The Law of the Wall:

Buffer Layer and Roughness Effects

Dr. Junke Guo

Assistant Professor, Dept. of Civil Engineering
Director of FHWA Flow Simulation Lab
University of Nebraska-Lincoln

Contents

Overview 4

Buffer Layer: Arctangent Law 7

Proposed Law of the Wall 13

Effect of Wall Roughness 15

Application to Bedload Initiation **20**

Conclusions **21**

Overview

1. What we know:
   - the linear law: $u^+ = y^+, \quad y^+ \leq 5$
   - the log law: $u^+ = \frac{1}{\kappa} \ln y^+ + B, \quad y^+ \geq 30$
   - roughness deduction: $u^+ = \frac{1}{\kappa} \ln y^+ + B - \Delta B$
   - the quartic asymptote:

$$u^+ = y^+ - \frac{1}{4\,(1150)} y^{+4} + \cdots, \quad 5 < y^+ \leq 8$$

2. What are the gaps:

   - no accurate buffer layer model
   - no accurate model for roughness effects

3. Why important:

   - bedload initiation and transport
   - boundary conditions for CFD modeling

4. What we propose:

   - a buffer layer model
   - a roughness function for transitional rough turbulence

5. Implication to bedload initiation

   - rational connection between the Shields diagram and the near wall velocity profiles

Buffer Layer: Arctangent Law

1. Idea:

   - The buffer layer law connects to the log law through the additive constant B.

   - The buffer layer law must tends to a constant in the log-law layer

2. Hypothesis:

- The buffer layer law might be approximated by a series of arctangent functions.

$$u^+ = \sum_{i=1}^{\infty} k_i \arctan^i \frac{y^+}{A}$$

- Expand the above to the 4th power.

$$\begin{aligned} u^+ &= \frac{k_1}{A} y^+ + \frac{k_2}{A^2} y^{+2} + \left(\frac{k_3}{A^3} - \frac{k_1}{3A^3} \right) y^{+3} \\ &\quad + \left(\frac{k_4}{A^4} - \frac{2k_2}{3A^4} \right) y^{+4} + \cdots \end{aligned}$$

- Comparing it with the quartic asymptote gives

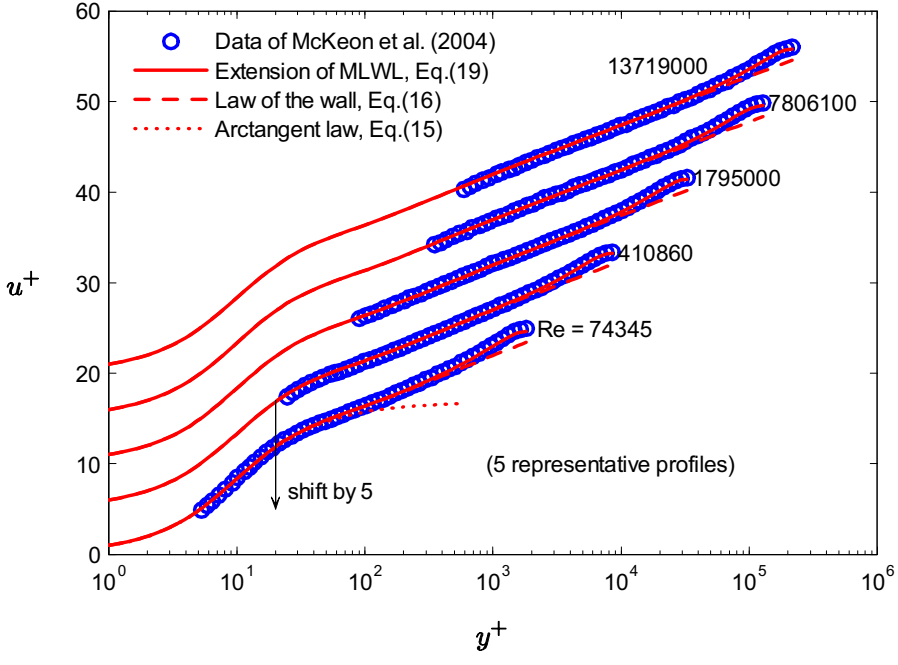
$$k_1 = A,\ k_2 = 0,\ k_3 = \frac{A}{3},\ k_4 = -\frac{A^4}{4\,(1150)}$$

- The arctangent law:

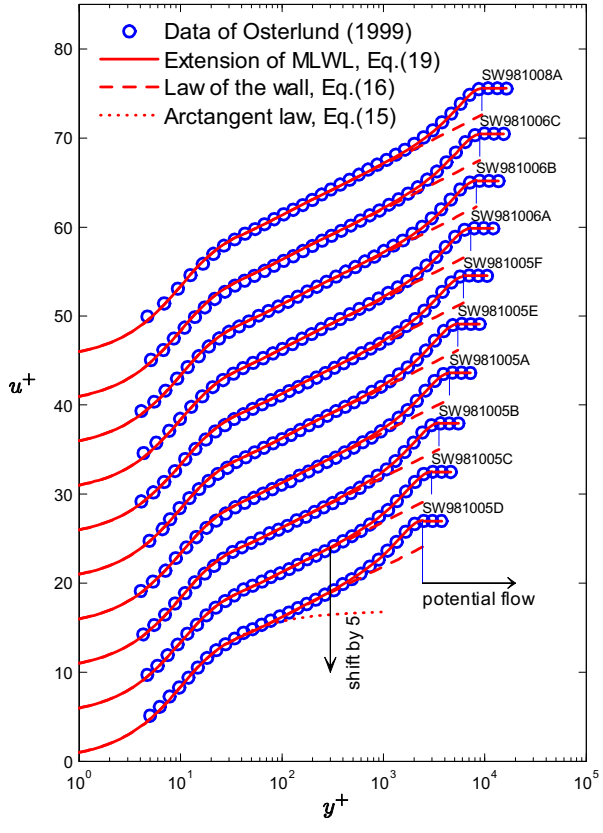
$$u^+ = A \arctan \frac{y^+}{A} + \frac{A}{3} \arctan^3 \frac{y^+}{A} - \frac{A^4}{4\,(1150)} \arctan^4 \frac{y^+}{A}$$

- Fitting: $A \approx 7$

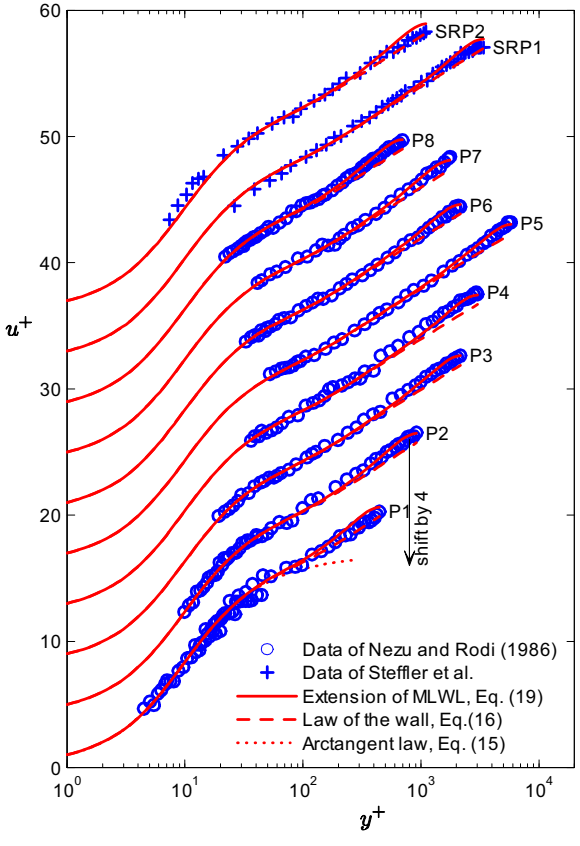
3. Test with pipe data:



4. Test with boundary layer data:



5. Test with open-channel data:



Proposed Law of the Wall

1. Hypothesis:

$$u^+ = 7 \arctan \frac{y^+}{7} + \frac{7}{3} \arctan^3 \frac{y^+}{7} - 0.52 \arctan^4 \frac{y^+}{7} + \ln \left[1 + \left(\frac{y^+}{C} \right)^{1/\kappa} \right]$$

   - small y^+, the arctangent law
   - large y^+, the log law. $C = \exp\left[\kappa \left(16.873 - B\right)\right]$

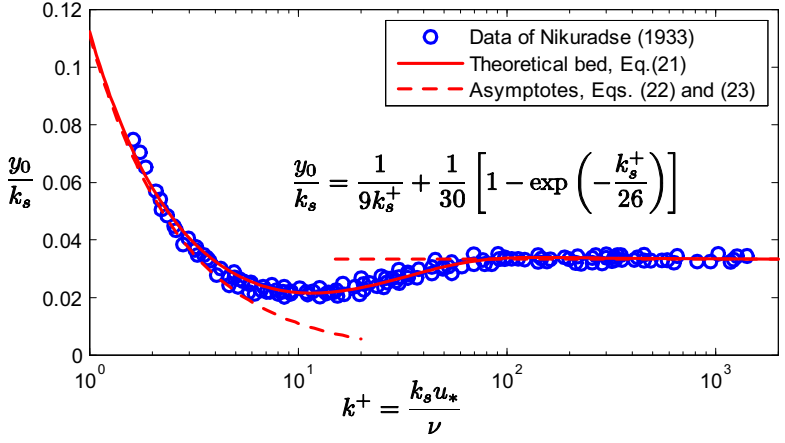
2. Combination with the modified log-wake law:

$$u^+ = 7\arctan\frac{y^+}{7} + \frac{7}{3}\arctan^3\frac{y^+}{7} - 0.52\arctan^4\frac{y^+}{7} + \ln\left[1+\left(\frac{y^+}{C}\right)^{1/\kappa}\right] + \underbrace{\frac{2\Pi}{\kappa}\sin^2\frac{\pi\xi}{2} - \frac{\xi^3}{3\kappa}}_{\text{the modified wake law}}$$

3. Test with data of pipes, boundary layers and open-channels

Effect of Wall Roughness

1. The log law for rough wall: $$u^+ = \frac{1}{\kappa} \ln \frac{y}{y_0}$$



2. Rearrange the above log law:

$$u^+ = \frac{1}{\kappa}\ln y^+ - \frac{1}{\kappa}\ln\left(\frac{y_0}{k_s}k_s^+\right)$$

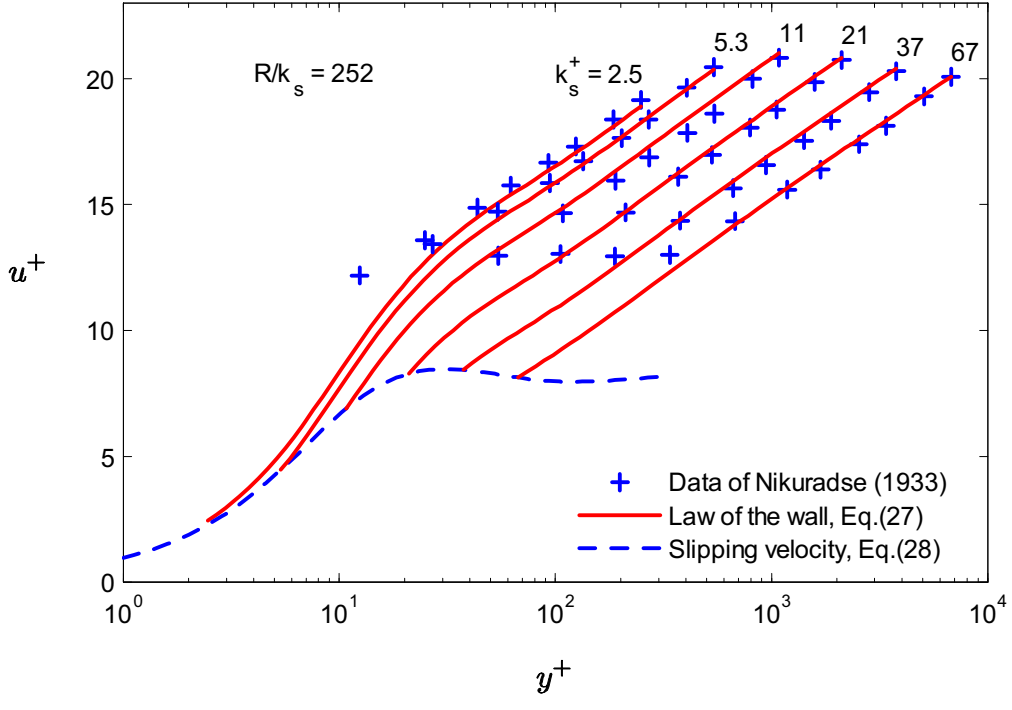
$$= \underbrace{\frac{1}{\kappa}\ln y^+ + B}_{\text{the log law}} - \underbrace{\left[B + \frac{1}{\kappa}\ln\left(\frac{y_0}{k_s}k_s^+\right)\right]}_{\text{roughness function } \Delta B\left(k_s^+\right)}$$

$$\Delta B = \frac{1}{\kappa}\ln\left\{1 + 0.3k_s^+\left[1 - \exp\left(-\frac{k_s^+}{26}\right)\right]\right\}$$

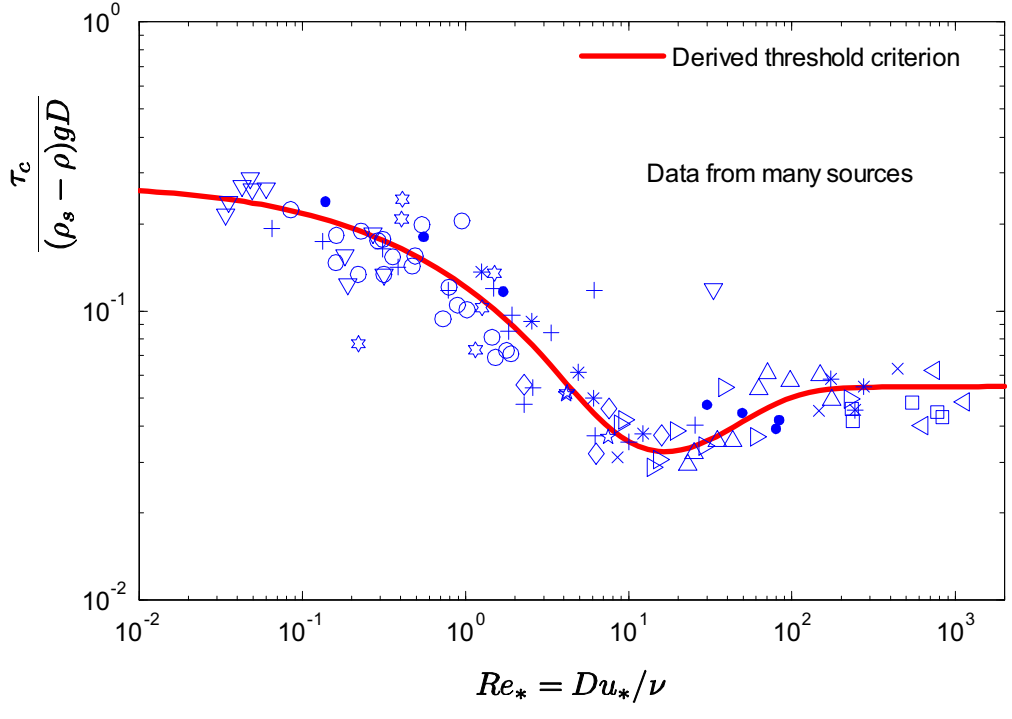
3. Velocity profile law with roughness effects:

$$
\begin{aligned}
u^+ &= 7 \arctan \frac{y^+}{7} + \frac{7}{3} \arctan^3 \frac{y^+}{7} - 0.52 \arctan^4 \frac{y^+}{7} \\
&\quad + \ln \left[1 + \left(\frac{y^+}{C} \right)^{1/\kappa} \right] - \Delta B
\end{aligned}
$$

4. Test with Nikuradse data:



Application to Bedload Initiation



Conclusions

- We have proposed an arctangent law to describe the velocity profile in the buffer layer.

- We have proposed a new roughness function to describe the transitional roughness effects on turbulent velocity profiles.

- Applying the proposed models can rationally derive the Shields diagram for bedload initiation.

Thanks!