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Writing in the Mathematics Classroom: Does It Have an Effect on Students' Mathematical Reasoning?

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Math in the Middle Institute Partnership
Action Research Project Report

In partial fulfillment of the MA Degree
Department of Teaching, Learning, and Teacher Education
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Writing in the math Classroom: Does it have an effect on student's mathematical reasoning?

Abstract

In this action research study of my classroom of 5th grade mathematics, I investigate how to improve students' written explanations to and reasoning of math problems. For this, I look at journal writing, dialogue, and collaborative grouping and its effects on students' conceptual understanding of the mathematics. In particular, I look at its effects on students' written explanations to various math problems throughout the semester. Throughout the study students worked on math problems in cooperative groups and then shared their solutions with classmates. Along with this I focus on the dialogue that occurred during these interactions and whether and how it moved students to a deeper level of conceptual understanding. Students also wrote responses about their learning in a weekly math journal. The purpose of this journal is two-fold. One is to have students write out their ideas. Second, is for me to provide the students with feedback on their responses. My research reveals that the integration of collaborative grouping, journaling, and active dialogue between students and teacher helps students develop a deeper understanding of mathematics concepts as well as an increase in their confidence as problem solvers. The use of journaling, dialogue, and collaborative grouping reveals themselves as promising learning tasks that can be integrated in a mathematics curriculum that seeks to cultivate students' thinking and reasoning.

Introduction

I am interested in improving students' written explanations in the math classroom. Writing is often taught in depth in language arts classes, but not as much emphasis is placed on writing in the math classroom. Students need to learn how to express their thoughts mathematically in words. In today's math classrooms students are required to not just provide a solution but explain how they arrived at that solution. It is imperative that students are taught these skills so that they can explain their mathematical thinking clearly and coherently. I want to look at and understand how students think about what they are doing while working on math concepts, individually and with others, and how this enhances their math learning and understanding. Students struggle with explaining, in writing, their math reasoning and justification of solutions in problem solving. I chose to look at this topic more in-depth by looking at the effects of collaborative grouping, dialogue between students and teacher, as well as journal writing, and how the three play and integral part in building the students' math understanding.

In my classroom students always struggle with explaining the why and how of their thinking process. When I have asked how they came to a certain solution they often have trouble writing the steps and details needed to clearly and coherently justify their solutions. Many of the responses that I heard from them were, "I don't get what you mean?" or "I just know it, why do I have to explain what I did?" Responses like these are clues to me that my students are either unable to explain or did not effectively understand just what it means to explain how they came up with a solution. Throughout the Houghton-Mifflin (2005) math curriculum that I use, there are questions that students are to answer that relate to math reasoning and the "why" behind certain mathematics concepts. These are always harder for the students to answer in a clear and

coherent manner, and often a point of frustration to them. I believe that it is imperative that I teach them how to express these ideas, both orally and written.

I work with a group of math students who have a passion for mathematics and are identified as gifted. They catch on quickly to math concepts and skills, but have not had as much experience with explaining or justifying the why behind it all. They had not made the connection between our group discussions and how they can use what they "say" to help them write out their explanations. This alerted me to the importance of providing my students with more opportunities to share and explain their understanding through written and oral discussions.

Since being a part of Math in the Middle I have learned that how one arrives at a solution is just as important as the solution itself, maybe more so. In my mind, I have an image of an ideal classroom: students would have the confidence and skills necessary to effectively communicate their reasoning and conjectures through a detailed explanation in writing at how they arrived at their solutions. As an educator I know the importance of teaching and modeling what I expect. To become more effective in this type of communication students need many opportunities to practice this type of writing. They need to be taught how to effectively communicate to themselves as well as others what they know and understand.

In my classroom during math class I have come to feel it is important to incorporate more lessons on how to write out clear justifications to problems and also have the students reflect on their understanding and thinking through journaling. Along with this idea, I also wanted to see the students sharing their explanations with their peers and I so that as a community of learners we could offer one another constructive feedback to fine tune and perfect their justifications.

Problem of Practice

There are several reasons why I believe this problem is worth knowing more about. In elementary school math classrooms I do not think there has been as much emphasis on students

explaining their thinking processes until recently, compared to what is expected in middle and high schools. I believe it is important that students leave my fifth grade classroom with the skills and tools necessary to effectively communicate in writing what they understand, since they will be expected to do this more and more as they become older.

I have come to see this as an extremely important tool in my assessment of the students understanding. I often look at the students' work if they have not arrived at a correct answer to see where their break down of understanding occurred, something I would not be able to do if they did not show their work and justification. Students' written explanations become an important teaching tool for my future lessons and a way to quickly redirect and address any misconceptions the students may have.

In relation to the local knowledge I could gain from this research, I believe that within my own classroom my students and I would gain a more in depth understanding of what it really means to provide justification and how to effectively communicate our understanding and reasoning in mathematics. The students will learn from their peers work as well as teacher modeling of effective reasoning and justification. This will help to enrich the math curriculum that is already being taught. I try to continually challenge and stretch my students' thinking about mathematics through more challenging problem solving, such as habits of mind, and as they become more proficient problem solvers and thinkers my hope is they will become more proficient with their reasoning. I believe this is an important part of becoming a well-rounded mathematician, and will give the students more confidence in their individual problem solving abilities, because it pushes their current level of reasoning.

The knowledge that I gain from this research can be beneficial to other educators that I work with, such as other teachers within my school and district. We all work with a wide range

of learning abilities, but I believe that all students would benefit from an increased knowledge of how to explain and understand their mathematical thinking. Working and sharing ideas and problems with other colleges can be a powerful tool for improving our instruction as well as our students understanding.

This problem of practice addresses a larger important process strand from the NCTM standards, specifically that of communication: the organizing and consolidating of mathematical thinking through communication and communicating mathematical thinking clearly and coherently to peers, teachers, and others. Effective communication is an important skill to mathematics because it is a way to share the knowledge and understanding that one has built. Without this communication, math is simply a set of rules that must be memorized and are unrelated. Through addressing this particular problem of practice I hope to create not only a classroom of mathematical thinkers that can effectively communicate their ideas, but also develop a classroom of effective communicators in life.

There have been many research studies related to the ideas of effectively communicating mathematical understanding. From the research literature I notice that three overarching ideas influence students' math understanding and thinking, collaborative grouping, journaling, and open dialogue between students and teachers. The research points to all three of these as being important in developing a deeper conceptual understanding. Through my action research I am seeking to understand my efforts as an educator to enhance students' mathematical reasoning through the use of collaborative grouping, dialogue, and journaling.

Literature Review

Asking students to write about the mathematics they are learning can be like pulling teeth. Students often ask, "Why do I have to write what I'm thinking, I just know how it works", or "I don't know how to explain what I did." These are common responses in my math

classroom, and often hear from other colleagues that they too struggle with these responses from their students, but the reality is that students need to be able to explain and communicate their reasoning and thinking in today's math curriculum. I reviewed the following pieces of literature related to writing and its importance in mathematics. Throughout the research literature the following categories were addressed: effects of writing on students mathematical thinking, use of writing as an assessment and teaching tool for classroom teachers, and the effects it has on students' reasoning skills.

Landi (2001) notes that in today's mathematics' classrooms, students are expected to explore problems, hypothesize solutions, and predict outcomes. In order to prepare students for the new century, she stresses that we must teach them to be good mathematical thinkers.

The role of communication, including writing, has been addressed in recent reports. The NCTM document *Curriculum and Evaluation Standards for School Mathematics* (2000) lists "Mathematics as Communication" as one of its strands. The definition of communication by the NCTM states that students must be able to organize and consolidate their mathematical thinking through communication, communicate the mathematical thinking coherently and clearly to peers, teachers, and others, analyze and evaluate the mathematical thinking and strategies of others, and use the language of mathematics to express mathematical ideas precisely.

Research indicates that writing can be a way for students to communicate these mathematical ideas. Shield and Galbraith (1998) state that much of the research literature has paid particular attention to the use of student writing activities as part of mathematical learning. They conducted a study in which they looked at the effects of writing in the math classroom, and if it enhanced students' learning. Specifically they focused on a scheme for coding student responses for analyzing student responses. More and more mathematics courses are requiring

students to not only understand how to follow a particular procedure to solve a math problem, but also to understand why that particular procedure works. Much of the research indicates that for students to make connections and understand the mathematics deeply they must understand the conceptual side of the mathematics also. One of the ways that students can move to a more reflective stage of understanding is through the process of writing down their reasoning and mathematical thinking.

Writing in mathematics can be a tool for students to use to express their mathematical thinking. One of the ways that the research literature addressed this was through the students written explanations of math problems. These explanations can provide insight into the ideas that students have constructed. The way these ideas are developed can happen in a variety of ways such as using questioning to clarify mathematical ideas, dialogue with peers and teacher, and sharing of ideas in collaborative groups.

Albert (2000) shows that when students worked collaboratively in a group they were able to write about and make sense of the mathematics they were exploring in groups. He goes on to suggest that learning to express mathematical understandings and concepts via writing is not an isolated process. Its expression depends upon being involved in active learning situations. Mathematics does not need to be learned in rote or rigid ways. Rather, students can apply their knowledge to specific tasks or situations that involve active, constructive, and innovative practices, which can be manifested through writing (Albert, 2000, p. 137).

Some research has suggested that it is not just the act of writing, but the communication it encourages that benefits students' mathematical thinking. Porter and Masingila (2000) suggest that the real benefits of writing to learn mathematics may be influenced, not by the actual act of

writing, but to the fact that it requires students to spend time thinking about mathematical ideas and then communicating them to others.

Through the use of questioning by both the teacher and students, research has found that this particular tool can help students become more proficient writers and develop a clearer understanding of the math concepts. Albert (2000) shares that the role of the researcher in his particular study was not to simply gather information about students' problem solving understandings, but rather their role was to facilitate the learning process through questioning and offering suggestions that assisted students in making sense of the mathematics. Through this questioning students were able to reflect and think about their own individual math understanding and in turn improve their written explanations.

The research has also addressed the importance of having students share their thinking with others to help develop their reasoning. Albert (2000) finds that students who worked collaboratively in groups gained more of an insight into their own mathematical thinking, which helped them, develop their written responses. Each student in the research group went from collective practices to independent practices in which they were able to write and make sense of the mathematics explored in their group. Through the related research writing has been viewed as a catalyst to improving and evoking more thoughtful and meaningful responses to mathematical ideas.

Writing has also been found to influence the teaching and learning that is happening in the math classroom. Teachers can gain a deeper insight into their students' math misconceptions and understandings of math concepts through the evaluation and interpretation of students written responses to various math problems.

In a study conducted by Burton (1984), students' responses were looked at closely paying attention to manipulation of ideas, sense of pattern, and articulation of problem. Burton described the movement between these areas as a student coming to a conclusion. He goes on to state that offering students overt opportunities for specializing, generalizing, conjecturing, and convincing enables the thinker to encounter aspects of his or her own thinking more deeply. Burton suggests that being aware of the operations of mathematical thinking helps both teachers and pupils to recognize their own power in thinking about mathematical experiences.

In a similar study, Shield and Galbraith (1998) examined students who were required to write two different forms of expository text in a mathematics classroom. They had to write a letter to a friend who was absent, to explain something about the mathematics that had been learned recently. Along with this, students also were required to write how they would explain a particular math task to a student who was having difficulty. Both these tasks required students to explain not only the procedures involved but also the meaning behind it.

The students' responses were evaluated using a scheme developed by Clarke, Waywood, and Stephens (1993). This coding scheme looked at three main categories in the students writing: recount, summary, and dialogue. After looking at individual student responses and coding each part of the students explanation, Shield and Galbraith (1998) conclude that it is reasonable to propose that a student who is able to write a generalized statement of a procedure, can demonstrate the procedure, can link the procedure to prior knowledge, and can justify the use of the procedure, has an effective understanding of that mathematical procedure. Being able to identify and name the parts of a student's written justification could enable teachers to assist students in writing more elaborate presentations, which stimulate deeper thought and understanding of the mathematical ideas being written about.

Along with this idea, Moska and Magone (2001) found that written tasks that are designed to allow student to choose their own mode of response, can give special insight into the students' thinking and reasoning. Moska and Magone refer to modes in this study as the manner in which information is communicated. Examples of these are pictorial representations, such as diagrams, written solutions or solutions shared orally in class, and the use of formal mathematical symbols in mathematical solutions.

Research suggests that careful evaluation of students' justifications can provide classroom teachers with an assessment to further develop students' written explanations and improve classroom instruction. Moska and Magone (2001) suggest that knowledge of students' appropriate and inappropriate relationships that are common to their different referent systems (related number systems) can help assist teachers in structuring instruction to advance students' knowledge of the particular math concept. Recognition of the appropriate relationships that students have established allows teachers to build upon their students' current knowledge.

Prior research (Musk, 1997) suggests that teachers do not naturally attend to the constructs of referent, relationships and modes when examining student responses. Instead, teachers often seek to make sense of the relationships that their students have established, but may not understand the basis of the misconception. This is likely to make it difficult for the teacher to select an appropriate instructional intervention.

Journal writing is a useful tool to gain insight into students deeper understanding and reasoning of mathematics concepts. In a study done on the effectiveness of journal writing to improve students' mathematical writing, Clarke, Waywood, and Stephens (1993) stated that regular monitoring of journals could inform teaching practice and provide the basis for discussion with students individually or with the class as a whole. They believe that the teacher

plays a critical role in helping students take control over their learning. As Clarke, Waywood and Stephens go on to explain:

> Getting students to articulate their own thinking at the point where they are coming to terms with a new idea, or meeting difficulty, is essential to helping many to move into the more reflective mode of writing, characterized as "dialogue." At the heart of the progression, *which a journal-writing program seeks to facilitate,* is the shift from a summary mode in which the focus is the compilation of cognitive knowledge and skills to the dialogue mode involving the acquisitions and utilization of metacognitive skills. The key appears to be to encourage students to question themselves when they do not understand rather than be dependent upon their teachers to tell them whether they understand (p. 248, emphasis added).

Teachers, then, must have a basis for evaluating students' written responses so that they can make use of the information these written explanations can give. There needs to be a clear set of expectations and formats.

Flores (2006) suggests that teachers need to understand the ways students use to justify so that those methods can be built upon to help students develop their ability to provide convincing arguments in mathematics. Flores states that it is important for students to develop methods to justify on their own correctness of a procedure or the truth of a fact. Without these methods, students will not be able to know when they are wrong.

Along with this idea, Evens and Houssart (2004) suggest that teachers need to assist children to express what they already know with more precision. The teachers can help students build on the initial answers that children provide. They can do this through discussion with peers and the teacher to see that examples are insufficient and some other form of justification is

required. If their justification is incomplete students may be helped by questions, which encourage them to improve their reasoning.

It is clear that there is a wealth of information that teachers can receive from students written explanations if they are looking for the right elements. Throughout much of the literature authors claim that writing about mathematics helped students to reflect on their own understanding and improve their reasoning and justifications. Clarke, Waywood, and Stephens (1993), for instance, claim that their research suggests that through the process of their journal writing students increasingly interpret mathematics in person terms: constructing meanings and make connections. Through these journal entries students began to question what they were doing, and showed increasing confidence in using their own words to link ideas. Through their writing they begin to show that they were actively constructing mathematical ideas (Clark, Waywood, & Stephens, 1993).

Other research suggests that how the traditional math classroom is set up needs to change so that students can learn in an environment that will fully help them develop accurate justifications. A study by Shield and Galbraith (1998) challenges the claim that the use of writing tasks in mathematics has promoted a deeper understanding of mathematical ideas. They believe that so far there has been little evidence to support this claim. They believe that a major shift in teaching practices and textbooks needs to come about so that students can be exposed to more proficient levels of written explanations.

Albert (2000) conducted a study of students writing based on Vygotsky's idea of the "zone of proximal development (1978)." Albert's study uses the idea that students' learning occurs when the students are involved with tasks or problems that go beyond their immediate individual capabilities in which teachers assist their performance, or in collaboration with more

knowledgeable peers. He suggests that from this stage students could move into a new zone, the zone of proximal practice (ZPP). In this zone students enter a new area of self-regulation and they begin to construct new meaning on their own. From the results of this study, Albert states that this study supports the notion that writing assists in the development of students' individual thought processes and helps students construct knowledge about mathematical ideas and concepts.

Flores (2006) states that students need to get used to questioning and explaining their own answers so that, gradually, they will think about what is presented to them and not just learn the new content with blind faith. It is through questioning and explaining that students further their own mathematical knowledge and become more confident in their own abilities.

Much of the research has shown how beneficial writing can be on students math reasoning. It helps move them from a dependent thinker to an independent thinker who is able to construct ideas and make sense of the mathematics concepts they are learning. One idea that came across clearly in the various literature pieces is the benefit that writing has on student's mathematical reasoning and communication of concepts. Flores (2006) states that the practice of justifying in class will help students remember what they are supposed to learn by highlighting what the important ideas in a lesson are, the concepts on which the exercises and computations are based, and the connections of these concepts with previously learned concepts.

Along with this idea, Albert (2000) discusses the importance for students to think about what they are doing, how they are doing it, and why they are doing it. One of the ways that students can focus on these ideas is through their written explanations. Written explanations afford the teacher and the student the ability to communicate clearly and coherently with one another. Much of the research states that more research needs to be conducted to create a broader

picture of how writing can improve and influence student's conceptual understanding of the math concepts being taught. Through my research I hope to fill in some of the gaps that exist in the current research and show the importance of using writing in a math program to help not only the students, but also the benefits it holds for the instructor.

Research Purpose

The purpose of my action research is to evaluate the effect writing has on students' mathematical thinking. In this inquiry I am seeking to gain a better understanding of my own teaching of mathematics, whether and how my efforts to enhance students mathematical reasoning through the use of collaborative grouping, dialogue, and journaling has any meaningful consequences for student and their learning mathematics.

As I seek to understand writings effects in the math classroom, I will have students reflect, explain, and summarize the concepts they are learning in mathematics, to improve and build upon the students' reasoning and justification skills. I believe that this research will be beneficial not only to me as an educator, but will help others in the education field understand the importance of having students communicate their mathematical thinking to increase their conceptual understanding of mathematics.

I am seeking to make sense out of the use of collaborative group discussion and sharing, journaling, and dialogue influence students' conceptual understanding and reasoning. I want to learn how I can effectively share with my students how writing their thoughts down will help them move to a more independent level of learning and give them more control over their own understanding. In turn I would also like to learn more effective methods for teaching my students how to write a quality response that really reflects what they are thinking, and can clearly be understood by others.

I examine the variables of students' conceptual understanding of math concepts as well as the quality of students written responses in seeking to answer the following research questions:

* How does journaling influence students' conceptual understanding of mathematical concepts and the quality of students' written responses?

*What is the influence of dialogue and collaborative grouping upon students' mathematical solutions?

*How does oral communication between students and teachers support students' reflective responses?

While each of these research questions is singly challenging, I am interested in how all three shape in a student's math reasoning. In my practice, these are not easily separated; there is always a co-occurrence of written and oral communication. I will show that it is through collaboration with peers and teachers as well as oral discussion that help to move students to more reflective thoughtful responses to math problems.

Methods

The data that I collected throughout the semester to help me pursue my research questions include students' class work, pre-tests and posttests, student journal responses, teacher observations, and student interviews.

My original plan to conduct student interviews was to hold an interview with students during the first week of my action research, one mid way through, and one at the end. My thoughts are to use the data to show how the students' ideas about writing in mathematics have changed, from the beginning of the study to the end. I did not receive my IRB permission letter to conduct interviews until mid March, and that next week our school had spring break. Coincidently I was not able to use an interview in the beginning or middle of the study. Subsequently I chose instead to use the data from interviews as a tool to look at how students

viewed writing in mathematics, and how it helped them improve their understanding and written solutions to various math problems.

I had a fellow colleague randomly assign students in groups of four for me to interview. I then had her choose four of the groups for me to conduct and interview with. For two of the groups I asked them questions (See Appendix A) relating to their overall ideas and feelings about writing in mathematics and how it has influenced their general understanding of the math concepts they had learned about. With the other two groups I asked questions (see Appendix B) specifically about a math problem that they had worked on together, to evaluate their work as a group, and how it helped them understand the problem at a deeper level. I recorded each of the interviews and afterwards composed notes about what I had learned from the students' responses. I then transcribed each of the interviews to get better insights into the themes that were emerging.

Throughout each math lesson in our Houghton Mifflin math curriculum there are questions that ask students to explain their thinking. I chose to use these during each math lesson as an opportunity for students to reflect back on the concept they had learned that day, and also for another form of data I could use to assess students understanding.

Along with these questions I also collected samples of students class work, both individual and group related. The group work I collected was from three habits of mind problems that the students worked on. I have included examples of the three problems in the appendices section (See Appendices E & F). The students often worked in small groups during daily lessons, but I wanted to have three instances in which they specifically worked on a problem, which would require much discussion and thinking.

The daily classwork varied depending on the week and lesson content, some lessons were more rote practice of skills and did not require as much explanation. I collected work between two to three times per week. I assessed the students' work using a 1- 4 grading scale rubric (See Appendix B) when written explanations were required. The classwork gave a glimpse into what math concepts the students learned about that day, and was useful in determining if they had a clear understanding of that days content. For work requiring skill practice I took percentages based on the number of correct answers.

During the time period of my action research students completed five chapter tests, in which I gave a pretest and posttest for each chapter. The chapters and topics students covered were, chapter 15-*Geometry,* 16-*Perimeter&Area,* 17/18-*Ratio,* and 19-*Percents,* and chapter 20-*Probability*. I recorded a percentage grade for each of the pre tests given, and used the district math rubric to grade the chapter tests. This rubric also was on a 1-4 scale, 1 meaning they did not meet district objectives and 4 meaning they exceeded district objectives. I stated earlier that this particular group of fifth graders was a high achieving gifted group, and prior to conducting this research most were receiving scores of 3 or 4 on their chapter test. While I did notice a definite increase in understanding from the pre to post test, I did not feel that the tests were a large indicator as to whether the writing students were doing was increasing their conceptual understanding.

For journaling data, I had students respond to journal prompts in a math journal (See Appendix C). Students wrote in their journals approximately two times per week. This differed from my original plan. I had wanted to have the students write daily in their journals, but daily life in my classroom did not always allow this to happen. Some factors included lessons going longer than expected, students not understanding and needing more explicit instruction, and

some students not finishing their responses. I had the students write in their math journals as part of the closure to the lesson at the end of math class, usually the last 10 minutes, or at the beginning of class the next day. I used the 4-point rubric described earlier to evaluate the quality of the students' responses.

One of the difficulties in conducting the research is lack of time. This is something all classroom teachers struggle with throughout the year. There are always other responsibilities and duties to think about. The main obstacle to time for me was lack of class time to collect data, and its influence on the flow of the research. The months of March and April held many obstacles to this. In March we had a week off for spring break, and I was out of the class on March 23 for school improvement, and we had a snow day on March 1. In April the following days of math class were lost due to plan days, MAT testing, and school programs (April 11, April 16-19). I lost a total of 12 days of instruction over the course of my research project, which meant no data to collect for those days as well as loss of flow in journal writing. It was difficult to establish a routine of journal writing. Time had to be spent playing "catch up" rather than focusing on research objectives.

Along with the student journals, I also kept a weekly journal of what was happening in the math classroom. I used the template provided to us during the first week of our class (see Appendix D). I used this journal to record thoughts and observations about what had been happening in the math classroom that week and how this related to my research questions. I would write ideas down each day and at the end of the week picked two or three of them to describe in more detail.

Findings

When starting my research I had sought to find how writing in mathematics could further develop students' conceptual understanding, and get them thinking more about the "why" behind

the mathematics. I chose to develop this understanding by incorporating journaling, dialogue, and cooperative grouping to help with students' written explanations.

As a result of incorporating journaling I believe students began to think more about the math concepts and their own understanding of it. This was evident in some of the students' responses from an interview I conducted on journaling. Some of the students felt that journaling helped them think about what they were doing in relation to the mathematics. The following student comments show evidence of this.

> It helps me, because you may understand some things about the chapter, but journaling that makes you think more about what your doing and how you get the answer. Wendy[1]

> You might have just memorized how to do the problem and might not know exactly why you did it and then if you journal about it you won't forget how to do it. Jane

As an educator, hearing these comments was very powerful. They showed me that the students were making connections between the concepts they were learning about in the classroom and how journaling helped them clarify these concepts.

In my journal for the week of March 5th, I wrote that, "Students seem to be more aware of how writing in mathematics helps them think about what they are doing rather that just memorizing how to perform a specific procedure." Several of the students' comments from a small group interview also provide evidence of their new thought processes.

> Yeah, because first we just learned how to do the problem then that was pretty much all we knew and in journaling we learned more about how to explain it instead of just doing the problem. Jon

> It's more stuck in your mind after you journal about it. Ann

The use of journals also provided valuable evidence for me of students understanding. I observed that students who had trouble explaining their thinking in a journal response, also had

[1] All names are pseudonyms

difficulty understanding the concept. After looking at some of the data from the students' class assignments I ran across a particular problem that 7 out of my 25 students had struggled with. I chose to discuss this problem more in depth with this group of students to help them understand the problem better. The students were to respond to the following question: *Is ½ *0.4 equal to 2/5 *0.5? Explain why or why not.*

The following two responses from Hannah and Matt were given a 1 on the 4-point rubric due to lack of support or evidence.

"They are equal, it's just a flip flop problem."

"Yes, because they are switched around."

Incidentally, these students, who had trouble giving a clear explanation, also did not do well on their assignment connected to multiplying decimals. I chose to sit down with these students to find out where their break down of understanding occurred. Through some reteaching of decimals and fractions and questions between the students and myself, they came to see what they were confused about, and saw the importance of explaining their steps clearly to show what they meant. After our discussion I asked them to try to write up a new explanation to this problem. The following is what each turned in:

"Yes it is because ½=. 5, and 2/5=. 4"

"Equal because ½ =5/10=0.5 and 0.4=4/10=2/5. It's two of the same problem, but written differently."

The two students both came back with a much clearer explanation of their thoughts.

Looking closely at students' explanations also proved to be a powerful indicator to me that if the students could not clearly explain their solution, they probably did not have a deep understanding of the problem.

Through my classroom observations and journaling I noticed another theme emerging. Students were more accountable for their own learning when they had to journal about a specific topic. When starting this study in February, a majority of the students lacked confidence in their abilities to explain their thinking. They felt as though writing down what they did was just a waste of time. I wrote the following in my first journal entry from February 20, *"Students seem unsure of abilities to explain their ideas, their confidence needs to be built up. With practice and encouragement I am hoping this will change."*

Towards the end of the study I witnessed more confidence coming from the students. I observed that they were more willing to take risks by getting up in front of the class to explain their solutions. I wrote in my April journal that, *"I am seeing an increased confidence in the students writing abilities. I am no longer hearing, what do you mean explain? I now have students coming up and asking if they need to add more to their explanation. They are becoming more self sufficient in their learning."*

Students were taking charge of their own learning through their journal questions. One of the areas students could add to in their journal response was examples and questions they had about the math concept. Throughout various responses, some of the students wrote, "I still don't understand this part," and "This makes sense to me now!" A comment written by a student in her journal provides evidence of this. Hannah wrote about what she had learned in our chapter on geometry:

> In chapter 15 I learned how to classify figures. It's still a little fuzzy, but I improved the most on that. Another thing I improved on and learned was how to use tick marks to show congruent sides of a figure.

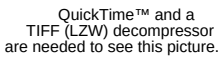
This shows she was aware of some of the topics we had covered, but was not quite confident in the deeper understanding of it. It is also a useful tool for me as an educator, because I was able to go back to her after reading it, and help to clarify some of what she did not yet clearly understand about classifying polygons.

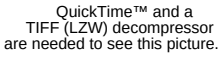
I found that through the use of collaborative grouping students became more confident and skilled in their mathematical thinking which improved their written explanations. Throughout the study I had students work in groups of two to four to solve various math problems. Some of these problems were story problems relating the days math concept and others were habits of mind problems to challenge and extend the students mathematical understanding. I found through these activities that students were more confident and effective problem solvers when they could interact in a group setting to discuss their thoughts and ideas. A piece of evidence that supports how collaborative grouping improved student written explanations was from a small groups response to the "Pouring with Pails" problem (See Appendix F). This group's written explanation was enhanced when they shared their work in small group and whole group. Students were asked to come up with a solution to the following problem:

> *How can you bring up from the river exactly six quarts of water when you have two containers, a nine-quart pail and a four-quart pail, to measure with?*

Some of the groups at first were very frustrated, I heard "I don't get it!" and "This is too hard, I can't do it." When I first heard these responses I tried to find a way to shift the students' negative approaches to a more positive approach. I chose to share some of the frustrations I felt when working on similar problems in my math in the middle experiences. I stressed to the students that it was important for them to try, and that if their solution wasn't correct that was o.k. and that

they might get some clarification from others in their group. This seemed to put the students at ease, and move them into the problem. They now had a place to begin their discussion. I suggested that each group member try something and then come back together to discuss what he or she had come up with. The following is an example of what one of the group's solutions following our discussion.





From this dialogue with the students they began some great discussions in which they began to come up with some very valuable information and a solution to the problem. This dialogue helped give the groups somewhere to start. Sometimes just getting the students to attempt a problem is half the battle.

During the student interviews I asked questions about how working in a group and sharing solutions with the class helped students write out an effective solution. The majority of

responses that I received showed again how group sharing was an important factor in increasing students understanding. The following are some of the common responses students gave during the interviews.

> Yes, I think it's helpful because you can learn what others are thinking and see if it's at all what your thinking, and it will help you come up with more solutions to a problem if there are any. Ross
>
> It helps me because when you're stuck on the problem and somebody has a couple different answers we can try to combine our answers to get a better answer or explanation. Kevin
>
> It gives you other people to count on, not just yourself. It's not just your problem and you're the only one who has to figure it out. People in a group they can help too. Ann
>
> If you don't get how to do it and the group members do they'll probably help you learn how to do it, or how they got their answers. Jon

These responses show evidence that students felt less pressure when working in a group, they felt as though all the responsibility was not just on them, and that if they were struggling they had others to rely on for help.

The last theme that emerged from the data was that questioning improved students understanding. I noted in my journal on April 30 that, *"Students are more willing to ask questions in class when they don't understand something."* This particular observation came about during a review before a test. Students wanted to make sure they understood the concepts taught in chapter 19 on ratio, before going on to the test. They began asking thoughtful questions about specific concepts from the chapter that they did not understand; such as what is the difference between rates versus proportion, and how do you find a rate using cross products? They were asking for further clarification before the test rather than just saying I don't get it and accepting defeat. This was not the case for many of these students at the beginning of the year. Many of them felt that if you had to ask a question you were "dumb."

Throughout my study I used questioning to help guide students thinking to help move them to a deeper understanding of the math concepts they were learning about. From this I found that thoughtful questioning helps guide students' mathematical thinking without giving them the answer. One piece of evidence to support this came from student work on a habits of mind problem entitled the locker problem. Students were to come up with a solution to the following problem.

> *Imagine a high school with 1000 numbered, closed lockers, one for each of its students. Suppose all the students line up and go through the locker are to perform a specific task. Student #1 opens all the lockers; student 32 closes every even-numbered locker. Student #3 "reverses" every third locker (if open, he closed; if closed, he opens). Each student goes through, reversing the lockers that correspond to his or her position in line. After the 1000th student passes through, which lockers will be open? Solve this problem by looking at a simpler case. Consider a school with 20 students and 20 lockers. Complete the chart to discover which lockers will remain open. (Use Foundation, 2005)*

Students were given a chart to simulate the 20 lockers and 20 students (See Appendix E). Looking over my observation notes from this problem, I noted that asking the following questions helped students use the information they had found to answer the question. The following questions were used to guide the students thinking.

"What important information about squares did I share with you last week?"

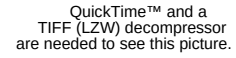
"Can you use what you learned about perfect squares to help you?"

After students had discovered that out of 20 lockers the following were left open, 1, 4, 9, 16, I asked them to think about these questions.

"What is special about those numbers?"

"Can you use this to help find a pattern?"

Through questioning students were able to effectively come up with accurate solutions to the locker problem and write out a clear explanation of their solutions. The following is an example of Mike, Nate, Riley, and Joel's solution.



Through this group's explanation they clearly show that they understood what the problem was asking and wrote a clear explanation of their steps. The group's explanation provided evidence for me of their learning. I was able to see that the students understood the problem and clearly could support their response. By asking the students' questions throughout the math class I was able to move the students from a more dependent level of thinking to an independent level in which they were able to come to a conclusion on their own in groups. They were no longer reliant on my feedback, and were able to use their own thoughts to produce a valid solution.

I used questioning in the students' journals as well to further students' understanding. In particular I used questions for students to further clarify their thoughts and written explanations. After teaching a lesson on congruency, I had the students write in their journals what new concepts had they learned about today. Almost all students stated that they learned that for a figure to be classified as congruent it had to be the same shape and size. Along with this students

also stated that they learned how to use "tick" marks. About 90% of the students did not explain what a tick mark was and how those tick marks show a figure is congruent.

I wrote the following question in each student's journal, *"What do the tick marks show you, and how do you use them?"* I then followed up with these questions in the next day's lesson, and had students shared their new explanations.

"Tick marks show which sides and angles are congruent to one another."

"You can use tick marks to show if one figure is congruent to another figure."

Through this oral form of questioning students were able to improve their written explanations of what they learned, as well as verify that they really knew what the tick marks stood for.

Students also stated during student interviews that questioning was beneficial when working on explanations to math problems. Here is an example of what the majority of students stated when asked the following questions.

What do you learn by watching and listening to others explanations of math problems?

"I learn different ways to solve a problem and sometimes it can help me with my explanations if we ever have to write something."

"And it just like um, like if I didn't think of doing something it gives me more ideas of how to solve something."

"Yeah, it helps me figure out the problem more easily."

What do you think about when your teacher asks questions during math class?

"How to solve it, if it's a problem how to solve it and start looking at it."

What about when you're stuck?

"They give you hints about what to do next maybe or how to do them a little bit, and help make the problem easier to understand."

"Helps you understand the problem more."

After reflecting back on what the students stated during the interview I came to the conclusion that questioning was a beneficial strategy to use with the students to help them further develop their math reasoning.

Throughout my journal I made numerous connections to how students were asking more questions during class, and wanting to know more about the "why" behind how concepts in mathematics work. I noted that, "*questioning and dialogue help students to further understand concepts, and holds them accountable for their own learning.*

Conclusions

After completing my research one thing was evident: the combination of dialogue, collaborative grouping, and journaling all had an influence on students' thinking and writing in math class. Each of them together helped move the students to a more independent level of thinking.

The most beneficial data I collected was from the student interviews. It was very eye-opening for me, because it was a verification that the strategies I was implementing in class were making a difference and the students were aware of it. From the data I found from the interviews, I thought it also may have been beneficial for me to conduct a pre- and post-survey of what the students thought about writing and journaling in mathematics before the study and after. I feel that this also would have been a powerful tool to use to answer my research questions. For instance I could use questions somewhat like those used in my student interview in more of a generic form to get a better idea of what the students thought about writing and journaling before the research took place and then compare those results to the post survey after implementing the strategies in class. This could help give some more evidence of how students' reasoning is influenced by writing in mathematics.

The answers that I found to my research questions through the research were that having students write out their explanations to math solutions was beneficial to both the students and me. The students stated this directly in the interviews that it helped the "ideas stick in their head." It was a way for the students to self assess their understanding as well as for the teacher to use as a tool for instruction. By writing down their thoughts and ideas students were more aware of their own understanding of math concepts when they were held accountable through written explanations. Students began to understand the importance of really "knowing" the concepts verses just memorizing how to perform a math procedure.

Overall, students had a much broader understanding of why asking questions, working in groups, and journaling helps them reflect and think about what they are doing rather than just accepting it because that is what the book or teacher tells them. The research I reviewed discussed journaling, use of dialogue, and group collaboration as effective tools for improving students written explanations and thinking about mathematics, but very few of them discussed the importance of incorporating all three. My current study helps to support others findings as well as add a new "twist" and way of thinking to use all three together to intertwine the workings of an effective mathematics classroom.

Implications

As a result of this study, I plan to continue using journaling in my math class next year. Starting out the year with journaling would help set the expectations of writing in math class for the students, as well as give them ample opportunities to refine and improve their explanations. I found this to be a powerful tool for my teaching as well, so I could use this to guide my instruction in the class. It could alert me immediately to problems students were having with certain concepts, so I could clarify the ideas right away before moving on to a new lesson.

I also plan to conduct interviews periodically throughout the school year to “hear” from the students what they are learning. This was such a powerful learning tool for me as an educator. So often we are focused on scores to tell us what the students are learning, but the students themselves hold great information for us, because they can tell us in their own words what is or is not working in our instruction. As a teacher I consistently feel as if I am being pulled in a million directions and before conducting this research would have laughed at someone if they suggested taking time to sit down and talk with the kids about what they were learning. As a result of my study I personally have found the value and importance in it. I have again been reminded of how teachers learn as much from their students, as students learn from their teachers. We are a team who together can achieve great things. Having students write up clear mathematical solutions will also be a focus of my instruction next year. I plan to continue to use collaborative groups and dialogue to help move students to more reflective responses.

One thing I did not do during my research that I would like to include next year is more benchmark examples to show students what I consider a quality response, and how it might look different in certain situations. This may help take away some of the anxiety that students feel when first asked to “explain” their thinking. They would have a concrete example of what a quality response looks like and help to take away some of the mystery behind explaining their thinking.

My colleagues who teach fifth grade and I will also be working together next year on researching ways to further improve students mathematical understanding, in particular with problem solving. We plan to find research related to problem solving and discuss how we can use these ideas in our classroom. My colleagues have also showed an interest in adding journaling and more collaborative grouping into their math classrooms since hearing about my

action research. I have learned many valuable pieces of information throughout this research, and am excited for the opportunity to expand and improve upon this valuable information.

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Appendix A

Student interview questions

1. What do you learn by watching & listening to others explanations of math problems?
2. What does it look like when you justify your answers on a homework assignment?
3. What are some benefits of justifying your answers on math homework, if any?
4. What do you think about when your teacher asks questions during math class?
5. How does working in group help you justify your solutions?
6. Do you think hearing others solutions is helpful?
7. Do you think it is important to explain your mathematical thinking to a math problem, why or why not?
8. Do you think journaling has helped you become a better writer of mathematics?
9. How does journaling help you express your thoughts and feelings?
10. Do questions raised by the teacher and your peers; help you when you are stuck on how to explain your thinking?

Interview questions related to a specific math problem

11. Could you explain this problem to a student who was absent from math class?
12. How would you explain what you know right now?
13. Is the solution reasonable, how do you know?
14. What made you think that was what you should do?
15. Why did you decide to organize your results like that?
16. Does anyone have the same answer but a different way to explain it?

Appendix B

Student journal prompts

What we did

What we learned

Examples and questions

Teacher journal prompts

Possible journal topics:	Description of 1-2 of the possibilities:
Monday	
Tuesday	
Wednesday	
Thursday	
Friday	

Reflection questions:

1. How does each of the two
Incidents I wrote about relate to my research questions?

2. What changes have I seen in my students this week?

3. What did I learn this week that will inform my teaching and /or journaling next week?

Appendix C

Math solution rubric

4- Solution exceeds expectations. Extensive explanation & support. Uses both words and pictures

3- Solution meets expectations. Explanation is clear and includes valid support. Uses words or pictures.

2- Solution almost meets expectations
writing has some explanation, but no valid support

1- Solution does not meet expectations
incorrect solution, no valid explanation or support

Appendix D

Math journal rubric

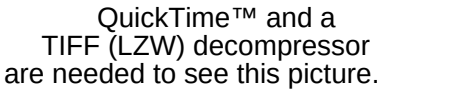
4- understanding is very evident

3 – there is some evidence of understanding

2- very little evidence of understanding

1- no evidence

Appendix E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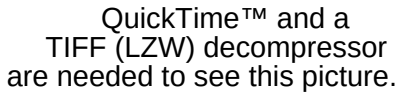
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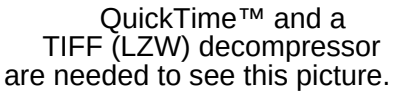
Appendix F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