were possible, one would plot them as falling directly on the x axis. But since $x < x_{min}$ is not possible, a discontinuity in the slope dy/dx at $x = x_{min}$ is not problematic. This concern brought by L19 does not apply directly to the rescaled formulation of Szilagyi et al. (2016) and Ma and Szilagyi (2019), because of the way they specified the boundary conditions, but all rescaled CR models do rely on x_{min} as the true lower limit of x.

4. Third Concern: A Formula in CQ18 Does Not Fit the Data

L19 noted that CQ18 included an equation that satisfies B15's BCs (i), (ii), and (iii) [and in the sense described above, also (iv)]. The concern is that it gives values of y > X, which does not describe the patterns found in experimental data sets. The present authors agree with this observation regarding fit to the data. However, we are not aware of any physical constraints that require y < X, although clearly there is a constraint that y < x. Note that individual data points frequently do plot above the line y = x, presumably due to experimental uncertainties; for example, several data points in B15's Figure 1 fall above this line. The present authors agree that the equation mentioned is unlikely to be a widely applicable formula. Actually, CQ18 never applied this equation with their data, but merely presented it in the appendix to show that additional boundary conditions can be easily incorporated in the rescaled CR. The concern raised here has no impact on the rescaled CR concept and played no role in the model testing or conclusions of the present authors.

5. Fourth Concern: The Parameter c in the GNAA Can Satisfy $y = 0$ at $x = x_{\min}$

B15 included a parameter c in the GNAA formulation, in order to increase the flexibility of the formula while maintaining the four boundary conditions. Szilagyi et al. (2016) noted that c needs to be in the range $-1 \leq c \leq 2$ so that y always increases monotonically with x and so that $y \leq x$. However, L19 noted that, if only positive values of y are plotted for $c > 2$, B15's equation forms curves that have $dy/dx = 1$ at $x = 1$ and that intersect the x axis at different points depending on the value of c. They noted the values of x at which $y = 0$ could be taken as values of $x_{\min}$, and they presented an equation relating c to the corresponding value of $x_{\min}$.

Note that all the curves from Figure 2 of L19 with $c > 2$ pass below the x axis for $x < x_{\min}$. If L19 found it problematic that dy/dx is not continuous across $x = x_{\min}$ (as discussed above in section 3), surely this scenario creates a much greater problem, since $y < 0$ for $x < x_{\min}$ in their Figure 2. While one might claim, mathematically speaking, that all four B15 boundary conditions are met for $c > 2$, boundary conditions (ii) and (iv) (defined in section 1 above) are stripped of physical meaning since y approaches the origin in an unphysical way (that is, from below the x axis), for $c > 2$. The concluding paragraph of B15 states that one advantage of the GNAA formulation is that "negative E values are made impossible," indicating that he would not advocate formulations that result in negative values (although there actually is a curve for $c = 5$ in Figure 3 of B15). Whatever utility the curves in Figure 2 of L19 might demonstrate (see next paragraph), they cannot be said to meaningfully match the four boundary conditions of B15.

Nevertheless, as discussed in the second concern above, there is no need to evaluate what happens when $x < x_{\min}$ if $x_{\min}$ is defined correctly, since x cannot go below $x_{\min}$. Therefore, L19's formulation relating c to $x_{\min}$ [their equation (10)] could conceivably be compatible with the rescaled CR. Specifically, once $x_{\min}$ is determined independently for each data point, their equation (10) could be used to find the corresponding value of c, and that value of c could be substituted into their equation (1) to obtain a value of y. When applied to the data set from the seven sites in Australia used by CQ18, however, this formulation of the rescaled CR performed significantly worse than $y = X$ (results not shown), so the present authors are not able to recommend it.

Finally, in Figure 3, L19 provide a graph of their formulation using a fixed $x_{\min}$ of 0.3 (corresponding to a c of 3.58). This does not account for the fact that $x_{\min}$ must be calculated for each individual data point. Thus, their Figure 2 misrepresents the role of $x_{\min}$ in the context of the rescaled CR.

6. Conclusions

The present authors appreciate this opportunity to clarify concepts related to the rescaled CR and address possible points of confusion. A common theme in this reply (specifically, in response to concerns 1, 2, and 4) is that $x_{\min}$ is a key variable in the CR, that it must be determined independently for each data point, and that it forms a firm lower bound to x. Once this point is understood, none of the concerns affect the viability of the rescaled CR concept.

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