3-15-1985

Self-consistent phases in topological particle theory

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(Received 17 August 1984)

To be a self-contained theory, topological particle theory should explain from the basis of its own stated framework of assumptions (nonlinear self-consistency equations, pole factorization, crossing symmetry, and Hermitian analyticity) all the mathematical properties and numerical values of scattering amplitudes. This paper attempts to move the theory in this direction by showing that the phases of the zero-entropy amplitudes in the theory are, in fact, determined by the above framework of assumptions except for trivial ambiguities that appear to have no physical consequences. This extends previous work on this subject and removes the need for certain extra assumptions. Once the results in this paper have been established it can be shown that the conventional connection between spin and statistics as well as parity, time-reversal, and charge-conjugation invariance in strong interactions also follow from the above framework of assumptions. These latter results will be fully discussed in the following paper.

I. INTRODUCTION

In a recent paper[1] (referred to hereafter as JU) the self-consistent spin structures introduced by Stapp[2] and employed in the topological bootstrap theory[3] were studied in some detail for mesons. The individual zero-entropy terms in the topological expansion are self-reproducing and the bootstrap problem for hadrons is confined to these planar zero-entropy terms. In fact, the bootstrap problem is reduced to solving a nonlinear discontinuity equation for a scalar amplitude since the spin dependence of Stapp's amplitudes factors out and is separately self-reproducing.

The zero-entropy terms have to be summed in the topological expansion as a first step in generating the physical scattering amplitudes. It turns out that the individual zero-entropy terms are neither parity invariant nor time-reversal invariant. This fact opens up the possibility that the topological theory could be extended to electroweak processes, a possibility which is now being realized.[4] In hadron physics the zero-entropy terms of Stapp[2] were added with the appropriate relative phases to guarantee parity and time-reversal invariance for the sum. In Stapp's approach[2] these phases are given more or less automatically by a prescription based on building in the fermion character of individual quarks from the outset. One consequence of Stapp's prescription of phases for the zero-entropy terms is that the sign of the discontinuity for the zero-entropy scalar amplitude alternates with the number of closed quark loops. This result is not unexpected since in field theory a closed fermion loop generates a minus sign.

In Ref. 1 relative phases of the zero-entropy terms were determined not by a consideration of the fermion properties of the constituent quarks but by employing only general principles of S-matrix theory: pole factorization, crossing symmetry, and Hermitian analyticity. Obviously, it is important to know how much of Stapp's structure[2]—and, in particular, the relative phases of zero-entropy terms—can be fixed using only general S-matrix principles and self-consistency. A central result of JU was that the alternation of sign with the number of closed loops could be derived with no reference to the fermion statistics of the underlying constituent quarks. In fact, as mentioned above, only meson amplitudes were considered and yet the minus sign for a single closed quark loop could be derived. However, in the arguments of JU, parity and time-reversal invariance were "built-in" by adjusting part of the arbitrariness of the relative phases of the zero-entropy terms in the topological expansion to ensure the known parity and time-reversal invariance. After enforcing these requirements, the phases of zero-entropy amplitudes were determined apart from trivial ambiguities which had no physical consequences. By thoroughly discussing the phases of the zero-entropy amplitudes, this paper aims to set the stage for producing general proofs that parity, time-reversal, and charge-conjugation invariance must be valid for the sum of zero-entropy terms and also that mesons, baryons, etc., obey the correct statistics. These latter results[5] which will be presented elsewhere indicate that topological particle theory can provide an explanation (in terms of general S-matrix principles) for the occurrence of these discrete invariances in strong interactions. At the same time since *individual* zero-entropy terms are not invariant, the way is open to explain the absence of these invariances in more complicated terms in the topological expansion associated with electroweak processes.[4,6,7]

II. ZERO-ENTROPY AMPLITUDES

Here we introduce notation and discuss the properties of the zero-entropy terms in the topological expansion of the scattering amplitude. The diagram in Fig. 1 represents a zero-entropy term where the variables A, B, $C, \ldots$ designate the momentum, spin state, and flavor content of each particle. Implicit in Fig. 1 is a cyclic order of the variables which we will take to be counterclockwise. We designate the zero-entropy amplitude of Fig. 1 by $A^z(A,B,C,\ldots,K)$ or by any other of the N cyclic permutations of the variables (where N is the number of external particles). These N different ways of writing the amplitude in Fig. 1 refer to the same process and the same term in the topological expansion and can differ at most by a phase. As we shall see, for purely mesonic amplitudes, there is no difference in phase for the amplitudes with different cyclic orders. However, a phase is required when dealing with processes involving baryons. The superscript z above is a suppressed label standing for $\{z_A z_B \cdots z_K\}$ which specifies the particular spin-patch structure on the classical topological surface for each particle, as will be more fully described shortly.

The zero-entropy amplitudes of the type shown in Fig. 1 are assumed to satisfy a "planar" discontinuity equation[8] of the form

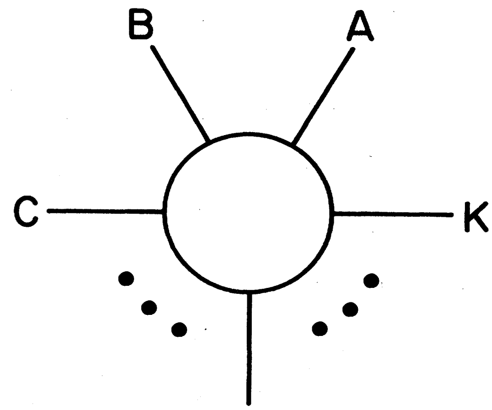

FIG. 1. Zero-entropy term.

$$A^{z_A\cdots z_K}(A\cdots K)_+ - A^{z_A\cdots z_K}(A\cdots K)_- = (2\pi)^4 i \sum_{\substack{z_L\cdots z_R\\ L\cdots R}} A^{z_A\cdots z_R}(A\cdots D,L\cdots R)_+ A^{z_{R'}\cdots z_K}(R'\cdots L',E\cdots K)_- \delta^4(p_L+\cdots+p_R-p_A-\cdots-p_D)\,. \tag{2.1}$$

The summation in (2.1) includes a spin summation for each intermediate particle, a sum over spin patches $\{z_L \cdots z_R\}$ for the intermediate state, and a momentum integration of the form

$$\prod_{x=L,\ldots,R} \int \frac{d^3\mathbf{p}_x}{(2\pi)^3 2E_x}\,.$$

The order of the variables in the amplitudes on the right side of (2.1) ensures a planar connected sum and this equation embodies the zero-entropy bootstrap. We adopt the convention in (2.1) that the $(R\cdots L)$ intermediate state is incoming and the primes on the variables $(L'\cdots R')$ are used to designate the corresponding outgoing state. In general notation like L and L' will be used to refer to the same multiple quark state where one is incoming and the other outgoing.

The self-consistent zero-entropy amplitudes in (2.1) (see Refs. 1 and 2) have the following general form in which the dependence on spin and spatial degrees of freedom is written in the product form:

$$A^{z_A\cdots z_K}(A\cdots K)_\pm = \Gamma(z_A\cdots z_K, A\cdots K) \times S^{z_A\cdots z_K}(A\cdots K) f(p_A\cdots p_K)_\pm\,. \tag{2.2}$$

In (2.2) f is an invariant scalar function of the momenta—the same one introduced in JU—and $\pm$ denotes above or below the cut in the variable corresponding to the intermediate state in (2.1). As discussed in JU, f is unchanged by a cyclic permutation of its variables or by a reversal of its order of variables. Γ denotes a phase factor whose determination constitutes the main point of this paper. Finally S^z is the spin structure factor discussed in JU for mesonic amplitudes and which as we now indicate generalizes in a straightforward way to amplitudes which, in addition to mesons, involve baryons, antibaryons, and baryonium. To understand the form of S^z in the general case, we magnify Fig. 1 to indicate the constituent quark structure of the particles. An example of such a quark diagram is shown in Fig. 2 and is a version of the so-called "classical surface"[3] for this term in the topological expansion.

The diagram in Fig. 2 consists of single quark lines, which by convention will always be taken to run clockwise, and disquark lines which will be taken to run counterclockwise. Each quark line in Fig. 2 has a definite flavor so physical particle amplitudes will in general involve sums over amplitudes of the type shown in Fig. 2. The

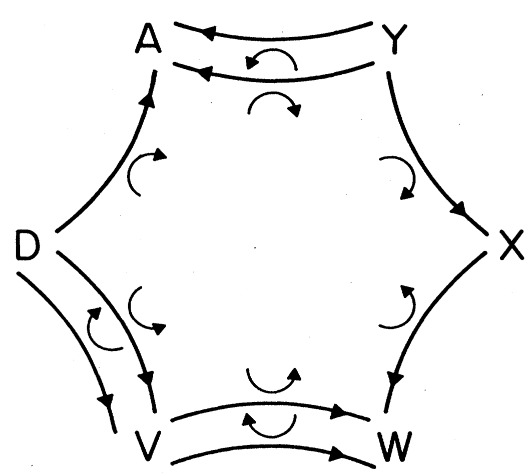

FIG. 2. Classical surface with patch structure.

small arrows next to each quark line in Fig. 2 orient a patch of the classical surface adjacent to the quark line.[9] It is this patch structure which determines the specific form of the spin structure factor S^z. If the spin-patch orientation is in agreement with the arrow on the adjacent quark line the quark is said to be an "ortho" quark and if the patch orientation is opposite to that of the adjacent quark arrow, it is said to be a "para" quark. Ortho and para quarks contribute differently to the spin structure factor S^z. Specifically, each quark line in the zero-entropy amplitudes contributes a factor to S^z consisting of a Lorentz-invariant scalar product of the two-component spinors associated with the head and tail of the quark line. These spinor pairs have dotted indices if the quark is ortho and undotted indices if the quark is para (Refs. 1 and 2 provide a brief but complete review of two-component spinor theory). The spin structure factor can also be expressed in terms of four-component Dirac spinors.[1–3]

We illustrate the form of S^z using Fig. 2 as an example of the general theory. The top quark line in the diquark connecting particles A and Y which is ortho and the second quark line in the diquark which is para contribute factors to S^z as follows:

$$\begin{aligned} S^{z_A\cdots z_Y}(A,D\cdots Y) &= \eta_{\dot\alpha}(v_A,\psi_{A1})\eta^{\dot\alpha}(v_Y,\phi_{Y1})\eta^{\beta}(v_A,\psi_{A2}) \\ &\quad\times\eta_\beta(v_Y,\phi_{Y2})\cdots\ , \\ p_A &= m_A v_A\ , \\ p_Y &= m_Y v_Y\ . \end{aligned} \tag{2.3}$$

The ψ's and ϕ's in (2.3) are two-component rest-frame spin states. The η's are obtained by simple boosts (whose parameters are determined by v_A and v_Y) on these rest-frame spin states (see Ref. 2 and JU). (We note that spinors for all quarks that are constituents of the same particle receive the same boost from their rest frame, namely, the one required to take the particle in question from its rest frame and give it momentum p_A or p_Y.) We also note that $\eta_{\dot\alpha}$ and η^β are associated with the heads of quark lines whereas $\eta^{\dot\alpha}$ and η_β are associated with the tails. The $z_A\cdots z_Y$ superscript on S is simply a set of labels which designates a particular ortho-para structure of the individual quark lines in the diagram. (Note, e.g., if A is a particle consisting of three quarks, the label z_A includes the ortho/para designations for each of those three quarks.) The two-component spinors η are normalized so that

$$\eta_{\dot\alpha}(v,\psi^*)\eta^{\dot\alpha}(v,\psi)=\eta^{\alpha}(v,\psi^*)\eta_\alpha(v,\psi)=\psi^\dagger\psi=1\ , \tag{2.4}$$

i.e., the η's have the same normalization as the rest-frame spin states which we take to be unity. We see from Fig. 2 and (2.3) that orthoquarks are associated with two-component spinors having dotted indices whereas paraquarks are associated with spinors having undotted indices.

We now list some important properties of the spin structure factors S^z which follow immediately from the properties of the two-component spinors discussed in JU and the definitions given above:

(a) S^z does not change under a cyclic permutation of its variables.

(b) The spin structure factors are self-reproducing in the sense that

$$\begin{aligned} S^{z_A\cdots z_K}(A\cdots K) &= 4^{-n}\sum_{\substack{\text{spin}(L\cdots R)\\ z_L\cdots z_R}} S^{z_A\cdots z_R}(A\cdots D,L\cdots R) \\ &\qquad\times S^{z_{R'}\cdots z_K}(R'\cdots L',E\cdots K)\ , \end{aligned} \tag{2.5}$$

where $\sum$ means summing over a complete set of rest-frame spin states for $L\cdots R$ particles. The summation in (2.5) differs from that in (2.1) only in the absence of the momentum integration in (2.5). The quantity n in (2.5) is the number of distinct quark lines between particles L and R on the classical surface. (For example, in Fig. 2 if $V=L$ and $Y=R$, $n=4$.) The sum over spin patches $\{z_L\cdots z_R\}$ can now be understood as allowing each of the n quark lines running from particle L to particle R to be either ortho or para. The factor 4^{-n} in (2.5) then simply arises because each of the n quarks which are summed over in the intermediate state can be either ortho or para and have spin up or down. When used in the bootstrap equation (2.1) to evaluate the sum over intermediate states on the right side, the quantity n is also the number of quark loops encountered in the intermediate state.

The result (2.5) is a consequence of the following facts:

$$\sum_i \eta^{\dot\alpha}(v,\phi_i)\eta_{\dot\beta}(v,\phi_i)=\delta^{\dot\alpha}{}_{\dot\beta}\ ,$$

$$\sum_i \eta^{\alpha}(v,\phi_i)\eta_{\beta}(v,\phi_i)=\delta^{\alpha}{}_{\beta}\ ,$$

where i is two-valued with

$$\phi_1=\begin{bmatrix}1\\0\end{bmatrix},\quad \phi_2=\begin{bmatrix}0\\1\end{bmatrix}.$$

(c) Finally, we have the important identity

$$[S^{z_A\cdots z_K}(A\cdots K)]^*=S^{\dot z_K\cdots \dot z_A}(K^*\cdots A^*)\ , \tag{2.6}$$

where dots over the z's on the right side mean that patch orientations have been reversed relative to the $\{z_A\cdots z_K\}$ patch structure, i.e., all orthoquarks are changed to paraquarks and vice versa. The complex conjugation of the arguments on the right side of (2.6) means that the two-component rest-frame spin state at the head and tail of each quark line is complex conjugated. The reversal of the order of variables follows from our conventions that individual quark lines run clockwise, diquark lines counterclockwise, and that spinors with lower dotted and upper undotted indices are associated with heads of quark lines while spinors with upper dotted and lower undotted indices are associated with tails. The result (2.6) is then a direct consequence of the following identities:

$$\begin{aligned} [\eta^{\dot\alpha}(v,\phi)]^* &= \eta^{\alpha}(v,\phi^*)\ , \\ [\eta_{\dot\alpha}(v,\phi)]^* &= \eta_{\alpha}(v,\phi^*)\ . \end{aligned} \tag{2.7}$$

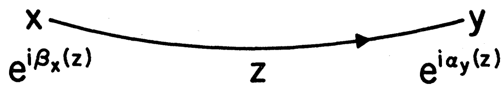

FIG. 3. Phases for single quark line.

We can now give a simple interpretation to the spin structure factor that appears on the right side of (2.6). Relative to the process represented by the structure factor $S^z(A \cdots K)$, $S^{\dot{z}}(K^* \cdots A^*)$ represents the structure factor for the process in which all incoming particles become outgoing particles in the same spin state and vice versa. Also orthoquarks become paraquarks and vice versa. This interpretation of (2.6) is important in understanding the property of Hermitian analyticity at the zero-entropy level of the topological expansion discussed in Sec. V.

Just as S^z has a fully factorized dependence on the head and tail of each quark line so the phase factor Γ will be written in a similarly factorized form.

For a single quark line in an amplitude, a phase $e^{i\alpha_y(z)}$ will be associated with the head of the quark line and a phase $e^{i\beta_x(z)}$ with the tail of a quark line as illustrated in Fig. 3. In the notation of Fig. 3, the label z designates whether the quark line is ortho or para while the labels x and y each include a designation of whether the quark line end is an in or out state and also whether the quark line terminates in a mesonic or baryonic state. In some of our calculations we shall find it useful to separate x and y into two labels such as $x=(i,\rho)$, where $i=2$ and $i=3$ designate mesonic and baryonic states, respectively, and the ρ label indicates in or out.

Figure 4 indicates the notation for phases in the case of diquarks. We do not assume that the phases associated with the heads and tails of quark lines are the same for single quark lines and diquark lines. In topological theory the two quarks in the diquark can be distinguished using the so-called quantum surface[3] and this distinction is indicated in Fig. 4 by the labels u and l. Labels z_1 and z_2 refer to the ortho-para character of the individual quarks mixing up the diquark.

As an example, the phase $\Gamma(AB \cdots Y)$ of Fig. 2 would have the form

$$\Gamma(AB \cdots Y) = \pi e^{i\alpha_A^u(0)} e^{i\alpha_A^l(P)} e^{i\alpha_A(0)} e^{i\beta_D(0)} \cdots , \qquad (2.8)$$

where π is a possible overall phase for the amplitude depending not on individual quark or diquark lines or their patch structure, but conceivably depending on the particular cyclic order of the variables. The use of labels $A, D, \ldots$ in (2.8) is not intended to suggest that these phases depend upon momentum; they depend just on the hadronic and in or out properties of the particles as previously discussed.

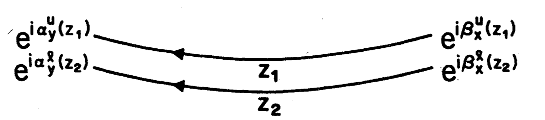

FIG. 4. Phases for diquark line.

III. CROSSING SYMMETRY FOR ZERO-ENTROPY AMPLITUDES

We wish now to begin a systematic determination of the individual quark phases which make up Γ, the total phase for the zero-entropy terms in Sec. II. First we consider the requirements of crossing. The spin-patch structure $\{z_A \cdots z_K\}$ of an amplitude such as (2.2) is not a physical observable and is summed over in the topological expansion. The expansion will thus include a summation over zero-entropy terms of the form

$$\sum_{\{z_A \cdots z_K\}} A^{z_A \cdots z_K}(A \cdots K) . \qquad (3.1)$$

The rules for writing each of the terms in (3.1) with its spin dependence completely factorized are given by (2.2) and the discussion following (2.2).

We now imagine constructing the sum (3.1) for some particular process and then crossing to some other channel by means of analysic continuation. Then the requirements of crossing symmetry dictate we must get the amplitude in the crossed channel—that is, if we calculate the amplitude in the crossed channel using the rules of Sec. II we must get the same result as when we analytically continue from the original channel to the crossed channel to within an overall phase. This requirement will put restrictions on the phases Γ as we now show.

In the crossing operation some of the momenta in (3.1) are analytically continued $p \to -p$. An appropriate *path* of continuation to the crossed channel can be found by examing the scalar function f in (2.2) because it contains no kinematic singularities. As pointed out in JU it is convenient to analytically continue the spin structure factors S^z along the path of continuation to the crossed channel even though kinematic singularities will be encountered. The advantage of carrying the spin structure factor along in the crossing continuation is that some simple general results can be obtained for crossing spinors which are independent of the particular path taken to achieve the continuation $p \to -p$. The results are that for a particular path of continuation we have (see JU Sec. III)

$$\eta_{\dot{\alpha}}(v,\phi) \underset{v \to -v}{\to} \eta_{\dot{\alpha}}(v,\phi') , \qquad (3.2a)$$

$$\eta^{\alpha}(v,\phi) \underset{v \to -v}{\to} -\eta^{\alpha}(v,\phi') , \qquad (3.2b)$$

$$\eta^{\dot{\alpha}}(v,\phi) \underset{v \to -v}{\to} \eta^{\dot{\alpha}}(v,\phi'') , \qquad (3.2c)$$

$$\eta_{\alpha}(v,\phi) \underset{v \to -v}{\to} -\eta_{\alpha}(v,\phi'') , \quad p = mv \qquad (3.2d)$$

where the important point is that although ϕ' and ϕ'' in (3.3) may depend upon the particular path of continuation used in the crossing process, the relative minus signs in the equations (3.2) do not. We note that the phase factor Γ in (2.2) has no functional dependence on the momenta (since this dependence is by definition in S^z and f) and hence Γ is unchanged by the analytic continuation to the crossed channel.

Additional insight into the meaning of (3.2) can be obtained by noting that a Dirac spinor whose components are made up, e.g., of the two-component spinors η_α and

$\eta^{\dot\alpha}$ will constitute a positive-energy solution to the Dirac equation, whereas a Dirac spinor made up of $-\eta_\alpha$ and $\eta^{\dot\alpha}$ will give a negative-energy solution to the Dirac equation.

We can now use (3.2) to infer a restriction on the phases Γ due to crossing. Suppose $\Gamma^{(i)}$ and $\Gamma^{(j)}$ are the phases of two of the terms in (3.1) for some process with certain particles ingoing and outgoing (where i and j denote the particular terms.) Let $\Gamma_c^{(i)}$ and $\Gamma_c^{(j)}$ be the corresponding phases for some particular crossed process where certain of the previous ingoing particles have become outgoing antiparticles, etc. Then the result (3.2) tells us that the rule for determining the phases Γ must be such that

$$\frac{\Gamma^{(i)}}{\Gamma^{(j)}}=\frac{\Gamma_c^{(i)}}{\Gamma_c^{(j)}}(-1)^{N^c}\,, \tag{3.3}$$

where N^c is the number of heads or tails of quark lines that are both (i) crossed *and* (ii) have opposite spin-patch orientations in the i and j terms of (3.1).

The result (3.3) can be readily translated into simple conditions on the individual quark line phases introduced in Sec. II. For single quark lines (3.3) requires

$$\frac{e^{i\alpha_x(z)}}{e^{i\alpha_x(\dot z)}}=(-1)\frac{e^{i\alpha_{\bar x}(z)}}{e^{i\alpha_{\bar x}(\dot z)}}\,, \tag{3.4a}$$

$$\frac{e^{i\beta_x(z)}}{e^{i\beta_x(\dot z)}}=(-1)\frac{e^{i\beta_{\bar x}(z)}}{e^{i\beta_{\bar x}(\dot z)}}\,. \tag{3.4b}$$

In (3.4) the notation $\bar x$ indicates a state with the opposite in-out character to x. For diquark lines, relations identical to (3.4) hold except that the superscripts u and l must be added to the phase α and β.

IV. POLE FACTORIZATION IN ZERO-ENTROPY AMPLITUDES

If we apply the discontinuity equation (2.1) to the case of intermediate poles (single-particle intermediate states) the discontinuity becomes a δ function and we have pole factorization. (At the pole, the residue of the zero-entropy amplitude A factorizes into two other A-type amplitudes corresponding to the transition from the initial state to intermediate particle and from the intermediate particle to final state. The sum over the spin states of the intermediate particle, of course, must be done.) Since the scalar function f in (2.2) is independent of the spin properties of the system it must factorize by itself, i.e.,

$$f(p_A\cdots p_K)\underset{p_E{}^2\to m_E{}^2}{\sim}\frac{f(p_A\cdots p_E)f(p_E\cdots p_K)}{m_E{}^2-p_E{}^2}\,.$$

Equation (2.5) with $n=0$ shows the spin structure functions also factor independently so we are led to the fact that the phases Γ defined by (2.2) must also factorize. Thus we require

$$\Gamma(z_A\cdots z_K,A\cdots K)=\Gamma(z_A..z_E,A\cdots E)\Gamma(z_E..z_K,E'\cdots K)\,. \tag{4.1}$$

The arguments of Γ indicate that it depends in general on the spin-patch structure of the amplitude, on the cyclic order of the variables, on the particle types (meson, baryon, baryonium), and on whether the particles are ingoing or outgoing. The phase Γ does *not* depend upon the spin states or four-momenta, this dependence being given by S^z and f.

Since the Γ phases in (4.1) are of the factorized form (2.8), the factorization requirement (4.1) can be written as a condition on the phases $e^{i\alpha}$ and $e^{i\beta}$ for individual quark lines. Using the conventions established in Sec. I, single quark lines which are constituents of particle poles in amplitudes occur in one of two ways shown in Figs. 5(a) and 5(b), depending upon whether the quark line is at the "top" or "bottom" of the zero-entropy diagram such as Fig. 2. Denoting the intermediate state E as in (4.1), pole factorization as indicated in Figs. 5(a) and 5(b) implies in our notation

$$e^{i\alpha_{\bar E}(z)}e^{i\beta_E(z)}=e^{i\theta_E}\,, \tag{4.2a}$$

$$e^{i\alpha_E(z)}e^{i\beta_{\bar E}(z)}=e^{i\theta_{\bar E}}\,. \tag{4.2b}$$

If E is an out state to the right of the dashed line as shown in Figs. 5(a) and 5(b), then (4.2a) corresponds to Fig. 5(a) and (4.2b) to Fig. 5(b). The presence of the phases $e^{i\theta_E}$ and $e^{i\theta_{\bar E}}$ in (4.2) needs careful explanation. Phases for an individual quark line will clearly factorize if $e^{i\theta_E}=e^{i\theta_{\bar E}}=1$. However, in arranging individual quark lines and diquark lines into zero-entropy amplitudes some adjustments may be necessary in light of the possible presence of the phase τ in (2.8). To anticipate this possible need we include the phases on the right of equations (4.2). However, no z dependence is allowed for θ_E or $\theta_{\bar E}$ since that dependence is accounted for completely in the phases $e^{i\alpha}$, $e^{i\beta}$, and in the spin structure factor S^z.

The preceding arguments can be repeated for diquark lines which as constituents of particle poles in amplitudes occur according to our conventions in Sec. II in one of the two ways depicted in Figs. 6(a) and 6(b). Pole factorization then leads to the conditions

$$e^{i\beta^u_{\bar E}(z_1)}e^{i\alpha^u_E(z_1)}e^{i\beta^l_{\bar E}(z_2)}e^{i\alpha^l_E(z_2)}=e^{i\eta_E}\,, \tag{4.3a}$$

$$e^{i\alpha^u_{\bar E}(z_1)}e^{i\beta^u_E(z_1)}e^{i\alpha^l_{\bar E}(z_2)}e^{i\beta^l_E(z_2)}=e^{i\eta_{\bar E}}\,, \tag{4.3b}$$

where again $e^{i\eta_E}$ and $e^{i\eta_{\bar E}}$ do not depend upon z_1 or z_2.

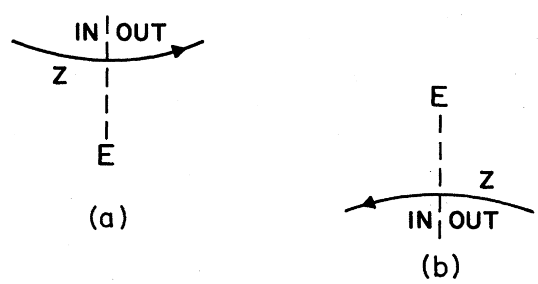

FIG. 5. Factorization of single quark lines.

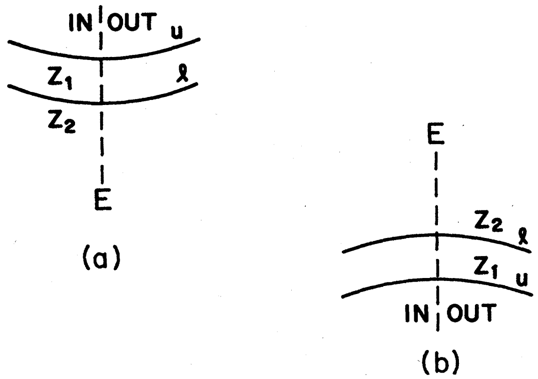

FIG. 6. Factorization of diquark lines.

V. HERMITIAN ANALYTICITY

Each zero-entropy term in the topological expansion is assumed to satisfy the discontinuity equation (2.1). This, however, is to assume *less* than planar unitarity for the zero-entropy amplitudes. In order for the zero-entropy amplitudes to be unitary (with planar connections of the intermediate states), they would have to satisfy Hermitian analyticity in addition to (2.1). Suppressing momentum, spin, and spin-patch labels, Hermitian analyticity would imply

$$A_{if}^{*}(+)=A_{fi}(-)\ , \tag{5.1}$$

where $\pm$ carries the meaning it has in (2.2) and the $i-f$ labels are used to indicate that in and out states are reversed in going from the left to the right side of (5.1). It was Stapp[2] who originally emphasized that zero-entropy amplitudes will not, in general, obey (5.1). This can be seen at once from (2.6) which indicates that the spin structure factors on the two sides of (5.1) will not, in general, agree.

The discussion following (2.6) indicates that the right side of (2.6) is the spin structure factor for the process in which in and out particles are reversed and spin-patch orientations are reversed. This means that although Hermitian analyticity cannot be obeyed by individual zero-entropy terms, pairs of terms with all spin-patch orientations opposite to one another could conceivably satisfy Hermitian analyticity. At least the spin structure factors will go into one another under the complex conjugation and interchange of in and out states as (2.6) shows. We now show that it is possible to achieve Hermitian analyticity for the pairs of zero-entropy terms indicated and, hence, for the zero-entropy sums as whole but that this enforces a further constraint on the phases Γ and, in turn, on $e^{i\alpha}$ and $e^{i\beta}$.

To proceed we pose the question whether the zero-entropy amplitude

$$[A^{z_A\cdots z_K}(A\cdots K)]^{*} \tag{5.2}$$

can be taken equal to the zero-entropy amplitude where (relative to the original process) both in and out and spin-patch orientations are reversed. (The physical spin states for individual quarks and particles remain the same when in and out are reversed here although the rest-frame spin states ϕ and ψ become complex conjugated.) In order to interpret (5.2) as another zero-entropy amplitude with in and out as well as patch orientations reversed, it must satisfy the appropriate discontinuity equation. To pursue this point further we consider the following amplitude:

$$A^{\dot z_K\cdots \dot z_A}(K\cdots A)\ , \tag{5.3}$$

which relative to the amplitude within the brackets in (5.2) [i.e., the amplitude (2.2)] has all its patch orientations reversed *and* the order of its variables reversed. The reversal of the order of variables is analogous to that in (2.6) and is necessary to be able to have (5.3) represent the amplitude where in and out states have been reversed relative to (2.2). This is because in order to reverse in and out states the arrows on the quark lines in a diagram (such as Fig. 2) must be reversed. This, in turn, means the order of variables must be reversed to maintain our convention that quark arrows run clockwise while diquark arrows run counterclockwise.

The discontinuity equation of the general form (2.1) satisfied by the amplitude (5.3) is

$$A^{\dot z_K\cdots \dot z_A}(K\cdots A)_{+}-A^{\dot z_K\cdots \dot z_A}(K\cdots A)_{-}=(2\pi)^4 i\sum_{\substack{\dot z_L\cdots \dot z_R\\ L\cdots R}} A^{\dot z_K\cdots \dot z_R}(K\cdots E,L\cdots R)_{+}A^{\dot z_{R'}\cdots \dot z_A}(R'\cdots L',D\cdots A)_{-}$$
$$\times\delta^4(p_L+\cdots+p_R-p_K-\cdots-p_E)\ . \tag{5.4}$$

We now wish to compare (5.4) with the complex conjugation of (2.1) bearing in mind (2.2), (2.6), and the fact that f, being an analytic scalar function of the Lorentz invariants, cannot change when in and out particles are reversed. (As emphasized in JU, f does not change its value either under a cyclic permutation of its variables or if the order of its variables is completely reversed.) It is then clear that the only way of consistently enforcing Hermitian analyticity at the level of the zero-entropy sum is to require

$$[A^{z_A\cdots z_K}(A\cdots K)_{\pm}]^{*}=A^{\dot z_K\cdots \dot z_A}(K'\cdots A')_{\mp}\ . \tag{5.5}$$

If the scalar function f is assumed to be real analytic,

$$f^{*}(p_A\cdots p_K)_{+}=f(p_A\cdots p_K)_{-}\ , \tag{5.6}$$

then the implication of (5.5) for the phases Γ is

$$[\Gamma(z_A\cdots z_K,A\cdots K)]^{*}=\Gamma(\dot z_K\cdots \dot z_A,K'\cdots A') \tag{5.7a}$$

or

$$\Gamma(z_A \cdots z_K, A \cdots K)\Gamma(\dot{z}_K \cdots \dot{z}_A, K' \cdots A') = 1 \ , \quad (5.7b)$$

where it must be remembered that on the right side of (5.7a) in and out states have been reversed relative to the left side of the equation.

Since the Γ phases in (5.7) completely factorize into the phases for the individual quark or diquark lines, we require (5.7) to hold for these individual phases. For single quark lines we then write the condition (5.7b)

$$e^{i\alpha_y(z)} e^{i\beta_x(z)} e^{i\alpha_{\bar{x}}(\dot{z})} e^{i\beta_{\bar{y}}(\dot{z})} = 1 \ . \quad (5.8)$$

For diquark lines we obtain

$$e^{i\alpha_y^u(z_1)} e^{i\beta_x^u(z_1)} e^{i\alpha_y^l(z_2)} e^{i\beta_x^l(z_2)} e^{i\alpha_{\bar{x}}^u(\dot{z}_1)} e^{i\beta_{\bar{y}}^u(\dot{z}_1)} e^{i\alpha_{\bar{x}}^l(\dot{z}_2)} e^{i\beta_{\bar{y}}^l(\dot{z}_2)} = 1 \ . \quad (5.9)$$

The τ factors discussed after (2.8) do not enter into the constraints (5.8) and (5.9). There exist symmetric amplitudes for which the τ factors are equal on both sides of (5.7a) and hence cancel leading to (5.8) and (5.9). Once (5.8) and (5.9) are established for individual quark and diquark lines we can conclude that τ factors must be equal on both sides of (5.7a) for all processes. When the τ factors are finally deduced they indeed obey this condition, as we shall see.

VI. DETERMINATION OF PHASES FOR SINGLE QUARK LINES

In the preceding sections, we found three general types of restriction on the phases $e^{i\alpha}$ and $e^{i\beta}$ associated with the heads and tails of single quark lines: (1) the crossing conditions (3.4); (2) the pole factorization condition (4.2); (3) the Hermitian analyticity condition (5.8). In this section we wish to determine the general form of these phases consistent with the above restrictions.

We begin by discussing the phases for the single quark lines. First we consider the Hermitian analyticity requirement (5.8) in the special case $y=\bar{x}$. This gives

$$e^{i\alpha_{\bar{x}}(z)} e^{i\beta_x(z)} e^{i\alpha_{\bar{x}}(\dot{z})} e^{i\beta_{\bar{y}}(\dot{z})} = 1 \ . \quad (6.1)$$

Using (4.2a) and remembering that $e^{i\theta_x}$ does not depend on patch structure, we find (6.1) is equivalent to the condition

$$(e^{i\theta_x})^2 = 1 \ . \quad (6.2)$$

Since x can be anything, it is also true that

$$(e^{i\theta_{\bar{x}}})^2 = 1 \ . \quad (6.3)$$

Next we take the factorization relations (4.2) and use them to eliminate $e^{i\alpha_x(z)}$ and $e^{i\alpha_{\bar{x}}(\dot{z})}$ in the crossing relation (3.4a). This gives

$$\frac{e^{i\theta_{\bar{x}}}}{e^{i\beta_{\bar{x}}(z)} e^{i\alpha_x(\dot{z})}} = (-1) \frac{e^{i\alpha_{\bar{x}}(z)} e^{i\beta_x(\dot{z})}}{e^{i\theta_x}} \ . \quad (6.4)$$

Incorporating the restriction (5.8) into (6.4) leads at once to the condition

$$e^{i\theta_{\bar{x}}} e^{i\theta_x} = -1 \ . \quad (6.5)$$

When (6.2), (6.3), and (6.5) are all considered, they imply that one of the two phases $e^{i\theta_x}$ or $e^{i\theta_{\bar{x}}}$ must be $+1$ and the other -1. There is however, only one acceptable possibility, because as discussed earlier the τ factor [see (2.8)] must compenstate for the required (-1) which now occurs in either (4.2a) or (4.2b) in order to ensure pole factorization. But τ as discussed in Sec. II can only depend upon the cyclic order of the variables in the amplitude.

Applying the discontinuity equation (2.1) with our in-out conventions at a single-particle intermediate state as discussed in Sec. IV we see that Fig. 5(a) depicts the quark line which is the first in the cyclic order of variables for both A-type amplitudes in (2.1) which are the residues of the particle pole. The quark line in Fig. 5(b), on the other hand, can occur anywhere in the cyclic order of variables in the two residue A amplitudes. Thus to ensure pole factorization we must associate the phase of (-1) discussed above with Fig. 5(a) and introduce a compensating (-1) τ factor for amplitudes which begin their cyclic order with a single quark line. Thus to summarize we must take

$$e^{i\theta_{j,\mathrm{out}}} = -1, \quad (6.6a)$$

$$e^{i\theta_{j,\mathrm{in}}} = +1 \ , \quad (6.6b)$$

and

$$\tau = -1 \text{(for amplitudes which begin their cyclic order with a single quark line)} \ . \quad (6.6c)$$

As an example of (6.6c), the amplitudes $A(Y,A,D \cdots)$, $A(D,V,W \cdots)$, or $A(X,Y,A, \cdots)$ of Fig. 2 have a τ factor of minus one. The other amplitudes in Fig. 2 have a cyclic order of variables which begins with a diquark and we must deduce what the τ factor will be in that case.

Introducing the requirements (6.6) into (4.2) we obtain

$$e^{i\alpha_{i,\mathrm{in}}(z)} e^{i\beta_{i,\mathrm{out}}(z)} = -1 \ ,$$
$$e^{i\alpha_{i,\mathrm{out}}(z)} e^{i\beta_{i,\mathrm{in}}(z)} = 1, \quad i=2,3 \quad (6.7)$$

where $i=2$ designates mesonic states and $i=3$ designates baryonic states. For a given patch orientation, say ortho, there are a total of eight α and β phases. Equations (6.7) enable four of the phases to be determined by the other four. Specifying the β orthophases as follows:

$$\beta_{2,\mathrm{in}} = \gamma_1 \ ,$$
$$\beta_{3,\mathrm{in}} = \gamma_2 \ ,$$
$$\beta_{2,\mathrm{out}} = \gamma_3 \ ,$$
$$\beta_{3,\mathrm{out}} = \gamma_4 \ , \quad (6.8)$$

all the orthophases are determined as shown in the second column of Table I.

The paraphases must also satisfy the conditions (6.7) but because the ortho and paraphases are related both by the crossing conditions (3.4) and the Hermitian analyticity conditions (5.8) only one additional parameter is needed in order to specify all the paraphases. To see this we first observe that (5.8) is true for arbitrary x and y. It then fol-

TABLE I. Phases for single quark lines.

	Ortho	Para
$e^{i\beta_{2,\text{in}}}$	$e^{i\gamma_1}$	$-e^{i\gamma_1}e^{-i\xi}$
$e^{i\beta_{3,\text{in}}}$	$e^{i\gamma_2}$	$-e^{i\gamma_2}e^{-i\xi}$
$e^{i\beta_{2,\text{out}}}$	$e^{i\gamma_3}$	$e^{i\gamma_3}e^{-i\xi}$
$e^{i\beta_{3,\text{out}}}$	$e^{i\gamma_4}$	$e^{i\gamma_4}e^{-i\xi}$
$e^{i\alpha_{2,\text{out}}}$	$e^{-i\gamma_1}$	$-e^{-i\gamma_1}e^{i\xi}$
$e^{i\alpha_{3,\text{out}}}$	$e^{-i\gamma_2}$	$-e^{-i\gamma_2}e^{i\xi}$
$e^{i\alpha_{2,\text{in}}}$	$-e^{-i\gamma_3}$	$-e^{-i\gamma_3}e^{i\xi}$
$e^{i\alpha_{3,\text{in}}}$	$-e^{-i\gamma_4}$	$-e^{-i\gamma_4}e^{i\xi}$

lows that $e^{i\beta_x(z)}e^{i\alpha_{\bar{x}}(z)}$ must be independent of x. We thus can set

$$e^{i\beta_x(0)}e^{i\alpha_{\bar{x}}(P)} = -e^{i\xi}\,, \tag{6.9a}$$

where ξ is independent of x and the minus sign is for convenience. It then follows from (5.8) that

$$e^{i\beta_x(P)}e^{i\alpha_{\bar{x}}(0)} = -e^{-i\xi} \tag{6.9b}$$

for all x. From (6.9a) and (6.9b) the paraphases can be completely determined in terms of the orthophases and the additional parameter ξ. These results are given in the third column of Table I. It may be readily verified that the paraphases in Table I satisfy (6.7).

VII. DETERMINATION OF PHASES FOR DIQUARK LINES

In a manner similar to what was done in the previous section we here determine the phases for diquark lines consistent with the crossing conditions (3.4), the factorization conditions (4.3), and the Hermitian analyticity requirement (5.9).

First considering the relation (5.9) in the special case $y=\bar{x}$ we can conclude from (4.3b) that

$$(e^{i\eta_{\bar{x}}})^2 = 1\,, \tag{7.1}$$

and since x can be anything, it is also true that

$$(e^{i\eta_x})^2 = 1\,. \tag{7.2}$$

Combining the requirements of (3.4), (4.3), and (5.9) it can be readily shown that

$$e^{i\eta_x}e^{i\eta_{\bar{x}}} = 1\,, \tag{7.3}$$

where there is a key sign difference here from the analogous equation (6.5) for single quark lines. Thus $e^{i\eta_x}$ must either be plus one for all values of x or minus one. In analogy with the argument given in Sec. VI, setting the phase equal to minus one requires a compensating adjustment in the τ phase. However, this cannot be achieved in the case of the diquark depicted in Fig. 6(b) because this diquark may occur essentially anywhere in the cyclic order for the amplitude. Therefore the only acceptable choice is

$$e^{i\eta_x} = 1\,. \tag{7.4}$$

Also in analogy with the discussion in Sec. VI if the situation depicted in Fig. 6(a) holds at a factorized particle pole, then the diquark line is first in the order of variables for both A-type amplitudes which are residues of the particle pole. Thus we can conclude as a result of (7.4)) that

$$\tau = 1 \text{ (for amplitudes which begin their cyclic order with a diquark line.)} \tag{7.5}$$

With the result (7.4), the relations (4.3) become

$$e^{i\beta_{\bar{x}}^u(z_1)}e^{i\alpha_x^u(z_1)}e^{i\beta_{\bar{x}}^l(z_2)}e^{i\alpha_x^l(z_2)} = 1\,. \tag{7.6}$$

Since (7.6) is true for arbitrary z_1 and z_2, it follows that the first two factors (as well as the last two) must be independent of patch structure. Thus if we set

$$e^{i\alpha_{i,\text{in}}^u(z)}e^{i\beta_{i,\text{out}}^u(z)} = e^{i\phi_1},\quad i=3,4 \tag{7.7a}$$

$$e^{i\alpha_{i,\text{out}}^u(z)}e^{i\beta_{i,\text{in}}^u(z)} = e^{i\phi_2},\quad i=3,4 \tag{7.7b}$$

where ϕ_1 and ϕ_2 are independent of z and $i=3$ denotes baryon or antibaryon states and $i=4$ denotes baryonium states. We also have from (7.6),

$$e^{i\alpha_{i,\text{in}}^l(z)}e^{i\beta_{i,\text{out}}^l(z)} = e^{-i\phi_1}\,, \tag{7.8a}$$

$$e^{i\alpha_{i,\text{out}}^l(z)}e^{i\beta_{i,\text{in}}^l(z)} = e^{-i\phi_2}\,. \tag{7.8b}$$

Now specifying the β orthophases as follows:

$$\begin{aligned}\beta_{3,\text{in}}^{u,l} &= \Gamma_1{}^{u,l}\,,\\ \beta_{4,\text{in}}^{u,l} &= \Gamma_2{}^{u,l}\,,\\ \beta_{3,\text{out}}^{u,l} &= \Gamma_3{}^{u,l}\,,\\ \beta_{4,\text{out}}^{u,l} &= \Gamma_4{}^{u,l}\,,\end{aligned} \tag{7.9}$$

the remaining orthophases are determined by (7.7) and (7.8). All the orthophases are given in the second column of Table II, where the $\pm$ signs are associated with the u and l labels, respectively.

The problem of determining the paraphases for diquark lines is somewhat more complex than the corresponding problem for single quark lines. The most direct approach is to begin with the crossing requirements (3.4) applied to diquark lines (where the α and β phases have a u or l su-

TABLE II. Phases for diquark lines.

	Ortho	Para
$e^{i\beta_{3,\text{in}}^{u,l}}$	$e^{i\Gamma_1{}^{u,l}}$	$-e^{i\Gamma_1^{u,l}}e^{-i\xi^{u,l}}$
$e^{i\beta_{4,\text{in}}^{u,l}}$	$e^{i\Gamma_2^{u,l}}$	$-e^{i\Gamma_2^{u,l}}\epsilon e^{-i\xi^{u,l}}$
$e^{i\beta_{3,\text{out}}^{u,l}}$	$e^{i\Gamma_3^{u,l}}$	$e^{i\Gamma_3^{u,l}}e^{-i\xi^{u,l}}$
$e^{i\beta_{4,\text{out}}^{u,l}}$	$e^{i\Gamma_4^{u,l}}$	$e^{i\Gamma_4^{u,l}}\epsilon e^{-i\xi^{u,l}}$
$e^{i\alpha_{3,\text{out}}^{u,l}}$	$e^{i\Gamma_1^{u,l}}e^{\pm i\phi_2}$	$-e^{-i\Gamma_1^{u,l}}e^{\pm i\phi_2}e^{i\xi^{u,l}}$
$e^{i\alpha_{4,\text{out}}^{u,l}}$	$e^{-i\Gamma_2^{u,l}}e^{\pm i\phi_2}$	$-e^{-i\Gamma_2^{u,l}}e^{\pm i\phi_2}\epsilon e^{+i\xi^{u,l}}$
$e^{i\alpha_{3,\text{in}}^{u,l}}$	$e^{-i\Gamma_3^{u,l}}e^{\pm i\phi_1}$	$e^{-i\Gamma_3^{u,l}}e^{\pm i\phi_1}e^{i\xi^{u,l}}$
$e^{i\alpha_{4,\text{in}}^{u,l}}$	$e^{-i\Gamma_4^{u,l}}e^{\pm i\phi_1}$	$e^{-i\Gamma_4^{u,l}}e^{\pm i\phi_1}\epsilon e^{i\xi^{u,l}}$

perscript). We use these relations (3.4) to express the 16 paraphases in terms of the 16 orthophases and 8 other phases $\xi_i^{u,l}$ and $\chi_i^{u,l}$ as follows:

$$e^{i\alpha_{i,\rho}^{u,l}(\dot{z})}=e^{i\alpha_{i,\rho}^{u,l}(z)}(-1)^{\delta_{\rho,\text{out}}}e^{i\xi_i^{u,l}(-1)\delta_{z,P}}\,, \tag{7.10a}$$

$$e^{i\beta_{i,\rho}^{u,l}(\dot{z})}=e^{i\beta_{i,\rho}^{u,l}(z)}(-1)^{\delta_{\rho,\text{in}}}e^{i\chi_i^{u,l}(-1)\delta_{z,P}}\,. \tag{7.10b}$$

The relations (7.10) guarantee that the crossing requirements (3.4) are satisfied. However the χ and ξ phases are not all independent. Requiring that condition (7.6) be satisfied where z_1 and/or z_2 are para as well as enforcing Hermitian analyticity (5.9) leads to the following restriction on the ξ and χ phases of (7.10):

$$e^{i(\xi_i^{u,l}+\chi_i^{u,l})}=1\,, \tag{7.11a}$$

$$e^{i(\chi_i^{u,l}-\chi_j^{u,l})}=\epsilon \;\text{ for }\; i\neq j\,, \tag{7.11b}$$

$$\epsilon=\pm 1\,. \tag{7.11c}$$

The results (7.11) mean that all the ξ and χ phases and, hence, the relation between the ortho and paraphases for quarks in a diquark line are determined by two independent phases which we label $\xi^{u,l}$ and by the factor ϵ which can be either ± 1. The following relations serve to define the $\xi^{u,l}$ phases and to determine the $\xi_i^{u,l}$ and $\chi_i^{u,l}$ phases in terms of them as a consequence of (7.11):

$$e^{i\xi_3^{u,l}}=e^{i\xi^{u,l}} \tag{7.12a}$$

$$e^{i\chi_3^{u,l}}=e^{-i\xi^{u,l}}\,, \tag{7.12b}$$

$$e^{i\xi_4^{u,l}}=\epsilon e^{i\xi^{u,l}}\,, \tag{7.12c}$$

$$e^{i\chi_4^{u,l}}=\epsilon e^{-i\xi^{u,l}}\,. \tag{7.12d}$$

In column three of Table II the paraphases for quarks in diquark lines are given in terms of the orthophases, the two parameters $\xi^{u,l}$ and ϵ.

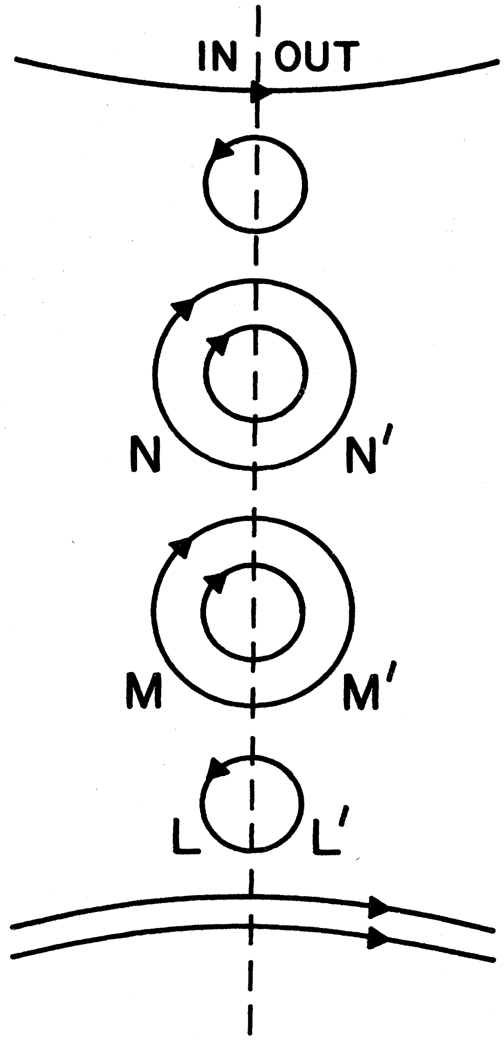

FIG. 7. Intermediate-state configuration.

VIII. DISCONTINUITY EQUATION FOR THE SCALAR ZERO-ENTROPY AMPLITUDES

We now wish to determine the discontinuity equations satisfied by the scalar amplitudes f of (2.2). It is the solving of these nonlinear equations that constitutes the zero-entropy bootstrap problem. To discover these equations we must begin with the full discontinuity equation (2.1) and substitute into it expressions of the form (2.2). Because both the phases Γ and the spin structure functions S^z completely factorize we shall be able to find a discontinuity equation involving f only which involves no spin dependence.

We illustrate in Fig. 7 a typical intermediate-state configuration for Eq. (2.1). When (2.2) is inserted in (2.1) the phases $e^{i\alpha}$ and $e^{i\beta}$ and the τ factor making up the Γ's are exactly the same on both sides of the discontinuity equation (due to factorization) with the exception of the phases for the intermediate single quark and diquark *loops* in Fig. 7 which occur on the right side of Eq. (2.1). For each single quark loop in Fig. 7 we get a factor of

$$e^{i\alpha_{i,\text{out}}(z)}e^{i\beta_{i,\text{in}}(z)}e^{i\alpha_{j,\text{in}}(z)}e^{i\beta_{j,\text{out}}(z)}\,,$$

which by (4.2) and (6.6a) and (6.6b) is just (-1). Thus for each single quark loop in the discontinuity equation, we get a factor of (-1) as originally deduced by Stapp.[2] For each diquark loop in Fig. 7 we get a factor of

$$e^{i\beta_{i,\text{out}}^{u}(z_1)}e^{i\beta_{i,\text{out}}^{l}(z_2)}e^{i\alpha_{i,\text{in}}^{l}(z_2)}e^{i\alpha_{i,\text{in}}^{u}(z_1)}e^{i\beta_{j,\text{in}}^{u}(z_1)}e^{i\beta_{j,\text{in}}^{l}(z_2)} \times e^{i\alpha_{j,\text{out}}^{u}(z_1)}e^{i\alpha_{j,\text{out}}^{l}(z_2)}$$

which by (4.3) and (7.6) is just $(+1)$. Thus we have a factor remaining on the right side of the discontinuity equation after cancellation of the phase factors of $(-1)^n$ where n is the number of quark loops in the intermediate state. Recalling (2.5) we see that there is also a factor of 4^n remaining on the right side of the discontinuity equation after the spin structure factors have been canceled. Finally, if the individual quarks come in N_f different flavors the sum over intermediate state sin (2.1) produces a factor of $(N_f)^n$.

So the discontinuity equation (2.1) written in terms of the scalar amplitude becomes

$$f(+)-f(-)=(2\pi)^4 i \sum_{L\cdots R}(-1)^n(4N_f)^n f(+)f(-)\times\delta^4(p_L+\cdots+p_R-p_A-p_B-\cdots)\,, \tag{8.1}$$

where $\sum$ now refers only to the intermediate momentum integrations. It is the solution of all equations of form (8.1) for scalar amplitudes involving arbitrary numbers of external particles that constitutes the zero-entropy bootstrap problem.

IX. PHASES FOR ZERO-ENTROPY AMPLITUDES

Using the results in the preceding sections, we are now in a position to write down the form of the phase Γ [introduced in (2.2)] for an arbitrary zero-entropy amplitude. We shall write the total phase Γ as a product:

$$\Gamma=\tau\Gamma^S\Gamma^D\,, \tag{9.1}$$

where τ is the overall phase factor discussed earlier depending only on the cyclic order of the variables in the amplitude and having a value of either plus or minus one. The factor Γ^S is the total phase associated with the single quark lines in the zero-entropy amplitude and Γ^D represents the total phase associated with the diquark lines in the amplitude. Recalling that the subscripts 2 and 3 appearing in the left column refer to mesonic and baryonic states, respectively, we can read off from Table I that

$$\Gamma^S=(-1)^{N_{\rm in}(\bar{Q},0)}(-1)^{N_{\rm in}(Q,P)}(-1)^{N_{\rm out}(Q,P)}(-1)^{N_{\rm in}(\bar{Q},P)}\times e^{i\gamma_i[N_{\rm in}(M)-N_{\rm out}(M)]}e^{i\gamma_3[N_{\rm out}(M)-N_{\rm in}(M)]}\times e^{i\gamma_2[N_{\rm in}(B)-N_{\rm out}(B)]}e^{i\gamma_4[N_{\rm out}(\bar{B})-N_{\rm in}(\bar{B})]}\,, \tag{9.2}$$

where $N_{\rm in}(\bar{Q},0)$ designates the number of in antiquarks (associated with single quark lines) which are ortho, $N_{\rm in}(Q,P)$ the number of in quarks which are para, etc., $N_{\rm in}(M)$ designates the number of in meson states irrespective of patch structure, B denotes baryons and, of course, bars denote antiparticles or antiquarks.

To simplify (9.2) somewhat, we first observe that

$$(-1)^{N_{\rm in}(Q,P)+N_{\rm out}(Q,P)}=(-1)^{N_L(P)}\,, \tag{9.3}$$

where $N_L(P)$ denotes the number of "like" paraquark lines, meaning that the head and tail of the quark line are either both incoming or both outgoing. A careful distinction must be made in the counting here because $N_L(P)$ refers to entire quark lines whereas $N_{\rm in}(Q,P)$ and $N_{\rm out}(Q,P)$ are determined by counting heads and tails of quark lines separately. Another simplification can be introduced through the observation that zero-entropy amplitudes automatically conserve baryon number. Thus we have

$$N_{\rm in}(B)-N_{\rm in}(\bar{B})=N_{\rm out}(B)-N_{\rm out}(\bar{B})\,, \tag{9.4}$$

and (9.2) becomes

$$\Gamma^S=(-1)^{N_L(P)}(-1)^{N_{\rm in}(\bar{Q})}e^{i(\gamma_1-\gamma_3)[N_{\rm in}(M)-N_{\rm out}(M)]}e^{i(\gamma_2-\gamma_4)[N_{\rm in}(B)-N_{\rm out}(B)]}\,, \tag{9.5}$$

where $N_{\rm in}(\bar{Q})$ denotes the number of incoming antiquarks irrespective of patch structure.

To determine Γ^D in (9.1) we consult Table II and find

$$\Gamma^D=(-1)^{n_{\rm in}(Q,P)+n_{\rm out}(Q,P)}\epsilon^{n(34,P)}e^{i\Gamma_1^+[N_{\rm in}(B)-N_{\rm out}(B)]}e^{i\Gamma_3^+[N_{\rm out}(\bar{B})-N_{\rm in}(\bar{B})]}e^{i\Gamma_2^+[N_{\rm in}(\mathscr{B})-N_{\rm out}(\mathscr{B})]}e^{i\Gamma_4^+[N_{\rm out}(\mathscr{B})-N_{\rm in}(\mathscr{B})]}\,, \tag{9.6}$$

where

$$\Gamma_i^+=\Gamma_i^l+\Gamma_i^u\,, \tag{9.7}$$

and where $\mathscr{B}$ denotes baryonium. In (9.6), $n(34,P)$ is the number of paraquarks or antiquarks within diquark lines involved in a (3,4) transition (baryon or antibaryon to baryonium). The symbols $n_{\rm in}(Q,P)$ and $n_{\rm out}(Q,P)$ denote the number of in and out paraquarks within diquark lines. These quantities are exactly analogous to the corresponding quantities discussed above for single quark lines except in the latter case a capital N was used. In analogy with (9.3) we thus have

$$(-1)^{n_{\rm in}(Q,P)+n_{\rm out}(Q,P)}=(-1)^{n_L(P)}\,, \tag{9.8}$$

where $n_L(P)$ is the number of like paraquark lines with diquark lines. Using (9.4) in (9.6) together with (9.8) and combining the phases for both single quark and diquark lines we get for the total phase:

$$\Gamma=\tau(-1)^{N_L^+(P)}(-1)^{N_{\rm in}(M)+N_{\rm in}(\bar{B})}\epsilon^{n(34,P)}e^{i(\gamma_1-\gamma_3)[N_{\rm in}(M)-N_{\rm out}(M)]}e^{i(\gamma_2+\Gamma_1^+-\gamma_4-\Gamma_3^+)[N_{\rm in}(B)-N_{\rm out}(B)]}e^{i(\Gamma_2^+-\Gamma_4^+)[N_{\rm in}(\mathscr{B})-N_{\rm out}(\mathscr{B})]}\,, \tag{9.9}$$

where $N_L^+(P)$ is the total number of like paraquark lines in the amplitude whether occurring as single quark lines or within diquarks. We have also made use in (9.9) of the identity

$$(-1)^{N_{\rm in}(\bar{Q})}=(-1)^{N_{\rm in}(M)+N_{\rm in}(\bar{B})}\,. \tag{9.10}$$

X. DISCUSSION

The result (9.9) gives a complete characterization of the phases associated with zero-entropy amplitudes, consistent with crossing, Hermitian analyticity, and pole factorization. The entire dependence of this phase on patch structure is contained in the two factors

$$(-1)^{N_L^+(P)}\epsilon^{n(34,P)} . \tag{10.1}$$

The other factors in (9.9) with the exception of τ depend only on the particular reaction involved and are common to each term in the zero-entropy sum. It will be shown elsewhere that (10.1) which gives the relative phases between zero-entropy amplitudes of different patch structure[5] implies the existence of parity, time-reversal, and charge-conjugation invariance in strong interactions.

As a final observation, we note that a number of the phases introduced in Tables I and II to characterize the phase dependence for the heads and tails of quark lines drop out of the final expression (9.8). In particular the zero-entropy amplitudes are independent of the phases ξ, ξ_u, ξ_l, ϕ_1, and ϕ_2. Also the final phase Γ depends only on the differences $(\gamma_i-\gamma_j)$, $(\Gamma_i^u-\Gamma_j^u)$, and $(\Gamma_i^l-\Gamma_j^l)$. Thus if a constant phase is added to each of the γ_i, another added to the Γ_i^u, and still another added to the Γ_i^l the amplitude remains unchanged. Invariance of the zero-entropy amplitudes under these phase changes can be interpreted, of course, as a global gauge invariance. This gauge invariance for the phases γ and Γ appears on the surface to mean no more than that quark number is conserved.

The meaning of the gauge invariance associated with the change of the phases ξ, ξ_u, ξ_l, ϕ_1, and ϕ_2 is more obscure. In particular, whether any of this gauge invariance can be interpreted as some kind of global remnant of local SU(3) color is by no means clear. Nonetheless, understanding the relationship of QCD and the topological theory is an important problem and the gauge invariances noted here could provide clues for its solution. Finally, we note that the phases in Tables I and II can from the point of view of the zero-entropy amplitudes be picked arbitrarily. This differs from the original treatment of Stapp[2] whose model assigned specific values to these phases based in part on field theory analogies. In particular no consideration at the zero-entropy level appears to prevent us from taking the phases all to be zero. This would make the phases Γ of (9.1) all real.

ACKNOWLEDGMENT

This work was in part funded by National Science Foundation Grant No. PHY-8308053.

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