

2-1-1996

# Heat Transfer In Rarefied Gas At A Gas-Solid Interface

Y. Demirel

*University of Nebraska-Lincoln*, ydemirel2@unl.edu

Follow this and additional works at: <http://digitalcommons.unl.edu/chemengphysical>



Part of the [Chemical Engineering Commons](#)

---

Demirel, Y., "Heat Transfer In Rarefied Gas At A Gas-Solid Interface" (1996). *Papers in Physical Chemistry*. 6.  
<http://digitalcommons.unl.edu/chemengphysical/6>

This Article is brought to you for free and open access by the Chemical and Biomolecular Engineering Research and Publications at DigitalCommons@University of Nebraska - Lincoln. It has been accepted for inclusion in Papers in Physical Chemistry by an authorized administrator of DigitalCommons@University of Nebraska - Lincoln.

## ABSTRACT

Modified expressions for the heat-transfer fluxes at the interface of a solid surface and a gas are given for different Knudsen-number regimes. These expressions for diatomic gases contain both a gas-reflection and a thermal accommodation coefficient. Parallel-plate, coaxial-cylindrical and concentric-spherical surfaces are considered. A general expression for the thermal accommodation coefficient is based on experimental data. Sample numerical calculations of heat flux in different heat-transfer regimes are presented in terms of the reciprocal Knudsen number.

## INTRODUCTION

Heat transfer at a gas-solid interface is of practical importance in the thermal insulation of solar receivers,<sup>1</sup> evacuated window<sup>2,3</sup> and cooling a workpiece in vacuum-processing equipment. Proper formulation with account for a variety of controlling parameters has not been accomplished in spite of a large body of experimental data. Most of the pertinent experimental and theoretical literature is reviewed in Ref. 4. It is customary to examine the heat-transfer process in four distinct regimes, which are distinguished by numerical values assigned to the Knudsen number  $Kn$ , which is defined as the ratio of the mean free path  $\lambda$  of molecules to a characteristic length  $d$  that is usually taken to be the width of the gas layer, i.e.

$$Kn = \lambda/d \quad (1)$$

Where,

$$\lambda = [(\sqrt{2})n\pi\phi^2]^{-1}. \quad (2)$$

Here,  $n$  is the molecular number density and  $\phi$  the molecular diameter. The four different heat-transfer regimes are<sup>4-6</sup> the continuum regime ( $Kn < 0.001$ ), temperature jump regime ( $0.001 < Kn < 0.1$ ), intermediate or transition regime ( $0.1 < Kn < 10$ ), and free molecule regime ( $Kn > 10$ ). All four regimes are encountered for a given system as the gas pressure is reduced. The heat-transfer process is quantitatively described in terms of a coefficient of thermal accommodation  $\alpha$ , which is a measure of the efficiency of energy exchange between a gas stream and a solid surface, viz.,

$$\alpha = (E_r - E_i) / (E_s - E_i) \quad (3)$$

Here,  $E$ , represents the energy brought to the solid surface in unit time by the incident gas stream,  $E_r$  denotes the energy carried away by these molecules as they leave the surface after reflection, and  $E_s$  is the energy which the reflected stream would carry away if it had the same mean energy per molecule as a stream issuing from a gas in thermal equilibrium at the surface temperature  $T_s$ . If there is no accumulation of gas molecules at the solid surface and the energies are proportional to the corresponding gas stream temperatures, then

$$\alpha = (T_r - T_i) / (T_s - T_i). \quad (4)$$

Here,  $T_r$  and  $T_i$  are the temperatures of the reflected and incident gas streams, respectively.

In the next section, we present expressions for the computation of heat-transfer fluxes by employing a new procedure for computing the mean gas temperature while characterizing the surface by a reflection coefficient  $\sigma$  as well as  $\alpha$ . Parallel-plate, cylindrical and spherical surfaces are considered.

## HEAT-TRANSFER EXPRESSIONS

The thermal-energy flux in the free-molecule heat-conduction regime is<sup>6,7</sup>

$$Q_{FM} = \left[ \frac{1}{\alpha_1} + \left( \frac{r_1}{r_2} \right)^b \left( \frac{1}{\alpha_2} - 1 \right) \right]^{-1} \frac{P[C_v + (R/2)]}{(2\pi RT^* M_g)} (T_{s1} - T_{s2}). \quad (5)$$

Here,  $b$  is 0 for parallel plates, 1 for coaxial cylinders, and 2 for concentric spheres.  $T_{s1}$  and  $T_{s2}$  denote the temperatures of the surfaces of radii  $r_1$  and  $r_2$ , respectively, where  $r_2 > r_1$ ,  $P$  is the gas pressure,  $R$  the molar gas constant,  $C_v$  the molar heat capacity at constant volume,  $M_g$  the molecular weight of the gas,  $a_1$  and  $a_2$  refer to the bounding surfaces 1 and 2, respectively, and  $T^*$  is the effective gas temperature. Equation (5) is based on the assumption that gas molecules are diffusely reflected from the two surfaces. Sparrow and Kinney<sup>8</sup> characterized the surface not only by  $a$  but also by a reflection coefficient  $\sigma$ . The latter describes the directional distribution of gas molecules reflected at a surface;  $\sigma = 1$  denotes complete diffuse reflection.

$T^*$  in Eq. (5) may be computed from the relation<sup>8,10,11</sup>

$$Q_{FM} = \left[ \frac{1}{\alpha_1} + \left( \frac{r_1}{r_2} \right)^b \left( \frac{1}{\alpha_2} - 1 \right) \right]^{-1} \frac{P[C_v + (R/2)]}{(2\pi RT^* M_g)} (T_{s1} - T_{s2}). \quad (6)$$

$T_{r1}^*$  is obtained from

$$T_{r1}^* = \frac{[1 - (1 - \sigma_1)(1 - \sigma_2)]\{\alpha_1[1 - (1 - \alpha_2)(1 - \sigma_1)]T_{s1} + \alpha_2(\sigma_1 - \alpha_1)T_{s2}\}}{\sigma_1\{[1 - (1 - \alpha_2)(1 - \sigma_1)][1 - (1 - \alpha_1)(1 - \sigma_2)] - (\sigma_1 - \alpha_1)(\sigma_2 - \alpha_2)\}}. \quad (7)$$

$T_{r2}^*$  is obtained from Eq. (7) by interchanging the subscripts 1 and 2.

For parallel-plate geometry, Springer<sup>6</sup> gives the following expression for the heat flux  $Q_{TR}$  in the transition regime:

$$\frac{Q_{TR}}{Q_{FM}} = \left[ 1 + \frac{4B(d)}{15(\lambda)} \alpha_m \right]. \quad (8)$$

Here,  $B = 45/38$  for a diatomic gas,  $d$  is the thickness of gas layer and  $\alpha_m$  the mean value of  $\alpha$  which may be computed after Thomas<sup>9</sup> from the expression

$$\frac{\alpha_m}{2 - \alpha_m} = \frac{\alpha_1 \alpha_2}{\alpha_2 + \alpha_1(1 - \alpha_2)}. \quad (9)$$

Corresponding to Eq. (8), the expressions for coaxial cylinders and concentric spheres may be given in general form for diatomic gases by analogy with similar results obtained in the temperature jump regime, viz.,

$$\frac{Q_{TR}}{Q_{FM}} = \left[ 1 + \frac{4B}{15} \left( \frac{G}{\lambda} \right) \alpha_1 \right]^{-1} . \quad (10)$$

Here,  $G = d$  for parallel plates,  $G = r_1$ ,  $\ln(r_2/r_1)$  for concentric cylinders and  $G = r_1[1-(r_1/r_2)]$  for concentric spheres. Rearranging the results of the temperature-jump regime given by Kennard<sup>7</sup> and Springer<sup>6</sup> we write

$$\frac{Q_{TJ}}{Q_{FM}} = \left\{ \left[ \frac{1}{\alpha_1} + \left( \frac{r_1}{r_2} \right)^n \left( \frac{1}{\alpha_2} - 1 \right) \right]^{-1} \left( \frac{4}{15} B \frac{G}{\lambda} + \frac{1}{2} \left[ \frac{2-\alpha_1}{\alpha_1} + \left( \frac{r_1}{r_2} \right)^n \frac{2-\alpha_2}{\alpha_2} \right] \right) \right\}^{-1} . \quad (11)$$

Equation (11) is based on the assumption that the molecular flux arriving at the surface can be characterized by the distribution function of the gas away from the solid surface.<sup>8,17</sup>

The nature of the solid surface, as well as molecular weights and temperatures of the gas and the solid, may influence  $a$  for engineering surfaces.<sup>4,12</sup> Song and Yovanovich<sup>12</sup> analyzed the available data of  $a$  for a number of monatomic and polyatomic gases on different metal surfaces at temperatures in the range 273-1250 K and developed the correlation

$$\alpha = F \left( \frac{M_g}{6.8 + M_g} \right) + (1-F) \left( \frac{2.4\mu}{(1+\mu)^2} \right) , \quad (12)$$

Where  $\mu$  is the ratio of molecular weight of the gas to that of the solid and  $F$  stands for the fractional coverage of the adsorption layer which depends upon the nature of the surface.<sup>4</sup> Song and Yovanovich<sup>2</sup> assumed that  $F$  is determined primarily by the surface temperature. The nature of the surface, however, is characterized<sup>8</sup> by the thermal accommodation coefficient  $a$  and the reflection coefficient  $\sigma$ , which depend on the surface and reflected gas temperatures. Hence, a more general correlation for  $F$  would be<sup>10</sup>

$$F = \exp\{C(T_s - 273)/273\} . \quad (13)$$

Where  $C$  is a parameter related to the nature of the surface. With this modification,  $a$  becomes an implicit function of surface and gas temperatures. Therefore, to calculate  $a$  from Eqs. (12) and (13), an iterative procedure which is detailed by Demirel and Saxena<sup>10</sup> may be adopted.

## RESULTS AND DISCUSSIONS

Twelve different sets of  $a$  for the system nitrogen-tungsten ( $N_2$ -W) have been reported in the literature and tabulated by Saxena and Joshi<sup>4</sup>. These set data are shown in Fig. 1. Some overlapping data have been omitted to avoid overcrowding and only data up to 1200 K are considered. Discrepancies between data from different workers at the same temperature are large in several cases, probably due to different conditioning of solid surfaces. The constant  $C$  of Eq. (13) is adjusted to accommodate different solid

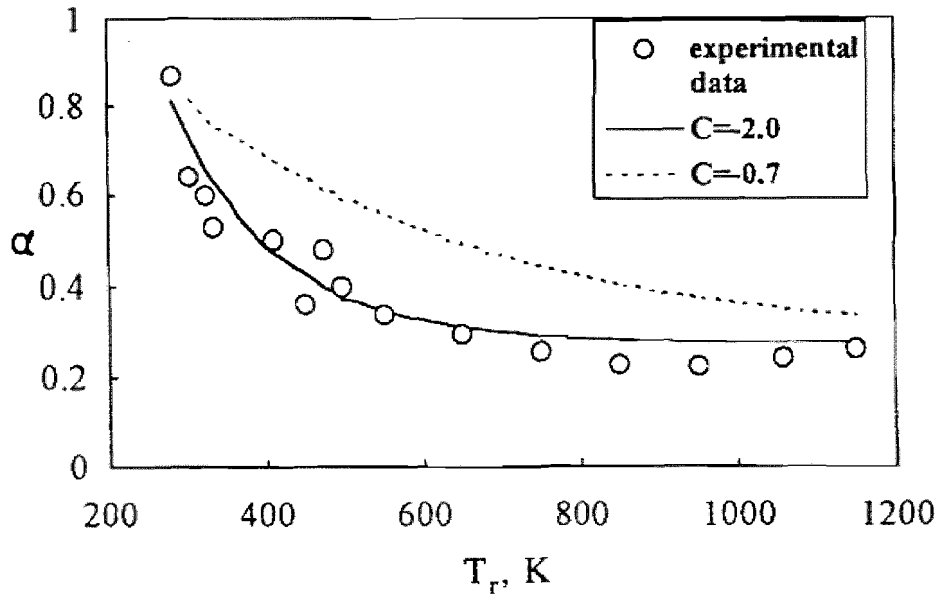


Fig. 1. Comparison between predicted and experimental values of thermal accommodation coefficients  $a$  for the system nitrogen-tungsten ( $N_2$ -W).

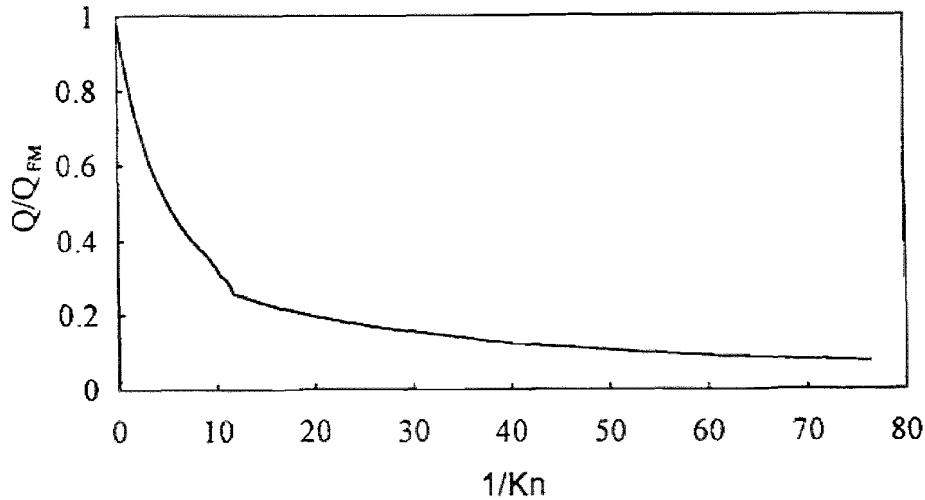


Fig. 2. Variation of  $Q/Q_{FM}$  for the parallel-plate geometry with reciprocal of the Knudsen number:  $Q = Q_{TR}$  for  $0.1 < 1/Kn < 10$ ,  $Q = Q_{TJ}$  for  $10 < 1/Kn < 1000$

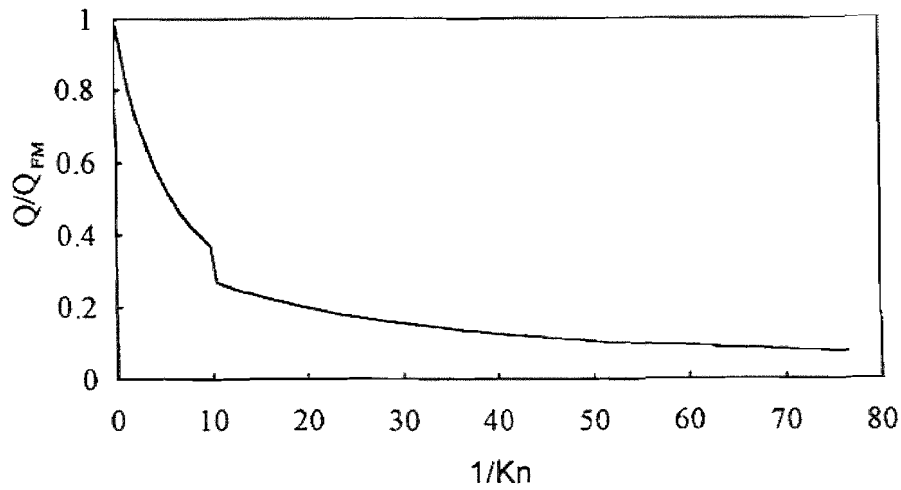


Fig.3. Variation of  $Q/Q_{FM}$  for coaxial cylinders with reciprocal of the Knudsen number:  $Q = Q_{TR}$  for  $0.1 < 1/Kn < 10$ ,  $Q = Q_{TJ}$  for  $10 < 1/Kn < 1000$

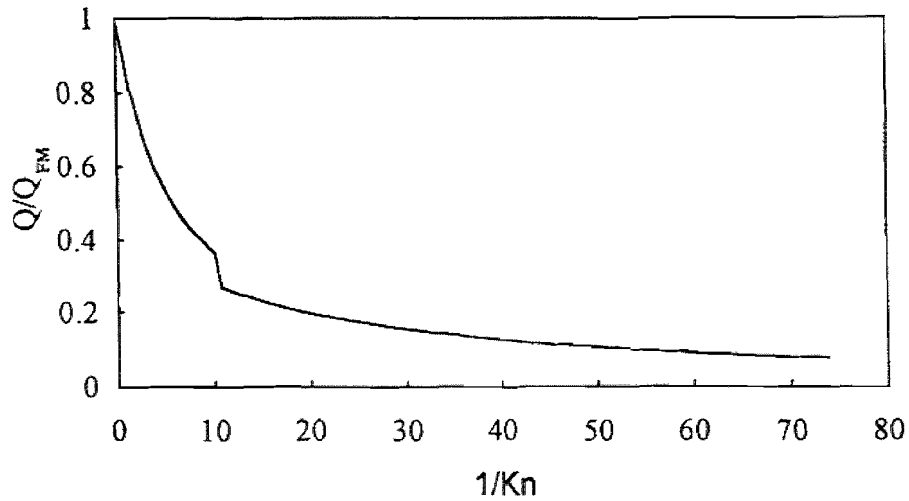


Fig.3. Variation of  $Q/Q_{FM}$  for concentric shapes with reciprocal of the Knudsen number:  $Q = Q_{TR}$  for  $0.1 < 1/Kn < 10$ ,  $Q = Q_{TJ}$  for  $10 < 1/Kn < 1000$

surface conditions. In Fig. 1, values of  $a$  for<sup>10</sup>  $C = -0.7$  and  $C = -2.0$  are shown. The latter correlates the experimental data adequately, especially their temperature dependence. Uncertainty of experimental values does not warrant fine tuning of the surface parameter  $C$ . Its sensitivity to  $a$  is evident from Fig. 1. In our further calculations,  $C = -2.0$  is employed.

Computed values of  $Q/Q_{FM}$  as a function of  $1/Kn$  are given for parallel plates in Fig. 2 and for the other two geometries in Figs. 3 and 4.  $Q = Q_{TR}$  in the transition regime is bounded by  $1/Kn$  values between 0.1 and 10;  $Q = Q_{TJ}$  in the temperature-jump regime lies between  $1/Kn$  values of 10 and 1000.

In the computations, the reflection coefficient has been assumed to be 0.8, whereas the distance between the surfaces is 0.0001 m. For all three geometries, the variations are steep in the transition regime and gradual in the temperature-jump regime. These qualitative trends provide significant guides in choosing the system dimensions for a preset value of heat loss or for estimating thermal losses from a given system.

## ACKNOWLEDGEMENT

The authors are grateful for the support provided by King Fahd University of Petroleum & Minerals

## REFERENCES

1. S. J. O'Shea and R. E. Collins, *Int. J. Heat Mass Transfer* **35**, 3431 (1992).
2. R. E. Collins and S. J. Robinson, *Sol. Energy* **47**, 27 (1991).
3. M. Sieradzki. *Nuclear Instruments and Methods in Physics Research B6*, 237 (1985).
4. S. C. Saxena and R. Joshi, *Thermal Accommodation and Adsorption Coefficients of Gases*, Y. S. Touloukian and C. Y. Ho. eds., pp. 295-298, McGraw-Hill, New York, NY (1981).
5. F. M. Devienne, "Low Density Heat Transfer" in *Advances in Heat Transfer*, J. P. Hartnett and T. F. Irvine, Jr., eds., Vol. 2. pp. 271-356, Academic Press, New York, NY (1965).
6. G. S. Springer. "Heat Transfer in Rarefied Gases", in *Advances in Heat Transfer*, J. P. Hartnett and T. F. Irvine, Jr... eds... Vol. 7, pp. 163-218, Academic Press, New York, NY (1971).
7. E. H. Kennard. *Kinetic Theory of Gases*, McGraw-Hill, New York, NY (1938).
8. E. M. Sparrow and R. B. Kinney, *Phys. Fluids* **7**, 473 (1964).
9. J. R. Thomas, "Transition Regime Heat Conduction: Influence of Unequal Surface Accommodation," Paper no. 214, *Twelfth Int. Symp. on Rarefied Gas Dynamics*, pp. 363-370, Charlottesville, VA (1980).
10. Y. Demirel and S. C. Saxena, *Int. J. Energy Res.*, **19** (1995).
11. R. J. Corruccini, *Vacuum VII & VIII* **19**, 19 (1958).
12. S. Song and M. M. Yovanovich, "Correlation of Thermal Accommodation Coefficient for Engineering Surfaces" in *Fundamentals of Conduction and Recent Developments in Contact Resistance*, M. Imber, G. P. Peterson, and M. M. Yovanovich, eds., Vol. 69, pp. 107-116, ASME, United Engineering Center, New York, NY (1987).



## NOMENCLATURE

- $A$  = Area ( $\text{m}^2$ )  
 $b$  = A constant in Eq. (5)  
 $B$  = 45/38 for diatomic gas  
 $C$  = A parameter in equation (13)  
 $C_v$  = Molar heat capacity at constant volume ( $\text{J}\cdot\text{mol}^{-1}\cdot\text{K}^{-1}$ )  
 $d$  = Characteristic length (m)  
 $E$  = Energy flux ( $\text{W}\cdot\text{m}^{-2}$ )  
 $F$  = Fraction of the surface covered by adsorption layer  
 $G$  = Parameter used in Eq. (10)  
 $\text{Kn}$  = Knudsen number  
 $M$  = Molecular weight of the gas ( $\text{kg}\cdot\text{kmol}^{-1}$ )  
 $n$  = Number of molecules per unit volume  
 $P$  = Gas pressure (Pa)  
 $Q$  = Heat flux ( $\text{W}\cdot\text{m}^{-2}$ )  
 $R$  = Molar gas constant ( $\text{m}^3\cdot\text{Pa}\cdot\text{mol}^{-1}\cdot\text{K}^{-1}$ )  
 $r_1, r_2$  = Radius of the outer and inner cylindrical or spherical surface, respectively (m)  
  
 $T, T^*$  = Effective gas temperature (K)

### *Greek symbols*

- $\alpha$  = Thermal accommodation coefficient  
 $\varphi$  = Diameter of molecule (m)  
 $\lambda$  = Mean free path (m)  
 $\mu$  = Ratio of molecular weights ( $M_g/M_s$ )  
 $\rho$  = Gas density ( $\text{kg}\cdot\text{m}^{-3}$ )  
 $\sigma$  = Surface reflectivity coefficient

### *Subscripts*

- FM = Free-molecule regime  
g = Gas  
i = Incident  
m = Mean  
r = Reflected  
TR = Transition regime  
s = Solid  
1,2 = Surfaces 1 and 2, respectively