1975

DIRECT-CURRENT CIRCUITS

DIRECT-CURRENT CIRCUITS

INTRODUCTION

One way to help you understand a new phenomenon is to show you that it is like something that you are already familiar with. This method is used very frequently in physics, e.g., the electric field is like the gravitational field. This module will introduce you to a simple class of RC circuits in which there are currents, charges, and voltages that decay exponentially. This may be your first detailed study of exponential decay, but it is like (analagous to) radioactive decay, Newton's law of cooling, the final depletion of a natural resource, the decrease in atmospheric pressure with altitude, and some other interesting phenomena. With a sign change, it is like a simple model of exponential growth, which is how population, energy consumption, and pollution generation seem to be growing. The module begins, however, with a few simple ideas applied to direct-current circuits. These are the basic ideas upon which you will later build an understanding of alternating-current circuits. This course will not cover electronics, but it will provide some of the introductory concepts that are needed for a study of electronic devices and circuits.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Apply Ohm's law to find potential difference from current and resistance (needed for Objective 1 of this module)	Ohm's Law Module
*Find potential difference of a capacitor from charge and capacitance (needed for Objective 4 of this module)	Capacitors Module
*Find the power developed in a resistor (needed for Objective 1 of this module)	Ohm's Law Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Loop equation - Analyze a single-loop direct-current (dc) circuit consisting of resistances and a seat of emf to find the loop current, the power developed in the circuit elements, and the terminal potential difference of the seat of emf.

2. Equivalent resistance - Determine an equivalent resistance for a series or parallel combination of resistances.

3. RC-loop equation - Write the differential equation for a single RC loop, and verify that particular assumed solutions satisfy this equation.

4. Exponential decay - Write the equation for the current, charge or voltage of a capacitor as a function of time in a single RC loop, and manipulate this equation to determine the value of one of the parameters when an appropriate set of other values is given.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Study the text, Chapter 21, Sections 21.8 and 21.10, and Chapter 22, Sections 22.1, 22.2, 22.3, and 22.4. Then read General Comment 1. Study Problems A and B of this study guide; then work Problem F and Problems 2, 4, 7, and 9 of Chapter 22 in your text. Study Sections 21.11 and 21.12 of the text; then read General Comment 2. Study Problems C, D, and E and work Problems G and H. Then work Problems 18, 19, 20, and 21 in Chapter 21.

When you think you have mastered the four learning objectives, take the Practice Test. If you need more help, work the Additional Problems before taking a Mastery Test.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 21.8 to 21.10, 22.2, General Comment 1	A	Illus.[a] 22.1	F	Chap. 22, Prob. 9	Chap. 22, Probs. 11, 14
2	Sec. 22.1, General Comment 2	B		F	Chap. 22, Probs. 2, 4, 7	Chap. 22, Probs. 1, 3, 5, 6, 20
3	Secs. 21.11, 21.12	C		G		
4	Secs. 21.11, 21.12	D, E		H	Chap. 21, Probs. 18, 19, 20, 21	I, J

[a]Illus. = Illustration(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Study Chapter 28, Sections 28-1 through 28-5 in the text. Then read General Comment 1 of this study guide. Study Problems A and B of this study guide before working Problems F and 5, 6, 7, and 20 of Chapter 28. Then study Section 28-6 and read General Comment 2. Study Problems C, D, and E before working Problems G and H and 32, 35, 36, 37 in Chapter 28.

Try the Practice Test, and work some of the Additional Problems if you have any difficulty, before taking a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text (Chap. 28)	(Chap. 28)
1	Secs. 28-1 to 28-5, General Comment 1	A	Ex.[a] 2, 3	F	5, 6, 7	1, 2, 3, 4, 8, 9, 10, 11, 12, 13, 14
2	Secs. 28-3, 28-5, General Comment 2	B	Ex. 1, 4	F	16, 20	15, 17
3	Sec. 28-6	C		G		
4	Sec. 28-6	D, E	Ex. 5	H	32, 35 36, 37	31, 33, 34, 38, 39, 40, 41; I, J

[a]Ex. = Example(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Study Chapter 28, Sections 28-5, 28-6, 28-7, and Chapter 29, Sections 29-1 through 29-6. Then read General Comment 1 of this study guide. Study Problems A and B and the Examples in Sections 28-5, 28-7, and 29-1 before working Problem F in this study guide and Problems 28-14, 28-15, 28-23, 28-24, 29-3, 29-4, and 29-6 in the text. Next study Section 29-7 (especially the Example), General Comment 2, and Problems C, D, and E before working Problems G, H, I, J, and 29-35.

Try the Practice Test, and work some of the Additional Problems if necessary, before taking a Mastery Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 28-5, 28-7, General Comment 1	A	Sec. 28-5, Ex.[a] 1 to 5; Sec. 28-7, Ex. 1 to 4	F	28-14, 28-15, 28-23, 28-24	28-16, 28-17, 28-18, 28-20, 28-21, 28-22
2	Sec. 29-1, General Comment 2	B	Sec. 29-1, Ex.	F	29-3, 29-4, 29-6	29-1, 29-2, 29-5, 29-7, 29-8, 29-9, 29-10, 29-11
3	Sec. 29-7	C		G		
4	Sec. 29-7	D, E	Sec. 29-7, Ex.	H, I, J	29-35	29-33, 29-34

[a]Ex. = Example(s).

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

Study Chapter 28, Sections 28-1 through 28-5, and read General Comment 1. Study Problems A and B before working Problem F in this study guide and Problems 28-1 through 28-6 and 28-10 in your text. Next study Section 27-7, and read General Comment 2. Study Problems C, D, and E. Then work Problems G, H, I, and J and 27-24.

Try the Practice Test and work some Additional Problems if necessary, before taking a Mastery Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 28-1, 28-2, 28-4, 28-5, General Comment 1	A	Ex.[a] 28-1	F	28-1 to 28-5	
2	Sec. 28-3, General Comment 2	B	Ex. 28-2	F	28-6, 28-10	28-7, 28-8, 28-9
3	Sec. 27-7	C		G		
4	Sec. 27-7	D, E		H, I J	27-24	27-25, 27-26

[a]Ex. = Example(s).

GENERAL COMMENTS

1. The Loop Equation

The suggested procedure begins with a discussion of a seat of emf and the distinction between an emf and a potential difference. Next, the loop equation is introduced and a sign convention established so that the equation can be applied to particular circuits. The loop equation goes under a variety of aliases - loop theorem, Kirchhoff's loop rule, etc., and is an application of the conservation of energy. The main difficulty that you may have in applying the loop equation will probably be in correctly using the sign convention. There are four different directions (polarities) that are involved, and you need to be sure that you understand these differences among them. First, there is the direction of an emf which is independent of the direction of any current flowing though the seat of emf and may be represented by an arrow o⟶ and/or the battery symbol +| |−. The second direction is the direction of the potential difference across a resistor or between two points in a circuit. Note that $V_{ab} = V_a - V_b = -(V_b - V_a) = -V_{ba}$, and that the potential drop across a resistor does depend on the direction of current flow through the resistor. The terminal of a resistor at which the current enters is at a higher potential than the terminal from which it leaves. The third direction is that in which the current flows in a particular circuit element. Clearly, you must know the current direction in a resistor before you can determine the sign of the potential difference across a resistor, and just as clearly you frequently do not know the current directions at the beginning of solving a problem. The paradox is only apparent; simply assume a direction for each current and any wrong assumptions will simply result in a negative value for each wrong assumption. Be sure, however, to mark your assumptions on a circuit diagram so that you will use it consistently. The fourth direction is the one you adopt to traverse a loop in applying the loop equation. This direction is arbitrary and is not usually indicated on the circuit diagram. To make the opposite choice, simply change the sign of each term in the loop equation. If you do not understand the distinctions among these directions, return to your study of the text and examples until you do.

Your text discusses a number of applications of the loop equation, including many cases of two connected loops. These circuits add another equation that results from the conservation of charge. You should study these sections as applications of the loop equation, but you will not be tested on multiple-loop circuits requiring the solution of simultaneous equations. This exception does not include series and parallel resistance combinations, which you will be expected to reduce to a single equivalent resistance. The important thing to remember, both for deriving the expressions for equivalent resistance and for applying the expressions to simplify a circuit, is that a set of resistors is in series if the same current flows through each of them, and it is in parallel when the same potential difference appears across them.

2. Application of the Loop Equation

In this section we will consider a further application of the loop equation. When the loop equation is applied to a circuit with resistors and capacitors, the resulting equation is not simply an algebraic equation, but is a differential equation that has an implicit time dependence as a result of the interdependence of the charge (Q) and current (I = dQ/dt). The solution of this differential equation is not a number but a time-dependent function. In this course, we will not deal with the mathematics of solving such equations, but we will learn the general form of the solution and then check the solution to see if it satisfies the differential equation. "Satisfies" means that when we substitute all pertinent derivatives of the proposed solution into the differential equation we get an identity or an expression that can be made into an identity by renaming some constants, and we thus have verified the solution. This procedure was used in discussing traveling waves and in discussing simple harmonic motion.

The solutions of this module for RC circuits are of two types, one for charging a capacitor, the other for discharging a capacitor, respectively:

$$Q = \mathcal{E}C(1 - e^{-t/RC}) \quad \text{and} \quad Q = Q_0 e^{-t/RC}.$$

Each of these can be directly related to the voltage on a capacitor:

$$V_C = \mathcal{E}(1 - e^{-t/RC}) \quad \text{and} \quad V_C = V_0 e^{-t/RC}.$$

Both can also be related to a decreasing current

$$I = \frac{dQ}{dt} = \left(\frac{V_0}{R}\right)e^{-t/RC} \quad \text{and} \quad I = \frac{dQ}{dt} = -\left(\frac{V_0}{R}\right)e^{-t/RC}.$$

PROBLEM SET WITH SOLUTIONS

A(1). In the circuit shown in Figure 1 $\mathcal{E}_1$ = 8.0 V, $\mathcal{E}_2$ = 6.0 V, r_1 = 1.00 Ω, r_2 = 2.00 Ω, and R = 4.0 Ω. Find the terminal voltage and the output power of each battery and the heat generated in the 4.0-Ω resistor. (The resistances r_1 and r_2 represent the internal resistances of $\mathcal{E}_1$ and $\mathcal{E}_2$.

Solution

To begin the problem we need to find the current that will flow in the loop. This can be done by applying the loop equation starting at a and proceeding clockwise, noting that both emfs are in the positive sense as we traverse the loop and that we have assumed the current direction shown in the figure:

$$\mathcal{E}_2 - Ir_2 + \mathcal{E}_1 - Ir_1 - IR = 0.$$

This equation can be solved for the current:

$$I = \frac{\mathcal{E}_1 + \mathcal{E}_2}{r_1 + r_2 + R} = \frac{8.0 + 6.0}{2.00 + 1.00 + 4.0} = 2.00\ \text{A}.$$

Any real battery consists of a seat of emf and an internal resistance, and we cannot have access to these separately. The terminal voltage of $\mathcal{E}_2$ is the potential difference between a and b:

$$V_{ba} = V_b - V_a = \mathcal{E}_2 - Ir_2 = 6.0 - (2.00)(2.00) = 2.00\ \text{V}.$$

Similarly,

$$V_{dc} = V_d - V_c = \mathcal{E}_1 - Ir_1 = 8.0 - (2.00)(1.00) = 6.0\ \text{V}.$$

The power delivered to the external circuit is just the product VI, hence

$$P_1 = (6.0)(2.00) = 12.0\ \text{W}, \qquad P_2 = (2.00)(2.00) = 4.0\ \text{W},$$

and the heat generated in the 4.0-Ω resistor is

$$P = I^2R = (2.00)^2(4.0) = 16.0\ \text{W},$$

which is conveniently the sum of the power supplied by the two batteries. Note also that emf $\mathcal{E}_2$ is supplying $\mathcal{E}_2 I = (6.0)(2.00) = 12.0$ W, but $(2.00)^2(2.00) =$ 8.0 W is generated as heat within the battery.

Figure 1

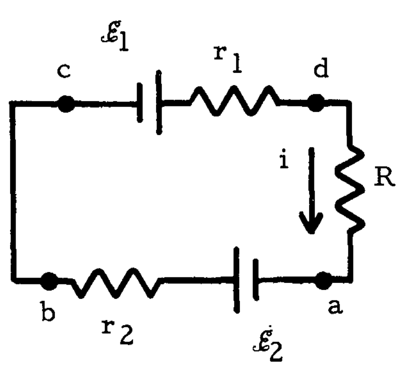
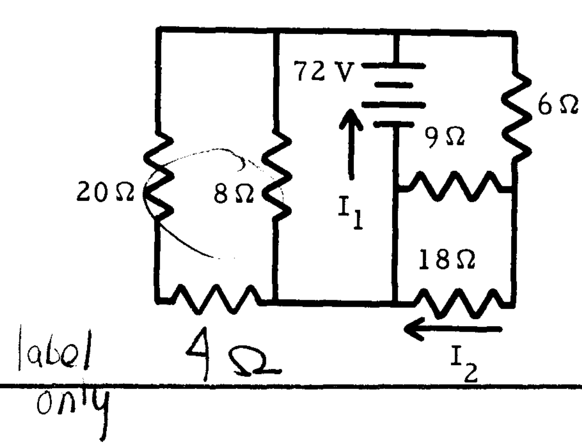


Figure 2



B(1, 2). For the circuit shown in Figure 2 find the equivalent resistance of the resistance network, and find currents I_1 and I_2.

Solution

Such network problems frequently appear more imposing than they really are. To solve this problem begin by combining any series or parallel combinations that you can identify and redraw the circuit as you go. Remember that the connecting lines represent wires with negligible resistance and can be rearranged any way that has an appearance that you like and preserves the electrical connections. This is not a general method, and there are resistance networks that cannot be reduced by these simple series-parallel equations.

You should find that the resistors to the left of the battery reduce to 6.0 Ω and those to the right to 12.0 Ω; these in turn are in parallel and equivalent to one resistor of 4.0 Ω. Thus, I_1 = 72/4.0 = 18.0 A. The parallel combination of 9.0 Ω and 18.0 Ω is equivalent to 6.0 Ω, which is in series with the original 6.0-Ω resistor across the 72-V emf. Thus the potential difference across the 9.0-Ω and 18.0-Ω parallel combination is 36 V, and the current I_2 = 36/18.0 = 2.00 A.

C(3). For the circuit shown in Figure 3 the loop equation gives $\mathcal{E} - IR - q/C = 0$, which with $I = dQ/dt$ leads to a differential equation: $\mathcal{E} = R(dQ/dt) + Q/C$. Show that $Q = K(1 - e^{-t/RC})$ is a solution.

Solution

To verify a solution we substitute it into the differential equation. If an identity results, the expression is a solution. First we find the derivative

$$dQ/dt = +K(1/RC)e^{-t/RC}$$

and substitute it and the expression for Q into the differential equation

$$\mathcal{E} = R[(K/RC)e^{-t/RC}] + (K/C)(1 - e^{-t/RC});$$

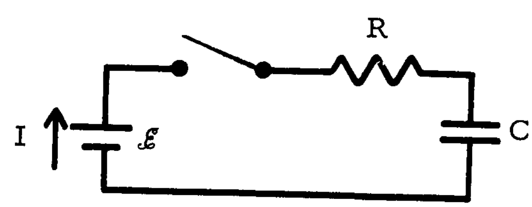
then

$$\mathcal{E} = (K/C)e^{-t/RC} + K/C - (K/C)e^{-t/RC},$$

which will be an identity if $K = \mathcal{E}C$. Thus,

$$Q = \mathcal{E}C(1 - e^{-t/RC})$$ is a solution.

Figure 3



D(4). For the circuit shown in Figure 4 find the voltage across the capacitor at 1.20 s after the switch is closed.

Solution

From the previous example the charge on the capacitor at any time is

$$q = \mathcal{E}C(1 - e^{-t/RC}).$$

Since the voltage on the capacitor is $V_c = q/C$,

$$V_c = (\mathcal{E}C/C)(1 - e^{-t/RC}) = (100\ V)(1 - e^{-t/0.60\ s}),$$

where the time constant $RC = (3.00 \times 10^5)(2.00 \times 10^{-6}) = 0.60$ s. At $t = 1.20$ s,

$$V_c = (100\ V)(1 - e^{-1.20/0.60\ s}) = (100\ V)(1 - 1/e^2) = 8.6\ V.$$

E(4). For the circuit shown in Figure 5 the switch was placed in position a for a long time and then quickly moved to position b; 12.0 s later the voltage across the capacitor was 1.00 V. Find the value of the capacitor.

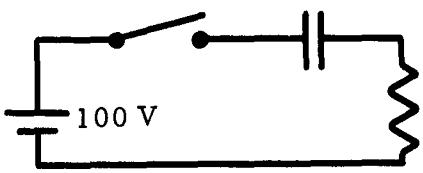


Figure 4

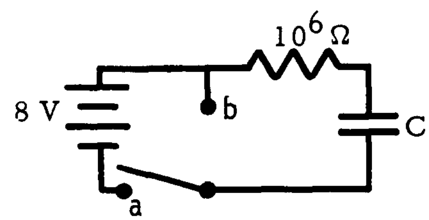


Figure 5

Solution

From the circuit we can conclude that the time constants for charging and discharging are about the same. Thus we can assume the capacitor was fully charged to 8.0 V when the switch was moved to b and we can write

$$V_c = \mathcal{E}e^{-t/RC} = 8.0e^{-t/10^6C},$$

and for the problem values

$$1.00 = (8.0)e^{-12.0/10^6C}.$$

To evaluate C we take the natural logarithm of this expression. Rewriting it slightly first we have

$$e^{-12.0/10^6 C} = 0.125, \quad -12.0/10^6 C = \ln 0.125,$$

$$C = -12/(10^6 \ln 0.125) = 5.8 \times 10^{-6}\ \text{F} = 5.8\ \mu\text{F}.$$

With a scientific calculator this evaluation is straightforward. However, for purposes of approximation many physicists like to estimate exponentials in terms of half-lives, and we note here that the reduction from 8.0 to 1.00 V represents the passing of three half-lives:

$$8 \xrightarrow{t_{1/2}} 4 \xrightarrow{t_{1/2}} 2 \xrightarrow{t_{1/2}} 1.$$

Thus the half-life is 4.0 s in this problem. For the expression $V = V_0 e^{-t/RC}$, we take

$$V = V_0/2 \quad \text{at } t = t_{1/2}, \quad V_0/2 = V_0 e^{-t_{1/2}/RC}$$

Again taking natural logarithms of both sides, we find

$$\ln(1/2) = -t_{1/2}/RC \quad \text{and} \quad t_{1/2} = RC \ln 2.0 = 0.69RC.$$

Thus in this case $RC = 4.0/0.69$ s and $C = 4/(10^6)(0.69) = 5.8 \times 10^{-6}$ F.

Problems

F(1, 2). For the circuit shown in Figure 6 find the current I, and the power dissipated in the 17.0-Ω resistor.

G(3). For the circuit shown in Figure 7 write the differential equation for the voltage on the capacitor, if the voltage was V_0 at $t = 0$. Show that $V = V_0 e^{-t/RC}$ is a solution for your differential equation.

Figure 6

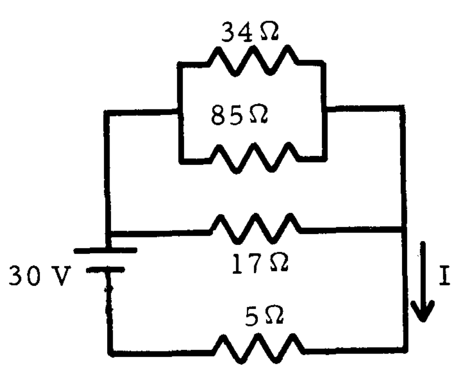
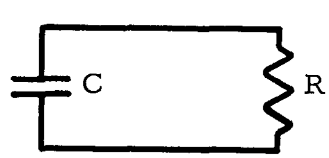


Figure 7



H(4). The switch in Figure 8 is left in position a for 2.00 h, then moved to position b.

(a) What is the charge in the capacitor 12.0 s after this is done?

(b) Two hours later, it is moved back to position a. What is the charge after 12.0 s now?

I(4). What is the current from the battery in Figure 9 5.0 s after the switch is closed?

J(4). Note that the circuit shown in Figure 10 can be analyzed in terms of series-parallel combinations of resistors, plus a capacitor.

(a) When the switch is first closed, the currents in various parts of the circuit change with time, as the capacitor charges up. But after a few seconds, the currents settle down to quite steady values. What is the steady current being drawn from the battery?

(b) What is then the potential across the capacitor?

(c) The switch is now opened. What is the potential across the capacitor after 30.0 ms?

Solutions

F(1, 2). I = 2.00 A. $P_{17.0}$ = 23.5 W.

G(3). R(dQ/dt) + Q/C = 0. Q = VC = $V_0 e^{-t/RC}$.

H(4). (a) 22.0 μC. (b) 38 μC.

I(4). 28.0 μA.

J(4). (a) 0.300 A. (b) 18.0 V. (c) 12.0 V.

Figure 8

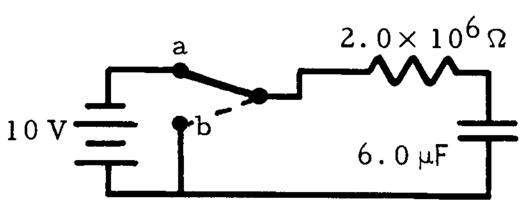


Figure 9

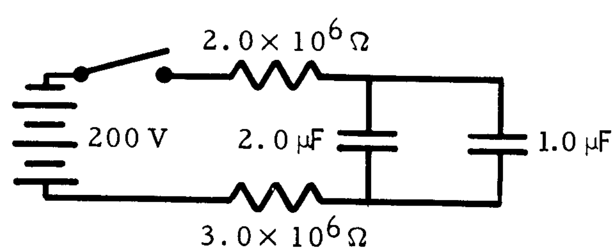
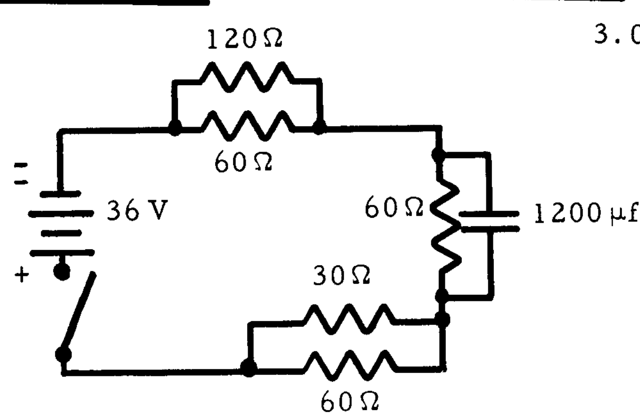


Figure 10



PRACTICE TEST

1. (a) For the circuit shown in Figure 11 find the current in the 5.0-Ω resistor.

   (b) Find the terminal voltage of the 24.0-V battery, V_{ab} (emf = 24.0 V; internal resistance = 1.00 Ω).

2. It has been suggested that an appropriate differential equation for the circuit shown in Figure 12 is $RQ = \mathcal{E} - (1/C)(dQ/dt)$. If this is true with the switch closed, show that $Q = Q_0(1 - e^{-t/RC})$ is a solution; if it is not true, correct the equation, and then show that $Q = Q_0(1 - e^{-t/RC})$ is a solution.

3. A capacitor is charged to 10.0 V and then connected in series with a resistor of 10^7 Ω. After 100 s the voltage on the capacitor is 2.50 V. Find the value of the capacitor.

Figure 11

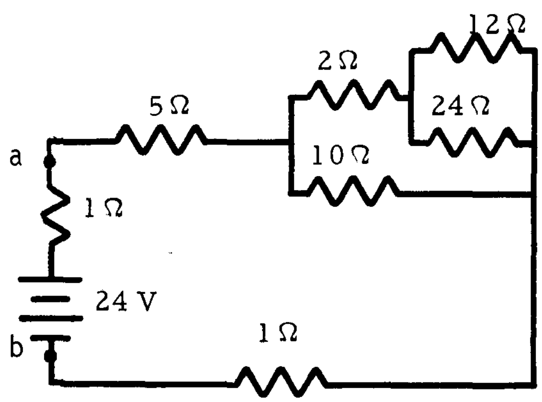
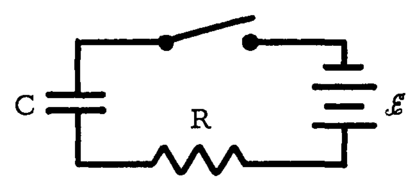


Figure 12



Practice Test Answers

1. (a) 2.00 A. (b) 22.0 V.
2. The Q and dQ/dt in the differential equation should be exchanged.
3. 7.2 μF.

DIRECT-CURRENT CIRCUITS

Mastery Test Form A

Name ______________________

Date ______________

pass recycle

1 2 3 4

Tutor ______________

1. For the circuit shown in Figure 1:

   (a) Find the current in the 10.0-Ω resistor.

   (b) Find the potential difference V_{ab}.

   (c) Find the total power generated in the 12.0- and 24.0-Ω resistors together.

Figure 1

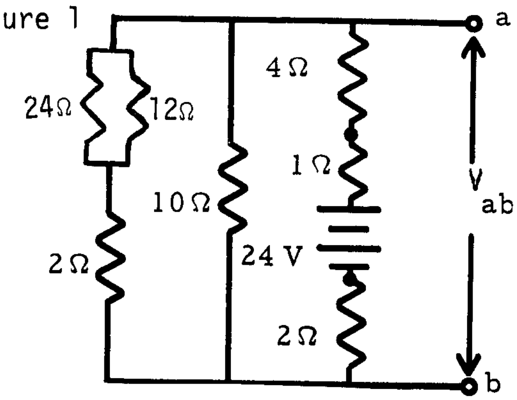
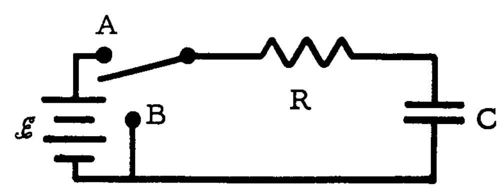


Figure 2



2. The circuit shown in Figure 2 had the switch in position A for a long time. At t = 0 it was moved to position B.

   (a) Is the following a correct differential equation for t > 0:

   $\mathcal{E} - R(dQ/dt) - Q/C = 0$?

   (b) If your answer was "yes," show that $Q = \mathcal{E}Ce^{-t/RC}$ is a solution. If your answer was "no," make the appropriate changes and show that $Q = \mathcal{E}Ce^{-t/RC}$ is a solution.

3. A 100-V battery, an uncharged 200-μF capacitor, and a 5.0×10^5 Ω resistor are connected in series at t = 0. Find the potential difference across the capacitor at t = 200 s.

DIRECT-CURRENT CIRCUITS

Mastery Test   Form B

Name ______________________

Date ______________________

pass   recycle

1   2   3   4

Tutor ______________________

1. For the circuit shown in Figure 1:

   (a) Find the current in the 1.00-Ω resistor.

   (b) Find the terminal voltage V_2 ($\mathcal{E}$ = 20.0 V, internal resistance 0.50 Ω).

   (c) Find the power supplied to the external circuit from terminals of V_2.

Figure 1

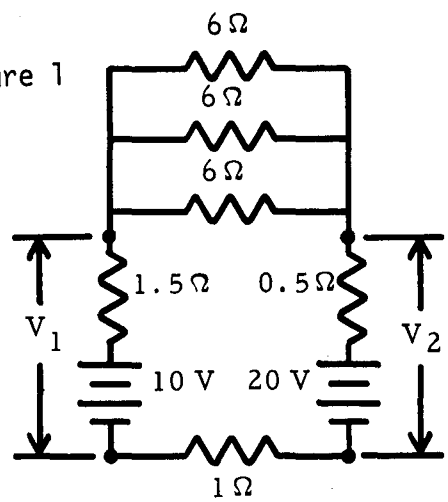
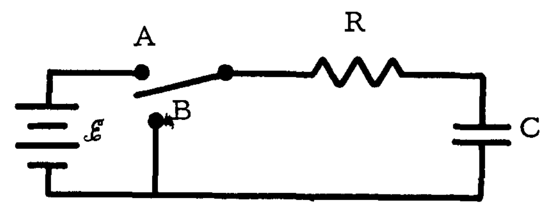


Figure 2



2. The circuit shown in Figure 2 had the switch in position A for a long time. At time = 0 it was moved to position B.

   (a) Is the following differential equation true for t > 0:

   $(1/C)(dQ/dt) + RQ = 0$?

   (b) Show that $V = V_0 e^{-t/RC}$ is a form of the solution of either the above equation or a corrected version of it.

2. A capacitor of 20.0 μF is fully charged to a potential difference V and quickly connected to a series resistor of 10^5 Ω. The first measurement of the current that can be made is 3.0×10^{-4} A at t = 4.0 s. Find the value V, the potential difference across the capacitor at t = 0.

DIRECT-CURRENT CIRCUITS

Mastery Test   Form C

Name ______________________

Date ______________________

pass   recycle

1 2 3 4

Tutor ______________________

1. For the circuit shown in Figure 1:

   (a) Find the indicated current I.

   (b) Find the terminal voltage V_1 ($\mathcal{E}$ = 24 V, internal R = 1.00 Ω).

   (c) Find the power generated in resistor R_1.

Figure 1

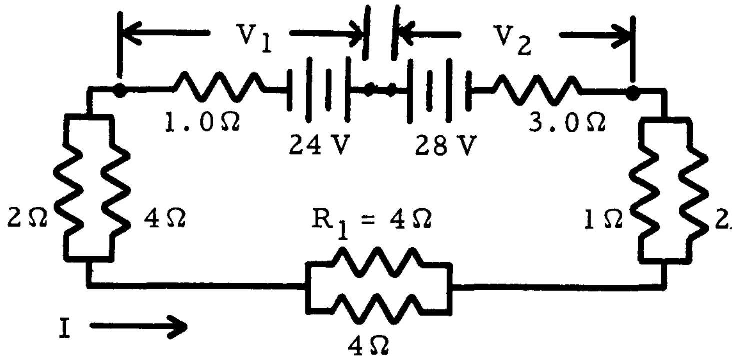
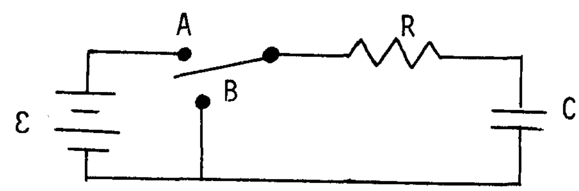


Figure 2



2. After a long time in position B the switch in the circuit shown in Figure 2 was moved to position A at t = 0.

   (b) Does the following differential equation represent the circuit for t > 0:

   $\mathcal{E} = -(dQ/dt)R - Q/C$?

   (b) Show that $Q = Q_0(1 - e^{-t/RC})$ is a solution of either the above equation or a corrected version of it.

3. A capacitor of 10.0 μF has a charge of Q_0 and is connected in series with a resistor R at t = 0. Find the value of the resistance R so that the charge on the capacitor will decrease to about $0.050Q_0$ within 10^{-3} s.

MASTERY TEST GRADING KEY - Form A

1. What To Look For: (a) Correct answer. (b) You may note that the easiest way to get the answer is from an intermediate step at which it is clear that the current through the 10.0-Ω resistor is 1.00 A, hence V_{ab} = 10.0 V. The implied 24.0 - (7.0)(2.00) = 10.0 V is, of course, acceptable. (c) The simplest calculation is, of course, from the equivalent resistance, 8.0 Ω and current, 1.00 A.

Solution: (a) 1.00 A. (b) 10.0 V. (c) $P = I^2R = 1^2(8) = 8$ W.

2. What To Look For: (a) If the answer is simply "no" ask "why"? (b) Be sure the correct differential equation is given and that the derivative is correct and the substitution is correctly done.

Solution: (a) The equation is not correct. It is for the switch in position A. The equation for position B is

$$R(dQ/dt) + Q/C = 0.$$

(b) $Q = \mathcal{E}Ce^{-t/RC}$, $\quad dQ/dt = -(C/RC)e^{-t/RC}$

Substituting in above equation, we find

$$(R\mathcal{E}C/RC)e^{-t/RC} + \mathcal{E}Ce^{-t/RC}/C = 0, \quad -\mathcal{E}e^{-t/RC} + \mathcal{E}e^{-t/RC} = 0,$$

which is an identity; therefore $Q = \mathcal{E}Ce^{-t/RC}$ is a solution.

3. What To Look For: If the answer is incorrect check to find out if the equation was wrong or if the student made an error in numerical computation.

Solution: $V = V_0(1 - e^{-t/RC}) = 100(1 - e^{-200/100}) = 86$ V.

$e^{-t/RC}$

MASTERY TEST GRADING KEY - Form B

1. What To Look For: (a) Check numerical answer. (b) The result should follow easily from (a). If part (a) is wrong, check work to see if a correct value of I would give the right answer. (c) Probably most easily calculated from $V_2 I$, but also acceptable is $\mathcal{E}I - I^2 R$.

Solution: (a) 6.0 A. (b) 19.0 V. (c) 38 W.

2 A

2. What To Look For: (b) Check for the correct differential equation, correct derivative dQ/dt, and correct substitution of one into the other.

Solution: (a) The equation given is incorrect. It should be

$$R(dQ/dt) + Q/C = 0.$$

(b) $V = Q/C = V_0 e^{-t/RC}$, $Q = CV_0 e^{-t/RC}$, $dQ/dt = -(CV_0/RC)e^{-t/RC}$.

Substituting, we find

$$-(RCV_0/RC)e^{-t/RC} + (1/C)CV_0 e^{-t/RC} = 0,$$

which is an identity; therefore $V = V_0 e^{-t/RC}$ is a solution.

3. What To Look For: If the answer is incorrect check to find out if the equation used was wrong or if the numerical evaluation was incorrect.

Solution: $I = (V/R)e^{-t/RC}$,

$$V = IRe^{-t/RC} = (3.0 \times 10^{-4})(10^5)e^{4.0/2.00} = 220 \text{ V}.$$

MASTERY TEST GRADING KEY - Form C

1. What To Look For: (a) Correct answer. (b) Neither 24 nor 23.5 are correct. (b) and (c) If the answer in (a) is wrong, check to see if the work is correct.

Solution: (a) 0.50 A. (b) $V = \mathcal{E} + IR = 24.5$ V. (c) $P = I^2R = (1/4)^2(4) = 0.250$ W.

2. What To Look For: Check for a correct differential equation, correct differentiation and correct substitution into the differential equation.

Solution: (a) The equation does not represent the circuit. It should be

$$\mathcal{E} - R(dQ/dt) - Q/C = 0.$$

(b) $Q = Q_0(1 - e^{-t/RC})$, $\quad \frac{dQ}{dt} = -Q_0(-\frac{1}{RC})e^{-t/RC} = (\frac{Q_0}{RC})e^{-t/RC}$.

Substituting, we find

$$\mathcal{E} - \frac{RQ_0}{RC}\, e^{-t/RC} - \frac{Q_0}{C} + (\frac{Q_0}{C})e^{-t/RC} = 0,$$

which is an identity if $Q_0 = \mathcal{E}C$.

3. What To Look For: If the answer is incorrect, check to see if the equation was chosen or used incorrectly or if the numerical evaluation was wrong.

Solution: $Q = Q_0e^{-t/RC}$, $\quad -Q/Q_0 = e^{-t/RC}$.

$$\ln Q/Q_0 = -t/RC,\ R = -\frac{t}{C\ \ln Q/Q_0} = \frac{t}{C\ \ln(Q_0/0)} = \frac{10^{-3}}{10^{-5}\ \ln(1.00/0.050)} = 33\ \Omega.$$