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Conic Sections Expository Paper

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Archimedean Solids

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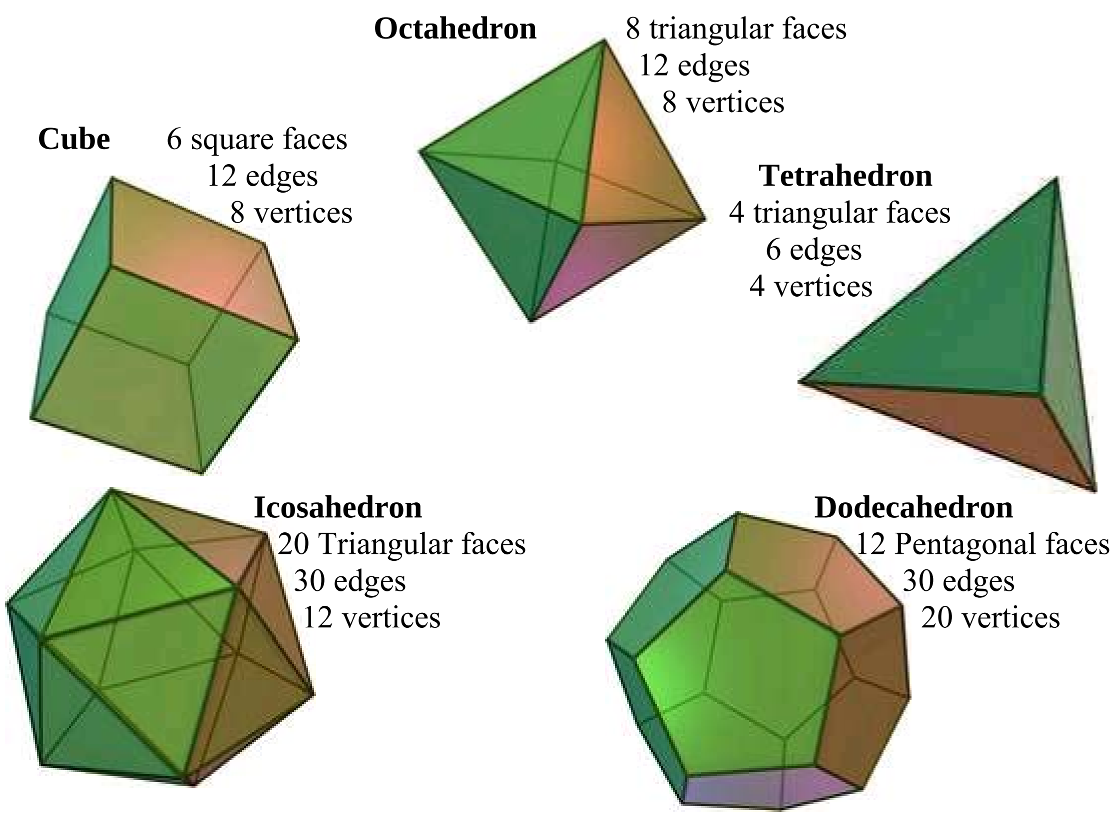
Archimedean Solids

A polygon is a simple, closed, planar figure with sides formed by joining line segments, where each line segment intersects exactly two others. If all of the sides have the same length and all of the angles are congruent, the polygon is called regular. The sum of the angles of a regular polygon with n sides, where n is 3 or more, is $180° \times (n - 2)$ degrees. If a regular polygon were connected with other regular polygons in three dimensional space, a polyhedron could be created. In geometry, a polyhedron is a three-dimensional solid which consists of a collection of polygons joined at their edges. The word polyhedron is derived from the Greek word *poly* (many) and the Indo-European term *hedron* (seat). The plural of polyhedron is "polyhedra" (or sometimes "polyhedrons"), and their discovery dates back to the ancient Greeks.

Pythagoras, famous for his Pythagorean Theorem, was thought to believe that there were an infinite number of regular polyhedra, polyhedra whose faces are congruent regular polygons which are assembled in the same way around each vertex. Plato also knew about regular polyhedra, as evidenced by his inclusion of five regular polyhedra in his work “the Timaeus”. He associated the cube with earth, the tetrahedron with fire, the octahedron with air, and the icosahedron with water. The model for the whole universe was the dodecahedron. These became known as the Platonic solids (for Plato). The Platonic Solids are convex polyhedra with each face congruent. A convex polyhedron can be defined as a polyhedron for which a line connecting any two (noncoplanar) points on the surface always lies in the interior of the polyhedron. Euclid, a student of Plato's, built on these platonic solids in his work "The Elements". He showed that there are exactly five regular convex polyhedra, known as the Platonic Solids. These are shown below. Each face of each Platonic solid is a convex regular polygon.



Platonic Solids

Each face of a Platonic solid is a regular polygon and all of the faces which are arranged to create the solid convex polyhedra are congruent. From these five Platonic solids the great Archimedes found that there are exactly thirteen semi-regular convex polyhedra. A solid is called semi-regular if its faces are all regular polygons and its corners are alike. These thirteen polyhedra are aptly called the Archimedean solids.

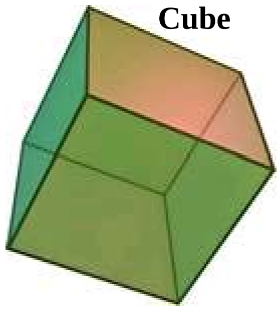
Archimedes was born at the end of Euclid's life in Syracuse, Sicily. He is known for the legendary cry of "Eureka" when introducing the theory of buoyancy, and for calculating the first accurate estimation of pi. He had many great mathematical discoveries, but his Archimedean solids are perhaps the primary contributors to his lasting fame.

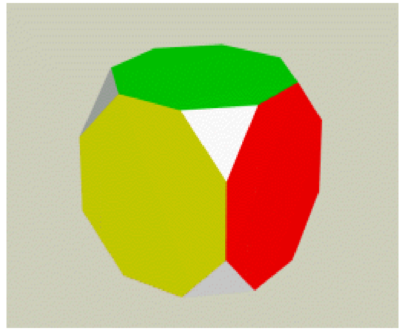
Archimedean solids, like Platonic solids, must be convex figures, but they are not exactly the same as Platonic solids. Archimedean Solids are the convex polyhedra that have a similar arrangement of nonintersecting regular convex polygons of two or more different types arranged in the same way about each vertex with all the edges the same length (Cromwell, 1997). The Platonic Solids are convex figures made up of one type of regular polygon. Archimedean solids are convex figures that can be made up of two or more types of regular polygons. All edge lengths of the polygons must be equal, and all of the vertices must be identical, meaning the polygons that meet at each vertex do so in the same way. Unlike prisms, which may have an arrangement of regular polygons at each vertex, Archimedean solids require that at all vertices the angles must be the same. The first five Archimedean solids are created by truncating the original Platonic solids. This allows more than one kind of regular polygon to be used for the faces.

The truncated cube is created by taking a cube (which is a Platonic solid) and literally cutting off the corners to create eight equilateral triangles. The truncated cube is shown below and has one triangle and two octagons around each vertex. It is then notated as (3,8,8) for the one triangle and two octagons at each vertex. Another notation is the Cundy and Rollett symbol which denotes the vertex in almost the same way, but uses exponents to avoid writing the same number multiple times. The Cundy and

Rollett symbol for the truncated cube is 3.8^2 for the triangle and two octagons at each vertex.

Cube



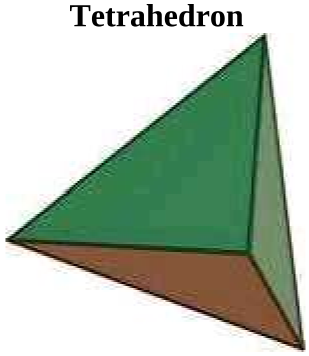


Truncated Cube
8 F_3 and 6 F_8
24 Vertices
36 Edges

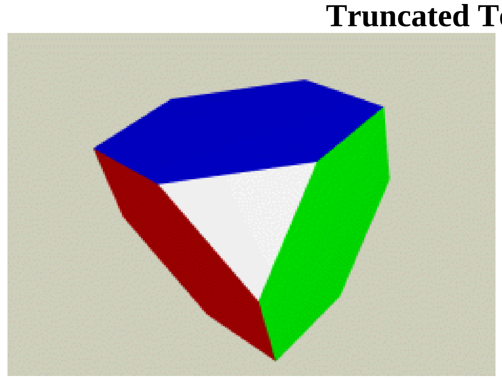
The Truncated Cube has eight faces of three sided figures (8 equilateral triangles) which the 8 F_3 indicates, and six faces of eight sides (6 regular octagons) which the 6 F_8 indicates. This has an interesting relationship with the original cube of 6 faces, 8 vertices, and 12 edges. The vertices and edges of the Truncated Cube are three times the number in the original cube while there are just 8 more faces (which was the number of cuts made).

Another Archimedean solid created from a Platonic solid is the Truncated Tetrahedron. This solid is created by cutting the vertices off the tetrahedron. At each vertex of the truncated tetrahedron would be an equilateral triangle and two regular hexagons. The symbol is then (3, 6, 6) or 3.6^2. Below is a picture of the original Tetrahedron and the Truncated Tetrahedron with its number of faces, edges, and vertices.

Tetrahedron



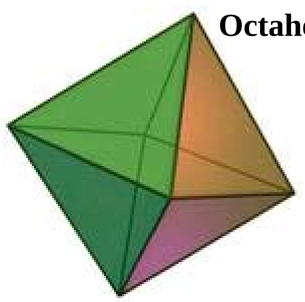
Truncated Tetrahedron



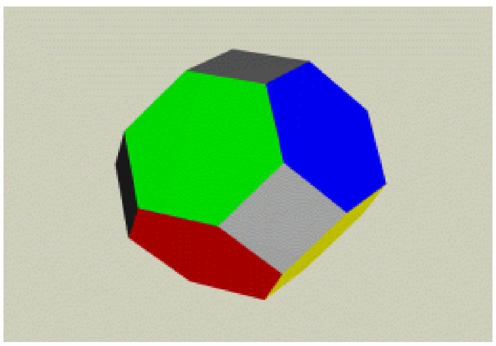
4 F_3, 4 F_6
12 Vertices
18 Edges

The Tetrahedron has 4 faces, 4 vertices, and 6 edges. The Truncated Tetrahedron has 4 equilateral triangular faces and 4 regular hexagonal faces which totals 8 faces, 12 vertices, and 18 edges. There are 4 more faces on the Truncated Tetrahedron due to the four cuts made. The number of vertices and edges are multiplied by three due to the addition of equilateral triangles (the same relationship that existed for the truncated cube).

The Octahedron can also be truncated to create an Archimedean solid. The Octahedron is shown below and has 8 faces, 6 vertices, and 12 edges. The Truncated Octahedron has the symbol (4, 6, 6) or 4.6^2 because at each vertex a square is joined by two regular hexagons. This square is formed by cutting off each vertex from the original Octahedron.



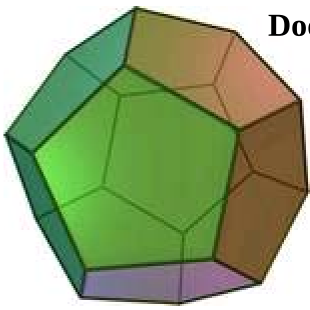
Octahedron



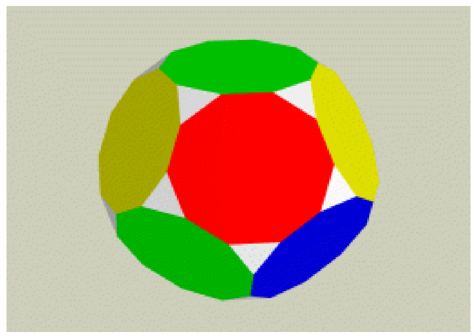
Truncated Octahedron
6 F_4, 8F_6
24 Vertices
36 Edges

The Octahedron has 8 faces, 6 vertices, and 12 edges. The Truncated Octahedron has 6 square faces and 8 regular hexagonal faces which give it 14 Faces, 24 Vertices, and 36 Edges. There were 6 cuts made on the Octahedron to create the total of 14 faces. The number of vertices was multiplied by 4 with the addition of squares, and the number of edges was increased by 24 as there were four additional edges for each of the 6 cuts. Note that the result is equivalent to multiplying the number of edges in the original octahedron by 3.

The Dodecahedron shown below has 12 faces, 20 vertices, and 30 edges. A cut can be made at each of the 20 vertices to create 20 equilateral triangles. The Truncated Dodecahedron shown below has a symbol of (3, 10, 10) or 3.10^2 because at each vertex an equilateral triangle is joined with two regular decagons.



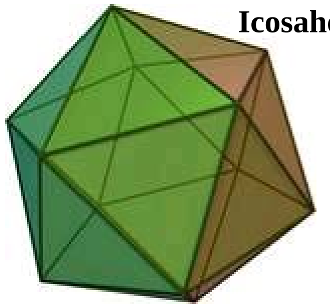
Dodecahedron



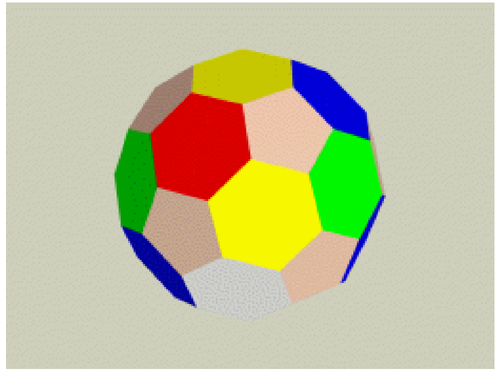
Truncated Dodecahedron
$20\ F_3$, $12\ F_{10}$
60 Vertices
90 Edges

The Truncated Dodecahedron has 20 equilateral triangular faces, and 12 regular decagonal faces. The 20 triangles come from the 20 cuts that were made at each vertex and are added to the 12 original dodecahedral faces to make 32 total faces. There are 60 Vertices and 90 Edges. These numbers come from multiplying the platonic solids' vertices and edges by three as equilateral triangles are created and their edges and vertices are added to the total.

The final Platonic solid is the Icosahedron with 20 faces, 12 vertices, and 30 edges. After truncating it along the 12 vertices a Truncated Icosahedron will be formed. The Truncated Icosahedron has the symbol (5, 6, 6) or 5.6^2 because at each vertex a regular pentagon is created and joined with two regular hexagons. The Truncated Icosahedron is also known as the soccer ball. Shown below are the Icosahedron and Truncated Icosahedron.



Icosahedron

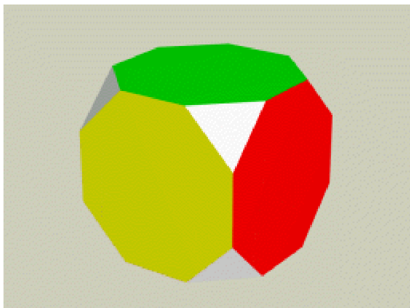


Truncated Icosahedron
12 F_5, 20 F_6
60 Vertices
90 Edges

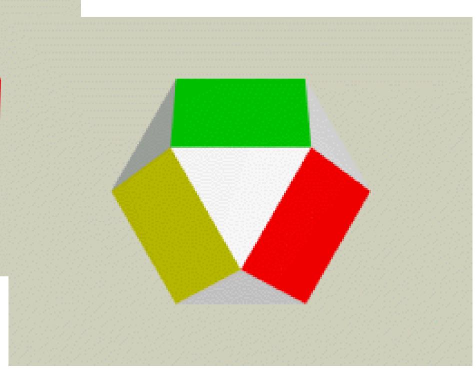
The Truncated Icosahedron has 12 regular pentagonal faces and 20 regular hexagonal faces for a total of 32 faces. There are 60 vertices and 90 edges. There were a total of 12 cuts to create the 12 pentagonal faces. When the 12 faces are added to the 20 original faces of the Icosahedron the 32 Faces are created. The regular pentagonal faces created from the cuts have five corners each and are multiplied by the original vertices of 12 to total 60 vertices. The original solid had 30 edges and 60 more edges are added with the addition of the 12 regular pentagons.

The next Archimedean solid can be created by truncation of either the Truncated Cube or the Cube. By slicing a bigger equilateral triangle on the Truncated Cube by connecting the midpoints of alternating edges of the octagons, a bigger equilateral triangle would be created and the shape left on the current octagon face would be four-sided and made to be a square. If one begins with the cube, by connecting the midpoints of the cube's edges, eight pyramids will be formed. Cutting off these pyramids will form the Cuboctahedron. These solids are shown below so the relationship can be easily seen. From the Truncated Cube which had 14 faces, 24 vertices, and 36 edges a cut is made at each of the 24 vertices to create the Cuboctahedron. The Cuboctahedron below has a symbol of (3, 3, 4, 4) or $3^2.4^2$ because at each vertex there are two equilateral triangles and two squares.

Truncated Cube

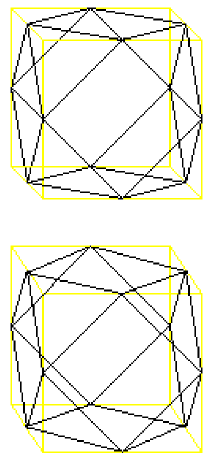


Cuboctahedron



8 F_3, 6 F_4
12 Vertices and 24 Edges

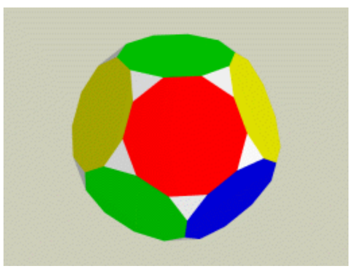
Cube (in yellow)



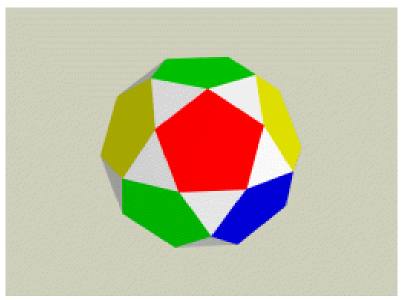
The Cuboctahedron has 8 equilateral triangular faces and 6 square faces. There are 14 faces on both the Truncated Cube and the Cuboctahedron. The number of vertices on the Cuboctahedron becomes 12 for each midpoint of an edge of the original cube. The number of edges is reduced to 24 since the regular hexagons of the Truncated Cube are changed to squares. The Cuboctahedron can also be obtained by truncating the original cube with triangles formed by the midpoints of each edge. The Cube has 6 faces, 8 vertices, and 12 edges. The number of faces is the original 6 squares plus the 8 cuts made. The number of vertices goes up by four as there are four midpoints on the squares. The number of edges goes up to 24 for the addition of the triangles.

Another Archimedean solid that can be obtained by truncating a Platonic Solid is the Icosidodecahedron. The Icosidodecahedron can be created from truncating either the Truncated Dodecahedron or the original Icosahedron. From the Truncated Dodecahedron which has 32 faces, 60 vertices, and 90 edges; cuts are made from to form larger equilateral triangles from the smaller triangles. After slicing the larger triangles, regular pentagons would be left behind to form the Icosidodecahedron which has a symbol of (3, 3, 5, 5) or $3^2.5^2$ since at each vertex two equilateral triangles are joined with two regular pentagons. Below the Truncated Dodecahedron, Icosidodecahedron, and original Icosahedron are shown. By finding the midpoints of the edges of the original Icosahedron and then making cuts, regular pentagonal faces and equilateral triangles will be formed.

Truncated Dodecahedron

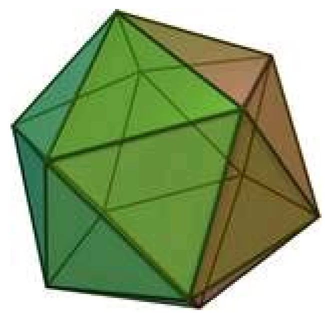


Icosidodecahedron



20 F_3, 12 F_5 30 Vertices
60 Edges

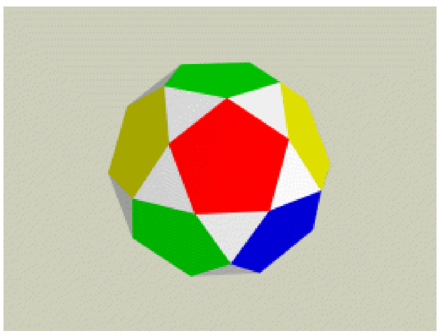
Original Icosahedron



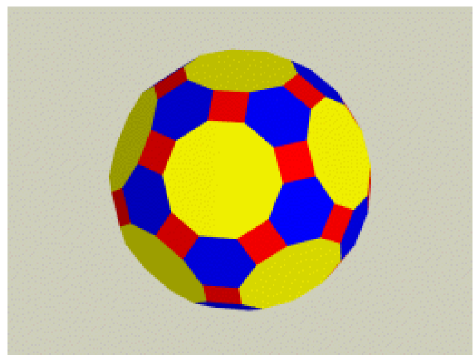
The Icosidodecahedron has 20 equilateral triangular faces and 12 regular pentagonal faces for a total of 32 Faces. The 30 vertices and 60 edges are created by the shared corners and sides of these equilateral triangles and regular pentagons. The Truncated Dodecahedron has 60 vertices and 90 edges. Since the shapes are equilateral triangles and regular decagons the number of shared corners and sides is more than the Icosidodecahedron. The original Icosahedron has 12 vertices and 30 edges because it is made up of equilateral triangles. The pairing of regular polygons with more sides leads to a larger amount of edges and vertices on the newly formed Icosidodecahedron.

No more Archimedean Solids can be obtained by a single truncation of a Platonic solid. However, by truncating the Icosidodecahedron and turning the resulting rectangles into squares, the Great Rhombicosidodecahedron is formed. The solids shown below show the relationship between the two. The symbol for the Great Rhombicosidodecahedron is 4.6.10 since at each vertex a square is joined with a regular hexagon and regular decagon.

Icosidodecahedron



Great Rhombicosidodecahedron



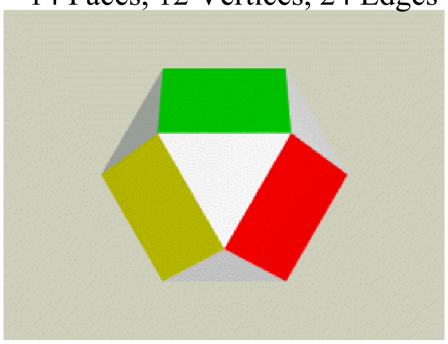
30 F_4 20 F_6 12 F_{10}
120 Vertices 180 Edges

The Great Rhombicosidodecahedron has 62 faces made up of 20 regular hexagons, 30 squares, and 12 regular decagons. It also has 120 vertices and 180 edges. It cannot be obtained merely by truncating another solid. After truncating the Icosidodecahedron, the resulting rectangles are distorted to make squares. The Icosidodecahedron has 32 faces, 30 vertices, and 60 edges. Cuts are made along those 30 vertices to make 30 new faces to add to the original 32 for a total of 62 faces. The number of vertices is multiplied by four for the addition of squares, and the number of edges is multiplied by three for the addition of the equilateral triangles.

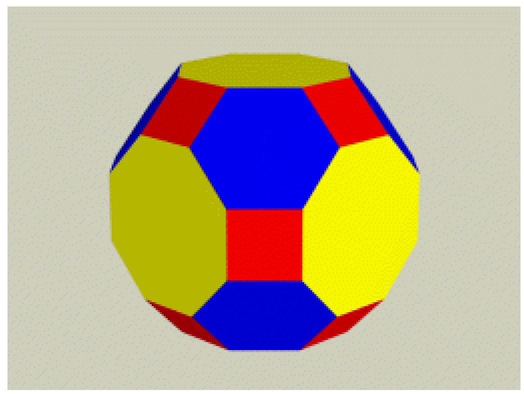
Another Archimedean solid created through truncation and distortion is the Great Rhombicuboctahedron. A cut is made along each vertex of the Cuboctahedron to create a solid with the symbol 4.6.8 for the square, regular hexagon, and regular octagon that meet at each vertex. The Cuboctahedron and Great Rhombicuboctahedron are shown below with their number of faces, vertices, and edges.

Cuboctahedron

14 Faces, 12 Vertices, 24 Edges



Great Rhombicuboctahedron

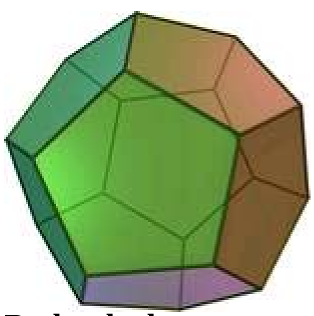


12 F_4,
8 F_6,
6 F_8

48 Vertices
72 Edges

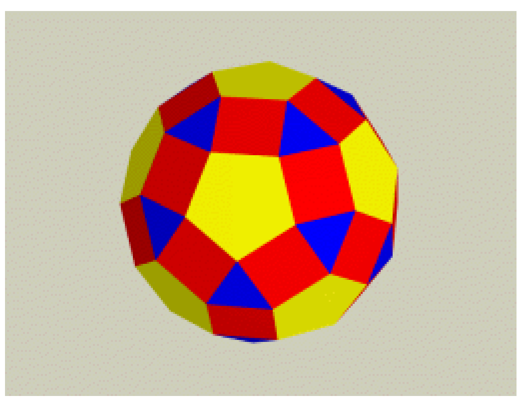
The Great Rhombicuboctahedron has 12 square faces, 8 regular hexagonal faces, and 6 regular octagonal faces for a total of 26 faces. This solid also has a total of 48 vertices and 72 edges. The original Cuboctahedron had 14 faces, 12 vertices, and 24 edges. After being truncated along the 12 vertices, 12 new rectangular faces are formed and distorted to become squares. These 12 new square faces added to the total of 14 old faces total the 26 faces. The original 12 vertices are multiplied by 4 for the squares added to get a total of 48 vertices. The original number of edges is multiplied by three since the original equilateral triangles are changed to hexagons.

Two Archimedean solids can be formed by expanding a Platonic solid rather than truncating it. Expansion is the process of radially displacing the edges or faces of a polyhedron (keeping their orientations and sizes constant) while filling in the gaps with new faces (Ball and Coxeter 1987, pp. 139-140). The first expanded solid is the Small Rhombicosidodecahedron. This Archimedean Solid is formed by the expansion of the Dodecahedron and Icosahedron, and the expansion process can be illustrated by examining this figure. Notice in the Rhombicosidodecahedron shown below, that if all pentagon and square faces were removed, the remaining triangular faces could be 'compressed' to form the Icosahedron. Similarly, if the square and triangular faces were removed, the remaining pentagonal faces could be compressed to form the Dodecahedron. The square quadrilateral faces could not be compressed to form a platonic solid; these are the faces which 'fill the gaps' when the faces from the Icosahedron and the Dodecahedron are combined to form a single solid. The symbol of the Small Rhombicosidodecahedron is 3.4.5.4 because an equilateral triangle, square, regular pentagon, and another square meet at each vertex. All three solids are shown on the next page with their numbers of faces, edges, and vertices.

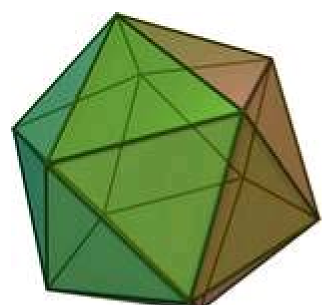


Dodecahedron
12 Faces, 20 Vertices
30 Edges

Small Rhombicosidodecahedron



$20\ F_3$, $30\ F_4$, $12\ F_5$ 60 Vertices
120 Edges



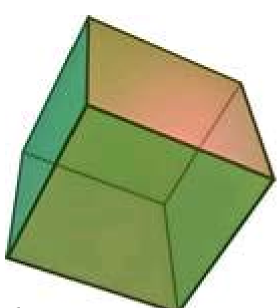
Icosahedron
20 Faces, 12 Vertices
30 Edges

The Small Rhombicosidodecahedron is made of 20 equilateral triangle faces, 30 square faces, and 12 regular pentagonal faces. These polygons create a total of 62 faces.

It appears that the 12 regular pentagonal faces of the dodecahedron are combined with the 20 equilateral triangle faces of the icosahedron and joined with squares to form the Small Rhombicosidodecahedron. The combination of these regular polygons creates the large amount of vertices and edges. The Dodecahedron had 20 vertices on its own then equilateral triangles are added with three corners each to make 60 vertices, or the Icosahedron had 12 vertices on its own then regular pentagons are added that have five corners each to create 60 total vertices. The number of edges on the Dodecahedron and Icosahedron is 30, and once the squares are added with four edges each the total of 120 is reached.

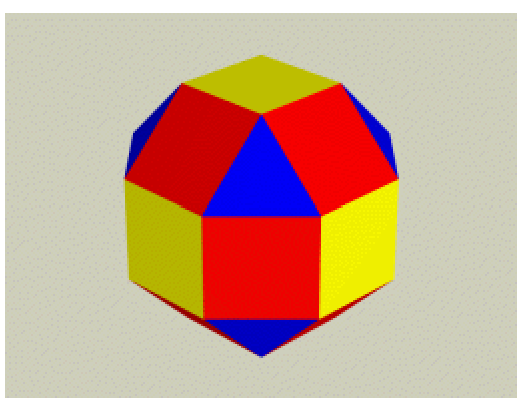
Another solid that can be created by expansion is the Small Rhombicuboctahedron. The Cube and Octahedron are expanded to create this solid. The Small Rhombicuboctahedron has a symbol of 3.4^3 because at each vertex three squares meet with one equilateral triangle. As can be seen at the top of the next page, the Cube and Octahedron are expanded and square faces are added to create the Small Rhombicuboctahedron.

Cube



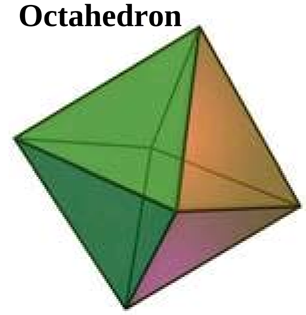
6 Faces
8 Vertices, 12 Edges

Small Rhombicuboctahedron



$8\ F_3$ $18\ F_4$
24 Vertices 48 Edges

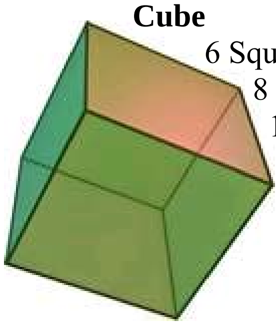
Octahedron



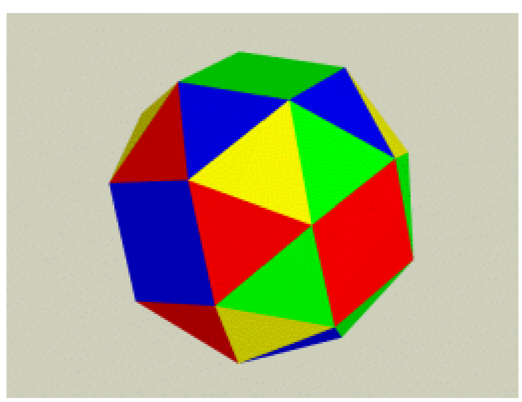
8 Faces
6 Vertices
12 Edges

The Small Rhombicuboctahedron is made of 8 equilateral triangle faces and 18 square faces. The Cube has 6 square faces and the Octahedron has 8 equilateral triangle faces. The 8 equilateral triangle faces are joined with the 6 square faces while 12 more squares are added to fill in the gaps. This creates the total of 26 faces on the Small Rhombicuboctahedron. The original Cube had 8 vertices while the Octahedron had 6 vertices. The 12 squares added share some corners with polygons already in place, but create 10 more vertices to add to the original 14. The number of edges increases by 24, which also comes from the sides added to the solid when the new squares are added. Some edges are shared with already formed polygons.

The final two Archimedean Solids are created by moving the faces of an existing Platonic solid outward while giving each face a twist. Both of these solids are known as chiral solids. Chiral means that the solids have different forms of handedness which are not mirror-symmetric. One form is called left-handed, and the other form is called right-handed. The first of these two chiral solids is the Snub Cube. The meeting of four equilateral triangles with one square at each vertex gives the Snub Cube a symbol of $3^4.4$. Looking at the Cube and Snub Cube below, one can see that the square faces of the Cube were expanded and twisted. The spaces created by this motion were filled in with ribbons of equilateral triangles.



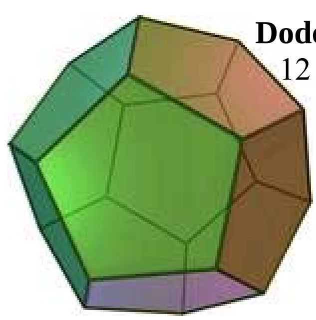
Cube
6 Square Face
8 Vertices
12 Edges



Snub Cube
32 F_3, 6 F_4
24 Vertices
60 Edges

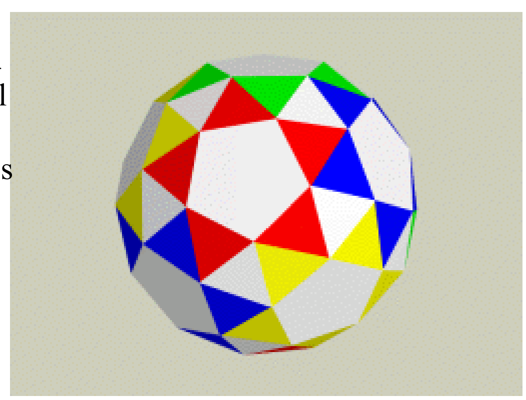
The Snub Cube contains the original 6 square faces of the Cube and 32 equilateral triangles are added to create a total of 38 faces. The number of vertices comes from the addition of the equilateral triangle faces that have three corners each. By taking the original vertices of the cube times three the number 24 vertices can be reached. The original Cube had only 12 edges. A square has four edges and when equilateral triangles are added 8 more edges are created. That is a total of 12 edges. By starting with one square of the cube and adding the edges of each square around the solid, all the edges will be accounted for.

Another Chiral solid that has a left-handed and right-handed appearance is the Snub Dodecahedron. The symbol of the Snub Dodecahedron is $3^4.5$ for the four equilateral triangles and one regular pentagon that meet at each vertex. The Dodecahedron's regular pentagons are expanded and twisted while the gaps in between them are filled with equilateral triangles. The Solids are shown below with their numbers of faces, vertices, and edges.



Dodecahedron
12 pentagonal Faces
20 Vertices
30 Edges

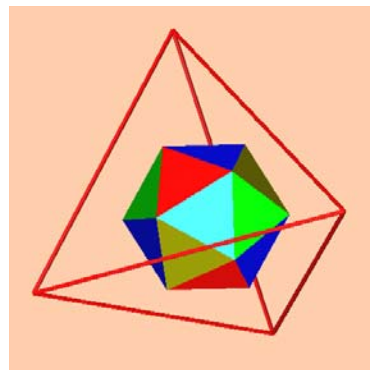
Snub Dodecahedron



80 F_3,
12 F_5,
60 Vertices
150 Edges

The Snub Dodecahedron contains the original Dodecahedron's 12 regular pentagonal faces, and the 80 equilateral triangle faces added to create the solid. This is a total of 92 faces. The original Dodecahedron has 20 vertices. Each equilateral triangle has three corners, and when the 20 vertices are multiplied by the three vertices for each triangle the total of 60 vertices is reached. The total of 150 edges comes from the original Dodecahedron's 30 edges multiplied by the 5 edges on each regular pentagon to get 150 total edges. This is because an equilateral triangle is connected to each side of the regular pentagon.

All Archimedean solids are capable of being circumscribed by a regular Tetrahedron. An example of this is shown here with a Platonic Solid. Four of the Platonic Solids' faces lie on the faces of that Tetrahedron.



Icosahedron circumscribed by a **Tetrahedron**

The Icosahedron in the figure has four faces that lie on the faces of the surrounding Tetrahedron. That is not the only similarity among the Archimedean Solids.

All the Archimedean solids satisfy the relationship $(2\pi - A)V = 4\pi$ where A is the sum of face's angles at a vertex and V is the number of vertices. (Ball and Coxeter, 1987). Ball and Coxeter also note that if A denotes the sum of the face-angles at a vertex, then A must be less than 2π in order to form an angle (otherwise joining the faces at a common vertex would cause them to lie flat). For example, the Snub Dodecahedron $3^4.5$ has 60 vertices, so the relationship between A and V gives us the following:

$$(2\pi - A) = \frac{4\pi}{60}$$

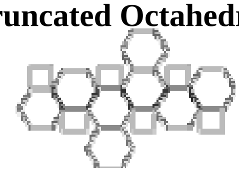
$$(2\pi - A) = \frac{\pi}{15}$$

$$2\pi = \frac{\pi}{15} + A$$
$$2\pi - \frac{\pi}{15} = A$$
$$\frac{30\pi}{15} - \frac{\pi}{15} = A$$
$$\frac{29\pi}{15} = A = \left(\frac{4}{3} + \frac{3}{5}\right)\pi$$

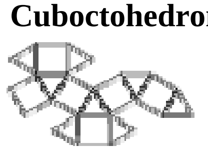
Here I have solved for A, the sum of the face-angles at a vertex (Ball and Coxeter left their expression as the sum of the two face-angles at each vertex).

In order to use the Archimedean Solids in my eighth grade classroom, I decided to study the nets of many of the Archimedean Solids. The state of Nebraska requires that eighth grade students understand Tessellations. The curriculum that my district uses does not include anything about Tessellations. While I supplemented the curriculum using internet activities, I didn't make a connection between Tessellations and Archimedean solids until I examined their nets. A net appears when the solid is unfolded to show a pattern of two-dimensional polygons rather than a three-dimensional polyhedron. A net is defined as an arrangement of edge-joined polygons in the plane which can be folded (along the edges) to become the faces of the polyhedron. It is conjectured, but not proven, that all convex polyhedra have nets; this is a statement known as Shephard's conjecture. Archimedean Solids are made up of faces of all regular polygons, and when some of these solids are unfolded they could tessellate. The possible nets (or possible unfolded appearance) for each of the Archimedean solids appear beneath the name of the solid to which the net corresponds.

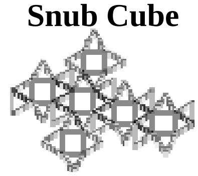
Truncated Octahedron



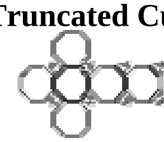
Cuboctohedron



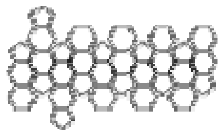
Snub Cube



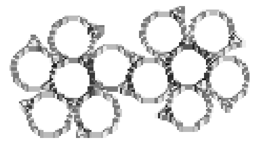
Truncated Cube



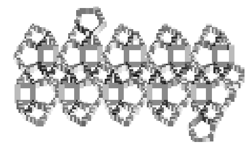
Truncated Icosahedron



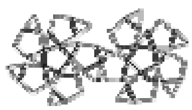
Truncated Dodecahedron



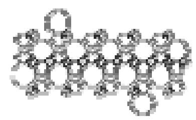
Small Rhombicosidodecahedron



Icosidodecahedron

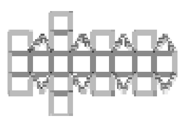


Great Rhombicosidodecahedron

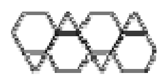


Snub Dodecahedron

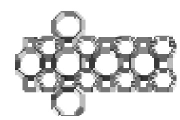
Small Rhombicuboctahedron



Truncated Tetrahedron



Great Rhombicuboctahedron



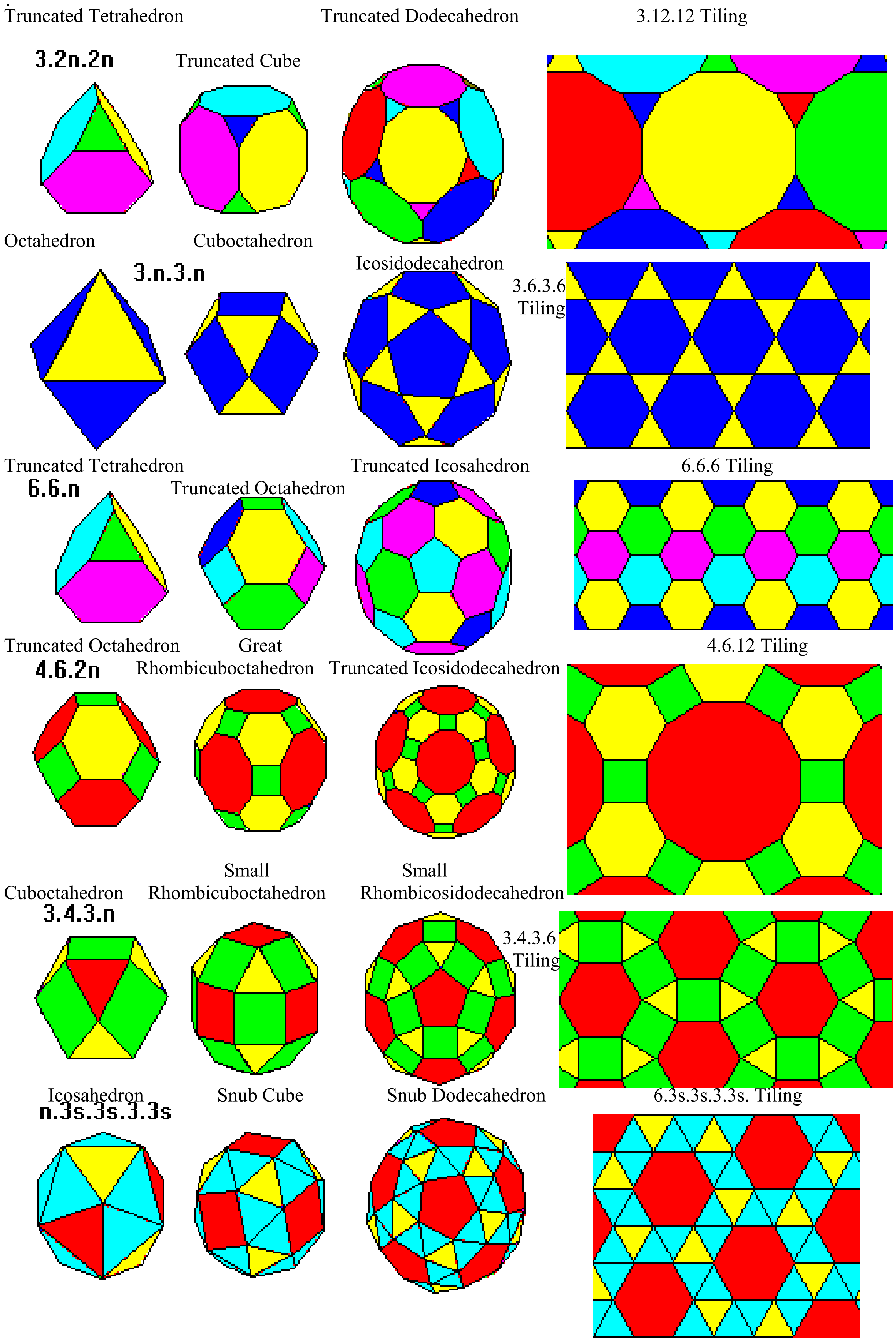
The patterns of these nets made me think of tessellations and how they relate to the Archimedean solids.

To create a tessellation using the net of an Archimedean solid, some adjustments must be made. A few of the nets above seem to tessellate on their own, but to truly tessellate the arrangement of polygons must repeat in a pattern without any gaps. The tilings appear to tessellate when the sum of the angles about a vertex is close enough to 360^{o} so that it appears that no gaps are formed in the nets. Gaps appear in the nets on the previous page, which does not allow the net of an Archimedean solid to be a true tessellation. The nets of the Archimedean Polyhedra can be adjusted to form what are known as Archimedean tilings, which do tessellate. An Archimedean tiling is a true tessellation where the number of edges of one face type has been increased enough to eliminate the gaps in the net.

In the illustration on the next page, the first row shows a Truncated Tetrahedron with vertices (3.2n.2n) or 3.6.6 for n=3. The Truncated Cube is next with vertices (3.2n.2n) or 3.8.8 for n=4, and last is a Truncated Dodecahedron with vertices (3.2n.2n) or 3.10.10 for n=5. When n=6, for the polyhedron (3.2n.2n) 3.12.12, the polygon angles at a common vertex sum to 360 degrees to obtain the 3.12.12 tiling. These tilings are displayed with their names and types of vertices on the next page, along with other Archimedean Solids and their corresponding tilings. When the number of edges of one face type is increased enough to eliminate gaps, the arrangement of regular polygons derived from the nets of the different Archimedean solids will tessellate.

I can use the study of Archimedean Solids, and their nets, to teach tessellations to my eighth grade math students. After creating the solids and defining the faces, edges, and vertices of each; the students will be able to create tessellations using the regular polygons found in each of the Archimedean solids. The Archimedean solids will become a valuable tool in my classroom in the teaching of the eighth grade curriculum.

The Archimedean Solids therefore can be shared with the eighth graders that I teach in order to help the students understand polyhedra, nets, and tessellations. The ability to create these solids by truncating or expanding other solids is vital to understanding where these solids come from. The nets of these solids can help students to define patterns and discover when they will tessellate. I look forward to using models to help the students understand how these solids are created and how they can be unfolded into various patterns. Archimedes was truly an amazing mathematician to discover these thirteen solids and to have his discovery stand the test of time for hundreds of years.



Truncated Tetrahedron
Truncated Dodecahedron
3.12.12 Tiling
3.2n.2n
Truncated Cube
Octahedron
Cuboctahedron
3.n.3.n
Icosidodecahedron
3.6.3.6 Tiling
Truncated Tetrahedron
Truncated Icosahedron
6.6.6 Tiling
6.6.n
Truncated Octahedron
Truncated Octahedron
Great Rhombicuboctahedron
4.6.2n
Truncated Icosidodecahedron
4.6.12 Tiling
Cuboctahedron
Small Rhombicuboctahedron
Small Rhombicosidodecahedron
3.4.3.n
3.4.3.6 Tiling
Icosahedron
Snub Cube
Snub Dodecahedron
6.3s.3s.3.3s. Tiling
n.3s.3s.3.3s

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