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Dispersionless forces and the Aharonov-Bohm effect

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Abstract – The independence of the Aharonov-Bohm phase shift on particle velocity is one of its defining properties. The classical counterpart to this dispersionless behavior is the absence of forces along the direction of motion of the particle. A reevaluation of the experimental demonstration that forces are absent in the AB physical system is given, including previously unpublished data. It is shown that the debate on the presence or absence of forces is not settled. Experiments that measure the influence of magnetic permeability on forces and search for dispersionless quantum forces are proposed.

Introduction. – Type-I Aharonov-Bohm effects [1] showcase the guiding principle of the Standard Model, local gauge invariance [2]. The Aharonov-Bohm effect is also a cornerstone phenomenon in quantum mechanics. It is thought to establish that the vector potential can cause measurable effects even when the fields are zero [3]. It is thus claimed to elevate the relevance of the vector potential from being a helpful mathematical construct to that of having direct physical reality [4]. However, Vaidman recently reconsidered this viewpoint [5]: "...the Aharonov-Bohm effect can be explained without the notion of potentials. It is explained by local action of the field of the electron on the source of the potential." The passing electron is shown to exert a force on the solenoid, while the solenoid does not exert a force on the passing electron. The first part of this argument agrees with Boyer's derivation [6]. Boyer claims that there is a force on the solenoid, but in contrast, he also claims that there is a back-action force on the electron that explains the AB-phase shift. McGregor *et al.* have shown [7] that both viewpoints can be maintained even if they appear to be at odds with each other. If the motion of the charge carriers in the solenoid is fully constrained, the solenoid experiences a force and the passing electron does not. If the charge carriers are completely free to move, the passing electron does experience a force. This supports the generally accepted interpretation of the Aharonov-Bohm effect: "phase without a force", in the case of fully constrained motion. This case has been shown to be an example of a Feynman paradox [7] on conservation of momentum. Missing momentum is stored in the combined electromagnetic field of the electron and solenoid in this case where there is no back-action force. Note that for the interaction of a charged particle and a magnetic flux (due to a solenoid, for example), the existence of hidden momentum is expected to affect the equation of motion [8]. For the alternative case of fully unconstrained motion, there is a back-action force, and momentum conservation does not require field momentum. The surprise is that the back-action force is exactly the correct magnitude to explain the AB-effect [5,6]. The two limits, constrained and unconstrained motion, considered in ref. [7], are not thought to represent a detailed realistic description of a physical system. A detailed model study of the response of the solenoid has currently not been completed [9]. A definitive theoretical answer is, thus, currently not available. This leaves concerns in the interpretation of the classical part of the analysis of the Aharonov-Bohm physical system. Do the experiments belong to the constrained or unconstrained case?

On the experimental side, a test showing the dispersionless nature of the Aharonov-Bohm effect with an electron wave interferometer [10,11] has never been performed. The next best approach is to rule out forces by time delay experiments. Caprez *et al.* have shown that an electron passing by a solenoid does not experience a force that causes a delay sufficiently large to explain the AB-effect [12,13]. It appears that this settles the issue.

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However, we address two concerns in this paper. The first considers the possibility of a different back-action for electrons in a solenoid as compared to electrons bound in atomic magnetic dipoles. Electrons in a conducting wire may, during the short interaction times, be effectively unconstrained, and, thus, provide a back-action [7]. But the core electrons are constrained much tighter by the atomic potential, and may, therefore, not provide a back-action. The second concern addressed is, whether or not there are forces that are approximately dispersionless. Here, as well as in refs. [2] and [6], the classical concept of force and the quantum-mechanical wave concept of dispersion are combined in a semi-classical fashion. The force gives rise to a position shift, Δx, in the propagation direction of the particle. This shift can be related to a phase shift through the expression, $\Delta\varphi = k\Delta x$, where k is the wave vector. If this phase shift is velocity independent, then the force is said to be dispersionless. Zeilinger [10] pointed out that the velocity independence of the phase shift is a defining feature of the AB-effect, as forces would shift the position of a particle. He continued by pointing out that a dispersionless interaction does not shift the centroid position of an electron wave packet. This view has been generally accepted [6]. But what if dispersionless forces exist? We will show that the Lorentz forces are approximately dispersionless for an electron passing a solenoid. These two concerns motivate our present reevalutation of currently proposed and performed experiments that test for the dispersionless nature or time delay.

The time delay experiment [12] is performed using a solenoid with a weak iron core. The response of the conduction electrons in the current carrying wire of the solenoid is possibly different from that of the iron core electrons that are bound in atomic states. Addressing the first concern, we consider whether the experimental data of ref. [12] is sufficient to rule out a back-action that is limited to the solenoid electrons. Addressing the second concern, we question whether the experiment is sufficient to rule out dispersionless forces.

Theory. – Consider an electron passing by a current carrying solenoid. The solenoid symmetry axis is chosen to coincide with the z-axis, while the electron moves parallel to the x-axis. The x-component of the Lorentz force on the solenoid with cross-sectional area A and magnetic field B_0 is given by the expression [7,14]

$$F_x = \frac{-B_0 Aqv(x_e, y_e)}{4\pi}\frac{4x_e y_e}{\left(x_e^2+y_e^2\right)^2}, \tag{1}$$

where v is the electron velocity along the x-direction and x_e and y_e are the xy-coordinates of the charge relative to the solenoid's z-axis. Assuming that there is an equal and opposite back-action, and using Newton's second law, this force can be integrated,

$$\Delta x = \frac{2}{m}\int_{-\infty}^{\infty}\int_{-\infty}^{t'} F_x(v, x_e, y_e)\mathrm{d}t'\mathrm{d}t \tag{2}$$

to yield a relative displacement Δx between electrons passing on opposite sides of the solenoid of $\Delta x = eB_0A/mv_0$. The approximation that $v = v_0$ is made assuming the force is weak. In a semi-classical approximation the resulting phase shift is $\Delta\varphi = k\Delta x = mv_0\Delta x/\hbar$ and equal to the Aharonov-Bohm phase shift

$$\Delta\varphi_{AB} = \frac{e}{\hbar}\int_C \vec{A}\cdot\mathrm{d}\vec{l} = \frac{e}{\hbar}\int \vec{B}\cdot\mathrm{d}\vec{S}, \tag{3}$$

which gives $\Delta\varphi_{AB} = eB_0A/\hbar$. It should be emphasised that the fact that such a force can be formulated at all, is surprising in view of the generally accepted interpretation of the effect. The proposed force does not only give rise to a phase shift in the semi-classical approximation, but also to a time delay for electrons passing by a solenoid in the classical picture [6]. This time delay was shown experimentally not to occur in the experiment mentioned above [12].

We improve on the approximation $v = v_0$ by calculating the effect of the force on the velocity. Combining $a_x = F_x/m$ with $a_x = \mathrm{d}v_x/\mathrm{d}t = (\mathrm{d}v_x/\mathrm{d}x)(\mathrm{d}x/\mathrm{d}t) = v_x(x_e, y_e)\,\mathrm{d}v_x/\mathrm{d}x$ gives $v_x = \frac{1}{m}\int F_x\mathrm{d}x$ leading to a velocity v^+ (v^-) of the electron passing on the side with $y_e > 0$ ($y_e < 0$) of

$$\begin{aligned} v^{\pm}(x_e, y_e) &= \frac{-BAq}{\pi m}\int_{-\infty}^{x_e}\frac{xy_e}{\left(x^2+y_e^2\right)^2}\mathrm{d}x \\ &= v_0 \pm \frac{BAq}{2\pi m}\frac{|y_e|}{x_e^2+y_e^2} = v_0 + \Delta v_x. \end{aligned} \tag{4}$$

The displacement of the electron

$$\Delta x^{\pm} = \int \Delta v_x \mathrm{d}t = \int (v^{\pm} - v_0)\frac{\mathrm{d}y}{v^{\pm}}$$

is given by

$$\begin{aligned} \Delta x^{\pm} &= \frac{BAq}{2\pi m}\int_{-\infty}^{\infty}\frac{1}{v_0\left(1+\frac{BAq}{2\pi m v_0}\frac{|y_e|}{x^2+y_e^2}\right)}\frac{|y_e|}{x^2+y_e^2}\mathrm{d}x \\ &\approx \pm\frac{BAq}{2mv_0} \mp \frac{1}{2|y_e|\pi}\left(\frac{BAq}{2mv_0}\right)^2, \end{aligned} \tag{5}$$

when $\Delta v = v^+ - v^- \ll v_0$, $\frac{BAq}{2\pi m v_0}\frac{|y_e|}{x^2+y_e^2} \ll 1$, and using $\int_{-\infty}^{\infty}\frac{\mathrm{d}x}{(x^2+y_e^2)^2} = \frac{\pi}{2y_e^3}$. The relative displacement between electrons that pass on opposite sides of the solenoid is

$$\Delta x = \Delta x^+ - \Delta x^- = \frac{BAq}{mv_0} - \frac{1}{\pi|y_e|}\left(\frac{BAq}{2mv_0}\right)^2. \tag{6}$$

The semi-classical phase shift now consists of the velocity-independent AB phase shift and a weak velocity-dependent term

$$\Delta\varphi = k\Delta x = \frac{BAq}{\hbar} - \frac{1}{\hbar\pi|y_e|mv_0}\left(\frac{BAq}{2}\right)^2. \tag{7}$$

The velocity-independent term (first term in eq. (7)) would now explain the usual observed AB phase shift for weak field interferometry experiments when the first term is dominant. The second term in eq. (7) causes the envelop of a wave packet to shift by the amount

$$\Delta x_{semi} = \frac{\partial \varphi}{\partial k} = \frac{1}{\hbar^2 k^2 \pi \,|y_e|} \left(\frac{BAq}{2}\right)^2. \tag{8}$$

The relation between the magnetic field and the solenoidal current is given by

$$B = \mu_r \mu_0 n I, \tag{9}$$

where μ_r is the relative permeability, μ_0 is the vacuum permeability, n is the number of windings per unit length, and I is the current. The iron core enhances the magnetic field by a factor of $\mu_r \approx 150$ [12]. For the case that the back-action of the iron core is absent, the relative permeability is set equal to one. The classical time delay follows from the first term of eq. (6),

$$\Delta t_{clas} = \frac{\Delta x}{v_0} = \frac{BAq}{m v_0^2}, \tag{10}$$

where the magnetic field is given by eq. (9). The semi-classical delay follows from eq. (8),

$$\Delta t_{semi} = \frac{\Delta x_{semi}}{v_0} = \frac{1}{\hbar^2 k^2 \pi \,|y_e|\, v_0} \left(\frac{BAq}{2}\right)^2. \tag{11}$$

The crucial assumption in the above argument, that there is an opposite and equal back-action of the solenoid on the electron, has been generally accepted to be incorrect. The reason is that the rate of change of hidden momentum modifies the equation of motion [8,15]. The force on a magnetic dipole $\vec{m}$ at rest in an external magnetic field $\vec{B}$ and an external electric field $\vec{E}$ is given by

$$\vec{F} = \vec{\nabla}\left(\vec{m}\cdot\vec{B}\right) - \frac{1}{c^2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\vec{m}\times\vec{E}\right). \tag{12}$$

This takes into account the hidden momentum $\vec{m}\times\vec{E}/c^2$. Vaidman presents three models [8] that further support this argument. This view is nicely codified in the book by Aharonov and Rohrlich [16]: "The paradox is crucial to clarifying the entirely quantum interactions of "fluxons" and charges..." This paradox is a classical relativistic one [17]. How can a stationary magnetic moment and charge have zero net momentum, while having linear field momentum? The answer is that the field momentum is canceled by the hidden momentum.

The existence of hidden momentum follows from the following lemma: any finite stationary distribution of matter has zero total momentum [8]. The term "stationary" is defined by $\partial_0 T^{\mu\nu} = 0$, where $T^{\mu\nu}$ is the electromagnetic stress tensor. A stationary distribution along with the conservation law $\partial_\mu T^{\mu\nu} = 0$ gives $\partial_j T^{j0} = 0$. As a consequence of the divergence theorem, the total momentum may be written as a surface integral [18],

$$\begin{aligned} p^i &= \frac{1}{c}\int T^{i0}\mathrm{d}\tau = \\ &\frac{1}{c}\int \left[\partial_j x_i T^{j0}\right]\mathrm{d}\tau = \frac{1}{c}\oint x_i T^{j0}\mathrm{d}S_j. \end{aligned} \tag{13}$$

For a finite localised distribution of matter the elements of the stress tensor must fall off as $1/r^{4+\delta}$ ($\delta \geq 0$). The above surface integral is thus zero, proving the lemma

$$p^i = \frac{1}{c}\oint x_i T^{j0}\mathrm{d}S_j = 0. \tag{14}$$

Given that a stationary charge in the vicinity of a rotating cylinder of charged matter is a stationary distribution of matter, its total momentum must be zero. This configuration has electromagnetic momentum, and taken together with the validity of the lemma, demands that there is another opposite and equal form of momentum. This "hidden momentum" is present in the internal motion of the physical system. One text-book example is that of a current carrying loop of wire, placed in a uniform external electric field [19]. Relevant for our present discussion, the electric field could be thought of as arising from the presence of a point charge.

The validity of the lemma is not under debate. However, the lemma should not be applied indiscriminately to dynamical systems as it is derived for stationary systems. In ref. [7] an example of a dynamical system is analyzed in detail, which shows that the equations of motion are not modifed by the presence of hidden momentum. This is thus an example where the lemma does not help to give the correct equation of motion. In general, to obtain the equations of motion of a dynamic physical system, a non-stationary system needs to be considered.

Time delay experiment. – In the time delay experiment [12] an electron passed by a macroscopic solenoid. A femtosecond laser pulse was used to extract electrons from a field emission tip [20,21]. The electron pulse then passed between two identical solenoids. The two solenoids were connected through high permeability magnet iron bars to form a square magnetic toroid. This arrangement reduces magnetic flux leakage and enhances the magnetic flux by μ_r. Finally, the arrival of the electron was detected with a channelplate, and a time-of-flight spectrum was obtained.

Time-of-flight spectra were fitted to find the electron's arrival time. In the left panel of fig. 1 the result of ref. [12] is repeated. The experimental time delay data is compared with the classical theory (eq. (10)). It shows that no delay occurs, ruling out the classical prediction. The right panel of fig. 1 shows the same data, but with the time scales expanded by three orders of magnitude. A comparison is made with the semi-classical theory without the iron core. For the applied current I, the 2.5 mm diameter solenoid gives a magnetic flux of $B_0 A = \mu_r \mu_0 I n A$, where $\mu_r \sim 150$ is the relative magnetic permeability of the iron core, μ_0 the permeability of free space, $n = 3/\text{mm}$ is the winding density, and $A = \pi r^2$ with $r = 1.25$ mm. For these parameters the classical time delay is indicated in the left panel by the solid line, while for the right panel $\mu_r = 1$.

The theoretical curves are close to the data when the iron core does not contribute. An experiment that is

Table 1: Comparison of experiments with our proposed experiment. See text for explanation.

Experiments	Electron energy (keV)	de Broglie wavelength (pm)	Coherence length (nm)	Phase shift ($\pi \times$ radians)	Shift (nm)	Magnetic flux (G cm^2)
Chambers [22]	20	8.7	1200	800	3500	1.7×10^{-4}
Möllenstedt [23]	40	6.1	1632	2	6.1	4.1×10^{-7}
Bayh [24]	40	6.1	1632	2	6.1	4.1×10^{-7}
Schaal [25]	50	5.5	1825	40	110	4.1×10^{-7}
Tonomura [26]	150	3.2	3200	5.5	8.8	2.4×10^{-6}
Proposed experiment	1	39	77	48000	9.4×10^5	9.9×10^{-3}

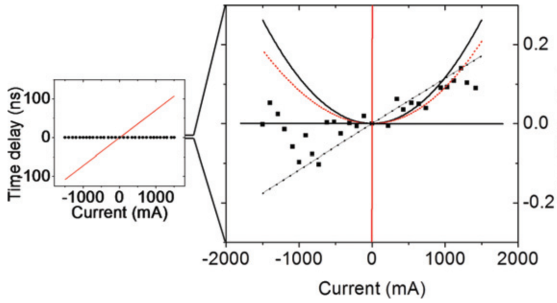

Fig. 1: (Colour online). Time-of-flight data. The left panel indicates that electrons passing by a current carrying solenoid experience a time delay (black dots) that is much smaller than the predicted classical time delay (eq. (10)) as indicated with the red solid line ($\mu_r = 150$). The right panel shows the same data with an expansion of the time scale. The horizontal black line is the generally accepted prediction, the dotted sloped line is the classical prediction without the iron core ($\mu_r = 1$), while the curves represent the analytic result (eq. (11), solid line) and the numerical result (dotted red curve) of the semi-classical theory. The relative strength between the classical (eq. (10)) and semi-classical (eq. (11)) predictions is one at a current of ~960 mA (right panel). The experimental data is not good enough to rule out any of the predictions.

similar to that of ref. [12], but with improved sensitivity (about 10 times) and without an iron core, is thus proposed to rule out the classical and semi-classical theory.

Other experiments. – It should be noted that the first experiment confirming the AB-effect, performed by Chambers [22], uses a magnetic whisker made of an iron core enclosed by the arms of an electron interferometer. If iron cores had no back-action, as considered in this paper, then the Chambers' experiment would apparently not have shown an AB-effect. However, as pointed out in Chambers' paper (attributed to Pryce), the field leakage from the magnetic whisker is exactly right to explain the observed effect in terms of a classical Lorentz force.

In the Möllenstedt experiment [23], electrons were passed by a small solenoid; no iron core was used. In this experiment, the back-action as proposed by Boyer, could explain the observed AB-effect. In Tonomura's famous experiment [26], the situation was more complicated. Magnetised toroids embedded in a super conducting field were used, and the AB-effect was observed. The Meisner effect was used to ensure that no magnetic leakage fields from the toroid could play a role. However, no model has been made of the response of the toroidal system to a passing electron and its potential back-action. Note that even though the Meisner effect shields the DC magnetic flux of the toroid, its shielding does not extend to fast pulsed fields (above the inverse plasmon frequency) as induced by the passing electron [27].

The dispersionless nature of the original magnetic AB-effect has not been observed yet. A similar test to the one discussed below has been proposed [28]. The requirement for the test is that the induced AB-phase shift, φ_{AB}, has to exceed the coherence length (in units of $2\pi/\lambda_{dB}$): $\varphi_{AB} > 2\pi L_{coh}/\lambda_{dB}$. Because the coherence length for previous experiments was typically 10^5 de Broglie wavelengths (table 1), and the induced phase shift was limited to several hundred times 2π, this requirement was never met. A comparison of parameters of several experiments and a proposed experiment that meets the above requirement is given in table 1. The proposed experiment is a modification of the Möllenstedt experiment with adjusted experimental parameters. The energy is lowered to 1 keV to decrease the coherence length, which is given by $L_{coh} = \frac{\lambda^2}{\Delta\lambda} = \frac{h}{\Delta E}\sqrt{\frac{2E}{m}}$ [29]. The magnetic flux is that of a 50 micron diameter solenoid, wound with 12 micron diameter gold wire that supports 0.1 A current. The electron interferometer with the largest beam separation ever achieved is 100 micron, which can enclose such a solenoid. Thus, the experiment is within reach of current technology.

Typically, two possible outcomes of the experiment testing the dispersionless nature are considered. A) There is no back-action in the AB-effect, and its usual interpretation is correct. In this case fringes will be observed outside the electron's coherence length. B) There is a back-action for solenoids of this type, the experiment is not an AB-effect, and fringes will not be observed outside the electron coherence length. The proposed test is generally expected to give outcome A) and demonstrate the dispersionless nature of the magnetic AB-effect [10,11,14,30]. But, what if dispersionless forces exist? In this case there is a third option C). If the time delay dt_{clas} in fig. 1 has a value giving $v\,dt_{clas} > L_{coh}$, but at a lower current where

$dt_{semi} < dt_{clas}$, then the possibility exist that $vdt_{semi} < L_{coh}$. In this case the observation of fringes rules out classical forces, but not the existence of semi-classical forces. For an experiment to rule out dispersionless forces the current must be high enough so that $vdt_{semi} > L_{coh}$. For all previous experiments (see table 1) using eq. (10) leads to $vdt_{clas} < L_{coh}$. For example, Tonomura's experiment has $vdt_{clas} \approx 2\times10^{-11}$ m $\ll L_{coh} \approx 3\times10^{-6}$ m. The proposed experiment has $vdt_{clas} \approx 10^{-6}$ m $> L_{coh} \approx 10^{-8}$ m, but using eq. (11) gives $vdt_{semi} \approx 10^{-9}$ m $< L_{coh} \approx 10^{-8}$ m. To rule out dispersionless forces, interference experiments need to be pushed to even higher enclosed magnetic fluxes.

Although the original Aharonov-Bohm effect has not been tested for its dispersionless nature, in a tour de force experiment, the scalar analogue of the AB-effect has been shown to be dispersionless [31]. Does this rule out the existence of dispersionless forces? In ref. [31], it was pointed out that these results cannot be generalised to the original electron-solenoid case. Moreover, the same question can be asked as stated above. Is there an approximately dispersionless force that could be responsible for these effects? This would require a detailed microscopic description of the interaction between both interacting constituents of the AB-effects for each case to predict the magnitude and thus test for it. Such detailed descriptions are not available in the literature, and the question whether or not dispersion forces exist can currently not be answered based on these experimental results.

Conclusion. – In the broader context of modern field theories [32], it may appear that searching for forces in the AB-effect is a philosophical throwback to classical physics. After all, local gauge invariance of potentials has become a central means by which to find the interactions between particles [33]. However, dispersionless forces do not give rise to delays or deflections, and can thus be considered quantum forces, similar in nature to the quantum forces consider by Shelankov [34], Berry [35], and Keating and Robbins [36]. The approximate nature of the dispersionless forces considered in this paper, ensures that weak but measurable classical delays do occur, and an accompanying Lorentz force interpretation is possible, if these forces are found. The usual explanation in terms of potentials of the AB-effect remains valid and the philosophical throwback does not occur, but instead new dispersionless quantum forces may be found.

In summary, two concerns in the interpretation of experiments on the Aharonov-Bohm effect are discussed. The first is the possibility that magnetised iron cores do not provide a classical back-action reducing the predicted time delay. The second is the possibility that dispersionless quantum forces exist. Both of these possibilities make the time delay experiment inconclusive. An experiment without an iron core is proposed to rule out classical forces and search for approximate dispersionless quantum forces. Finally, proposed tests of the dispersionless nature of the AB-effect can be performed in two regimes. In the first regime classical forces can be ruled out, and in the second, and harder to reach regime, approximate dispersionless forces can be tested for.

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