1975

Kinetic Theory of Gases

KINETIC THEORY OF GASES

INTRODUCTION

As you read this sentence you will experimentally demonstrate the general gas law at least once by breathing in and out. As you expand the volume in your lungs, the pressure drops and air comes in; as you decrease the lung volume the pressure rises and air goes out. Pressure (p) and volume (V) are related; at constant temperature, pV = const.

This relation, called Boyle's law, was well established before the atomic theory of matter was accepted. In this module you will learn how to apply much of your knowledge of Newton's laws, kinetic energy, momentum, and elastic collisions to molecular motion. The fact that you can derive the macroscopic general gas law from Newtonian mechanics applied to molecules should give you further insight into the remarkable simplification Newton wrought in our understanding of matter. You will show that his laws hold equally well for masses 10^{27} times smaller and dimensions 10^{10} times smaller than the blocks and inclined planes we have discussed in previous modules. Newton's laws work from planetary physics to microphysics, an astronomical range!

To be sure, we have overstated the case in order to make a point. The behavior of an ideal gas is just that, idealized, and real gases only approximately behave this way. At the molecular level the fundamental "graininess" or quantization of momentum and energy as well as matter becomes important so that kinetic theory is also an idealization of nature.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Solve mechanical problems using Newton's second law in the form: force = time rate of change of momentum (needed for Objective 3 of this module)	Newton's Laws Module
*Solve collision problems in which kinetic energy and momentum are conserved (needed for Objective 3 of this module)	Collisions Module
*Define the terms <u>temperature</u>, <u>heat</u>, and <u>specific heat</u> (needed for Objective 1 of this module)	Temperature, Heat, and Thermodynamics Module
*Differentiate a given polynomial (needed for Objectives 3 and 5 of this module)	Calculus Review
*Find the total rotational energy of a rigid body (needed for Objective 5 of this module)	Rotational Dynamics Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Ideal gas - macroscopic - Define an ideal gas from a macroscopic point of view using the equation of state, $pV = nRT$, and use this equation to solve problems involving the relations among the macroscopic variables p, V, n, and T.
2. Ideal gas - microscopic - Give the six postulates used in this module to define the microscopic kinetic-theory model of an ideal gas.
3. Kinetic theory - Solve problems based on the steps involved in the kinetic-theory derivation of the macroscopic pressure. These problems will be based on the relationships among pressure, force, velocity, momentum, and the number of particles.
4. Temperature - molecular speed - Use the definition of temperature in the kinetic model to show that the kinetic-theory pressure equation $pV = NmV_{rms}^2/3$ is equivalent to the equation of state; or solve problems involving the relationship among T, m, and v_{rms}.
5. Equipartition of energy - State the law of equipartition of energy and apply it to the calculation of C_V for gases; or apply it to the calculation of molecular speeds in containers holding a mixture of gases.

GENERAL COMMENTS

1. Order of Presentation

Kinetic theory may be defined as the statistical application of the laws of mechanics to assemblies of atoms using relatively simple averaging techniques. More advanced techniques are utilized in a branch of physics called statistical mechanics. This module introduces the kinetic theory of gases in the following order: (1) an ideal gas is defined from the macroscopic point of view as a gas that exactly follows the equation of state; (2) a list of postulates is given for a kinetic theory of gases from a microscopic point of view; (3) using these postulates, the macroscopic quantity pressure is calculated from the kinetic theory; and (4) by making an identification of temperature in terms of molecular kinetic energy, the macroscopic equation of state is derived from a molecular model using kinetic theory. Not all the texts handle the material in this order; however, if you will read the material assigned with the above outline in mind, you should have no difficulty with this module.

2. Postulates for the Kinetic Theory

Objective 2 asks you to list the assumptions that define the microscopic model of an ideal gas. Various texts list these in various orders and forms. The following list is given in terms of short phrases that will be easier to reproduce on the Mastery Test. Your written answers need not include more than this, but you may be asked to give more detail orally.

a. Large number of particles. The direction and speed of any molecule may change abruptly on collision, and any particular molecule will follow a zigzag path. However, the very large number of particles ensures a distribution of molecular velocities and randomness of the motion.

b. Negligible particle volume. The molecules of a gas are, on the average, separated by distances that are large compared with the molecular diameters. We assume the molecules have negligible volume compared with the volume of the container.

c. Random motion. The molecules move in all directions with equal probability and with a variety of speeds. The center of mass of the gas as a whole remains at rest, the total linear momentum being zero.

d. No forces except during collisions. Between collisions the molecules are free of forces and move with a constant speed. A collision takes place when one molecule is within the short-range intermolecular force of a second molecule. The duration of any collision is assumed to be very small compared with the time between collisions. Thus, although a pair of molecules will lose kinetic energy and gain potential energy during a collision, the potential energy can be ignored because a molecule spends a negligible fraction of its time in collisions.

e. Elastic collisions. The collisions between molecules are perfectly elastic.

f. Newtonian mechanics applies. In computing the properties of the motion, we assume that Newtonian mechanics works at the microscopic level. This assumption will stand or fall depending on whether or not the experimental facts it predicts are correct.

3. Temperature Dependence

Some texts do not place great emphasis on the result that the internal energy of an ideal gas, U, as derived using the kinetic theory, depends only on the temperature. The importance of this result will be seen in other modules (Second Law and Entropy).

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read the General Comment 1. This will make clearer the reason for the selection of text readings. Read Chapter 15, Sections 15.1, 15.2, and 15.4 and Chapter 16, Sections 16.4 and 16.7. Bueche does not introduce the equation of state, $pV = nRT$, as a macroscopic law, but proceeds to calculate it from kinetic theory. This accounts for the arrangement of readings with respect to the objectives of this module.

The author gives the assumptions of the kinetic theory required in Objective 2 in narrative form. For your convenience these are listed in General Comment 2. Read General Comment 3. Study Problems A through I. Then solve the Problems J through Q. If you need more practice, you may work some of the optional Additional Problems

Try the Practice Test.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	
1	General Comment 1, Sec. 15.4	A, B	Illus.[a] 15.1, 15.2	J, K	Chap. 15, Probs. 6, 7, 11-13
2	Sec. 15.1, General Comment 2				
3	Sec. 15.2	C, D		L, M	
4	Sec. 15.4, General Comment 3	E, F	Illus. 15.3	N, O	Chap. 15, Probs. 3, 4, 8, 9
5	Secs. 16.4, 16.7	G, H, I	Illus. 16.2	P, Q	Chap. 16, Probs. 10, 14, 27

[a]Illus. = Illustration(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read Chapter 20, Sections 20-1 through 20-7. In your reading of Section 20-6 on the specific heat, you should read the derivation of C_p as well as of C_v. Although only C_v will be studied in this unit, the next module (Second Law and Entropy) requires the definition of C_p. Your text uses the symbol μ for the number of moles, whereas we have used the more common symbol n ($\mu \equiv n$). Read General Comments 1 through 3. Study Problems A through I. Then solve problems J through Q. If you need more practice, you may work some of the optional Additional Problems from your text.

Try the Practice Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	(Chap. 20)
1	General Comment 1, Sec. 20-2	A, B	Ex.[a] 1, 2	J, K	2, 3, 4
2	General Comment 2, Sec. 20-3				
3	Sec. 20-4	C, D	Ex. 3, 4	L, M	13
4	Sec. 20-5, General Comment 3	E, F		N, O	14, 15, 16, 18, 21, 23, 24
5	Secs. 20-6, 20-7	G, H, I	Ex. 6	P, Q	22, 23, 43, 44

[a]Ex. = Example(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read the General Comment 1. This will make clearer the reason for the selection of the text readings. Read Chapter 18, Sections 8-1 and 8-2, and Chapter 20, Sections 20-1 to 20-4 and 20-7. Objective 2 deals with the specific assumptions in the kinetic-theory derivation of the equation of state. The text gives these in narrative style in Section 20-3 (pp. 287, 288). For your convenience in mastering Objective 2, the assumptions are listed in General Comment 2.

Read General Comment 3. Then study Problems A through I before solving Problems J through Q. If you need more practice, you may wish to work some of the optional Additional Problems from your text.

Try the Practice Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	
1	General Comment 1, Secs. 18-1, 18-2	A, B	Sec. 18-2, Ex.[a] 1	J, K	18-1 to 18-12, 20-1 to 20-3
2	General Comment 2, Sec. 20-3				
3	Secs. 20-1 to 20-3	C, D		L, M	20-4
4	Sec. 20-3, General Comment 3	E, F	Sec. 20-3, Ex. 1, 2, 3, 4	N, O	20-5, 20-8
5	Secs. 20-4, 20-7	G, H, I		P, Q	20-6, 20-7

[a]Ex. = Example(s).

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Read the General Comment 1. This will make clearer the reason for the selection of readings in your text. Read Chapter 18, Sections 18-5 and 18-6, Chapter 19, Sections 19-1 to 19-3 and 19-5, General Comment 2, and Chapter 20, Section 20-4. You may already have read Section 19-5 on specific heat in relation to the module Temperature, Heat, and Thermodynamics; however, the emphasis now is on the influence of extra degrees of freedom introduced by rotation and vibration.

Read General Comment 3. Then study Problems A through I before solving Problems J through Q. If you need more practice you may work some of the optional Additional Problems from your text.

Try the Practice Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	
1	General Comment 1, Secs. 18-5, 18-6	A, B	Ex.[a] 18-1, 18-2	J, K	18-2, 18-11, 18-12, 18-14 to 18-18, 18-20
2	General Comment 2, Secs. 19-1, 19-2				
3	Sec. 19-2	C, D	Ex. p. 381	L, M	19-2, 19-5
4	Sec. 19-3, General Comment 3	E, F	Ex. 19-1	N, O	19-6 to 19-8, 19-12
5	Secs. 19-5, 20-4	G, H, I		P, Q	19-19, 19-20

[a]Ex. = Example(s).

PROBLEM SET WITH SOLUTIONS

A(1). Show that the kinetic-theory model of an ideal gas obeys Avogadro's law: If two containers of gas have the same volume, pressure, and temperature, then there are equal numbers of molecules in them.

Solution

$$pV = nRT = (N/N_0)kN_0T = NkT$$

since $n = N/N_0$ and $R = kN_0$. Therefore,

$$N = pV/kT.$$

It is given that

$$p_1V_1/kT_1 = p_2V_2/kT_2 = \text{const},$$

therefore

$$N = pV/kT = \text{const}.$$

B(1). A tank having a volume of 30.0 ℓ contains nitrogen (N_2) gas at 20° C and at a gauge pressure of 3.00×10^5 Pa. The tank's valve is opened momentarily, and some N_2 escapes. After the valve is closed and the gas has returned to room temperature (20° C), the tank's pressure gauge reads 2.40×10^5 Pa. How much N_2 leaked out?

Solution

Before: $p_1V_1 = n_1RT_1$, After: $p_2V_2 = n_2RT_2$. We wish to find

$$n_1 - n_2 = p_1V_1/RT_1 - p_2V_2/RT_2.$$

But $T_1 = T_2$ and $V_1 = V_2$, therefore,

$$n_1 - n_2 = \frac{V}{RT}(p_1 - p_2) = \frac{(3.00 \times 10^{-2}\ \text{m}^3)(6.00 \times 10^4\ \text{Pa})}{(8.31\ \text{J/g mol K})(293\ \text{K})} = 7.4 \times 10^{-1}\ \text{mol}.$$

But since 1 mol of N_2 = 28.0 g = 0.0280 kg,

$$m = n(0.0280) = 7.4 \times 10^{-1} \times 2.80 \times 10^{-2} = 2.10 \times 10^{-2}\ \text{kg}.$$

C(3). Precisely one million molecules strike a surface with an area of $1.00 \times 10^{-2}\ m^2$ in one second and bounce elastically. Each of these molecules strikes at an angle of 60° from the normal, each has a mass of 3.30×10^{-27} kg, and the average speed of these million molecules is 330 m/s (the speed of sound at normal conditions).

(a) What is the magnitude of the total momentum of these molecules before they strike the surface? How large is its component perpendicular to the surface?

(b) What average pressure do these molecules exert on this surface during the one second they are striking it?

(c) Normal atmospheric pressure is about 10^5 Pa; why is this so different from your answer to (b)?

Solution

(a) The momentum of one molecule is mV; the momentum of 10^6 molecules is

$$Nmv = (10^6)(3.30 \times 10^{-27}\ kg)(330\ m/s) = 1.10 \times 10^{-18}\ kg\ m/s.$$

The component perpendicular to the surface is

$$Nmv \cos 60° = 0.55 \times 10^{-18}\ kg\ m/s.$$

(b) The pressure is force per unit area. The pressure on the wall is

$$p = \frac{-Nd(mv)}{A\ dt} = \frac{-N(mv_{x_f} - mv_{x_i})}{At} = \frac{-N(-mv \cos 60° - mv \cos 60°)}{A(1.00\ s)}$$

$$= \frac{2Nmv \cos 60°}{A} = \frac{2(1.10 \times 10^{-18}\ kg\ m/s)(0.50)}{(10^{-2}\ m^2)(1.00\ s)} = 1.10 \times 10^{-16}\ Pa.$$

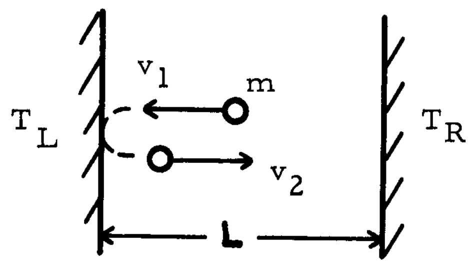
(c) There are approximately 3×10^{25} molecules per cubic meter in a gas at atmospheric pressure: the number hitting the wall per second is

$$(1/3)pAv = (3 \times 10^{25}\ molecules/3)(10^{-2} m^2)(3 \times 10^3\ m/s).$$

This equals about 10^{27} collisions per second, which is 10^{21} times more than we assumed. Therefore,

$$p_{atm} = (10^{21})(1.10 \times 10^{-16}\ Pa) \simeq 10^5\ Pa.$$

D(3). Assume that a particle bounces normally off a heated surface with a kinetic energy that is in thermal equilibrium with the surface temperature regardless of its approach energy. Calculate the average force F produced on the left-hand plate by a single molecule of mass m in bouncing back and forth.



Solution

$$F = \left(\frac{\text{number of collisions}}{\text{second}}\right)(\text{change in momentum of wall}) = \left(\frac{1.00\ \text{s}}{\text{round-trip time}}\right)\left(\frac{mv_2 + mv_1}{1.00\ \text{s}}\right).$$

The round-trip time is the sum of right to left and the return:

$$t = L/v_1 + L/v_2.$$

Therefore,

$$F = m(v_2 + v_1)v_1v_2/(Lv_2 + Lv_1) = mv_1v_2/L.$$

Note that this reduces to

$$F = mv^2/L \quad \text{if } v_1 = v_2.$$

From the kinetic definition of temperature

$$(1/2)kT = (1/2)mv_x^2,$$

we can evaluate v in terms of T:

$$v_1 = (kT_R/m)^{1/2}; \qquad v_2 = (kT_L/m)^{1/2}.$$

Thus

$$F = (mk/Lm)(T_RT_L)^{1/2} = (k/L)(T_RT_L)^{1/2}.$$

E(4). Escape velocity from the Moon is approximately 5310 miles per hour = 2.37×10^3 m/s. At certain spots on the surface of the Moon, the temperature of the surface material reaches 500 K. Suppose a molecule of hydrogen

($m = 3.33 \times 10^{-27}$ kg) were to hit the surface of the Moon and reach a speed equal to the root-mean-square (rms) speed for this temperature. What would happen to this molecule?

Solution

$$(mv^2/2)_{av} = (3/2)kT, \quad (v^2)_{av} = 3kT/m,$$

$$v_{rms} = \left(\frac{3kT}{m}\right)^{1/2} = \left[\frac{3(1.38 \times 10^{-23}\ \mathrm{J/K})(500\ \mathrm{K})}{3.33 \times 10^{-27}\ \mathrm{kg}}\right]^{1/2} = \left[\frac{3(1.38 \times 5 \times 10^{6}\ \mathrm{m^2/s^2})}{3.33}\right]^{1/2}$$

$$= 2.49 \times 10^{3}\ \mathrm{m/s}.$$

Since this speed is faster than escape velocity, the hydrogen molecule would escape from the Moon. Although other more massive molecules would not achieve escape velocity as easily, enough of them have so that the Moon now has no atmosphere.

F(4). The kinetic-theory derivation of the macroscopic pressure gave the result

$$p = (Nmv_{rms}^2)/3V.$$

It is easy to identify the average kinetic energy of the gas in this formula and reasonable to guess that temperature, which we know is related to heat energy, is proportional to the kinetic energy. Show that if

$$T = \frac{1}{2}\left(\frac{Mv_{rms}^2}{3nR/2}\right),$$

the pressure formula above reduces to the equation of state for an ideal gas.

Solution

Rearranging the pressure equation, we find

$$pV = \frac{Nmv_{rms}^2}{3} = \frac{Mv_{rms}^2}{3} = \frac{2}{3}\left(\frac{1}{2}Mv_{rms}^2\right).$$

From the definition of T, we see that $(1/2)Mv_{rms}^2 = (3/2)nRT$. Thus $pV = nRT$.

G(5). An air-table model of an ideal gas has pucks with masses of 2.00×10^{-2} kg and 8.0×10^{-2} kg. It is found that the 8.0×10^{-2} kg pucks have an rms speed of 0.50 m/s. An air table provides a level, nearly frictionless surface. The pucks, riding on a cushion of air, make nearly elastic

collisions with each other and with a taut wire fence at the boundaries.
(a) Approximately what will be the rms speed of the 2.00×10^{-2} kg pucks?
(b) The motion of the pucks in this model is, of course, provided by making the wire "fence" around the edges oscillate back and forth. However, if we raised the temperature of the pucks and the air table sufficiently, this motion of the pucks would occur naturally, as a thermal motion. (Assume the pucks and air table can stand the high temperature.) What temperature would be needed for this?

Solution

(a) By the equipartition theorem, each degree of freedom has the same energy [(1/2)kT] for each of the two translational axes, and therefore kT for the sum of the two. The "temperature" must be the same for the small pucks, therefore

$$(1/2)m_1 v^2_{(rms)1} = (1/2)m_2 v^2_{(rms)2}$$

or

$$v_{(rms)2} = \left(\frac{m_1}{m_2}\right)^{1/2} v_{(rms)1} = \left(\frac{8.0}{2.00}\right)^{1/2}(0.50) = 1.00 \text{ m/s}.$$

(b) If the motion were cased by temperature,

$$T = \frac{1}{2k} m_1 v^2_{rms} = \frac{(8.0 \times 10^{-2} \text{ kg})(0.50 \text{ m/s})^2}{2(1.38 \times 10^{-23} \text{ J/K})} = 7.2 \times 10^{20} \text{ K}.$$

H(5). (a) State the equipartition theorem.
(b) What is the specific molar heat at constant volume for a diatomic molecule that can be represented as a rigid dumbbell?

Solution

(a) There is one degree of freedom associated with each independent coordinate of the system of particles. Each degree of freedom has the same mean energy at thermal equilibrium [(1/2)kT].

(b) There are three degrees of freedom for translational kinetic energy. For a dumbbell there are only two rotational degrees of freedom, since for point masses the angle of rotation about an axis through the two masses is not a physical independent coordinate ($I_y = I_z$; $I_x = 0$). Since there are a total of five degrees of freedom,

$$U = (5/2)NkT,$$

$$C_V = \frac{dU}{dt} = \frac{d}{dt}(\frac{5}{2}N_0kT) = \frac{5}{2}N_0k = \frac{5}{2}R = \frac{5}{2}(8.31\text{ J/mol K}) = 20.8\text{ J/mol K}.$$

I(5). Calculate the specific heat of a three-dimensional solid.

Solution

Since a solid has its atoms in fixed locations the atoms are not free to translate. Neither do atoms have rotational degrees of freedom because of the structure of a solid. However, they do have vibrational degrees of freedom in each of three dimensions. Thus there are three degrees of freedom for vibrational potential energy and three degrees of freedom for vibrational kinetic energy for a total of six degrees of freedom. Thus

$$V = 6(1/2)N_0kT$$

for a mole of the solid, and

$$C_V = (dU/dt)_V = 3N_0k = 3R = 3(8.31\text{ J/mol K}) = 24.9\text{ J/mol K}.$$

Problems

J(1). An evacuated tank with a capacity of 40 ℓ is filled with helium at room temperature (20.0°C). The mass of the tank is thereby increased by 0.150 kg. What then is the pressure registered on the tank's pressure gauge?

K(1). Imagine a cube of gas kept at a temperature of 300 K and a pressure of 1.00×10^5 Pa. If this cube contains a number of molecules equal to the population of the world (4 000 000 000 people), what is the length of one of its edges?

L(3). The mass of a H_2 molecule is 3.32×10^{-27} kg. If 1.00×10^{23} hydrogen molecules per second strike $2.00 \times 10^{-4}\text{m}^2$ of wall at an angle of 45° with the normal when moving with a speed of 10^3 m/s, what pressure do they exert on the wall?

M(3). The temperature at the Sun's center is estimated to be about 14×10^6 K, whereas the density there is about $1.00 \times 10^5\text{ kg/m}^3$. Assuming the protons that constitute most of the Sun's core act like a perfect gas even at these very high densities, find the pressure of the proton gas at the Sun's core ($m_p = 1.67 \times 10^{27}$ kg).

N(4). The fragments from uranium atoms that have undergone unclear fission have an average kinetic energy of 1.10×10^{-11} J. What would be the approximate temperature of a gas consisting of such fission fragments?

O(4). Show that the kinetic-theory pressure equation is equivalent to the equation of state of an ideal gas if

$T = NK \quad T = \frac{NK}{3nR/2}$, where $K = (1/2)mv^2_{rms}$.

P(5). Assuming air to be composed of 78% nitrogen (N_2) and 22.0% oxygen (O_2) by weight, what should be the value of molar C_V for air?

Q(5). A flask contains a mixture of mercury vapor, neon, and helium. Compare (a) the average kinetic energies of the three types of atoms; and (b) the rms speeds.

Solutions

J(1). 23.0×10^5 Pa.

K(1). 5.5×10^{-6} m.

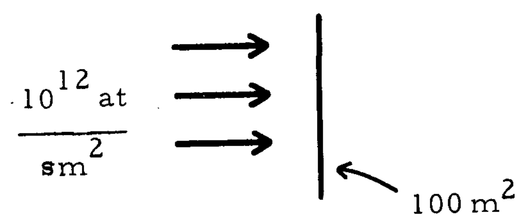
L(3). 2400 Pa.

M(3). 1.20×10^{16} Pa.

N(4). 5.3×10^{11} K.

O(4). $T = \frac{NK}{(3/2)nR} = \frac{N(1/2)mv_{rms}^2}{(3/2)nR} = \frac{Nmv_{rms}^2}{3nR}$; $nRT = Nmv_{rms}^2/3$. But $pV = Nmv_{rms}^2/3$, from the pressure equation. Thus $pV = nRT$.

P(5). 0.72 J/mol K.



PRACTICE TEST

1. (a) Give a macroscopic definition of an ideal gas.

   (b) A bubble of air rises from the bottom of a lake, where the pressure is 3.03×10^5 Pa, to the surface, where the pressure is 1.00×10^5 Pa. The temperature at the bottom of the lake is 7.0°C, and the temperature at the surface is 27.0°C. What is the ratio of the volume of the bubble as it reaches the surface to the volume of the bubble at the bottom?

2. There is a flow of gas radially outward from the Sun into space called the "solar wind." In the vicinity of the Earth the flow is primarily hydrogen atoms with a speed of 3.00×10^4 m/s. If the flux of atoms is 10^{12} atoms per second per square meter, calculate the acceleration of a 100-m^2 thin aluminum foil sail, normal to the flux of atoms, which has a mass of 10^{-2} kg($M_H = 1.67 \times 10^{-27}$ kg). (Assume elastic collisions.) (See figure above.)

3. Give six statements that define the microscopic kinetic-theory model of an ideal gas.

4. In some nuclear reactions, it is desirable to have "thermal neutrons," i.e., a large number of neutrons with a temperature T = 27.0°C.
   (a) What is your best estimate of the kinetic energy of such a neutron?
   (b) What is your best estimate of its speed?

5. Ammonia (NH_3) is a polyatomic gas. Use the kinetic theory to derive the value of C_V for ammonia.

Practice Test Answers

1.(a) An ideal gas is one that satisfies the equation of state pV = nRT.

(b) At the bottom: $p_1V_1 = nRT_1$; at the surface: $p_2V_2 = nRT_2$. Since n is a constant, $p_2V_2/p_1v_1 = T_2/T_1$, or

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}\frac{p_2}{p_1} = \frac{300\ \text{K}}{280\ \text{K}}\ \frac{3.03 \times 10^5\ \text{Pa}}{1.00 \times 10^5\ \text{Pa}} = 3.24.$$

2. $F = ma = \frac{dp}{dt} = A(2mV)\#/s\ m = A(\frac{\#}{s})(2mV)$,

$$a = \frac{A}{M}\left(\frac{\#}{s}\right)(2mV) = \left(\frac{10^2\ \text{m}^2}{10^{-2}\ \text{kg}}\right)\left(\frac{10^{12}}{\text{m}^2\ \text{s}}\right)2(1.67 \times 10^{-27}\ \text{kg})(3.00 \times 10^4\ \text{m/s}) = 1.00 \times 10^{-6}\ \text{m/s}^2.$$

3. Large numbers of particles, negligible particle volume, random motion, no forces except during collisions, elastic collisions, Newtonian mechanics applies.

4.(a) $(KE)_{av} = \frac{1}{2}N^{-2}_{rms} = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23}\ \text{J/K})(300\ \text{K}) = 6.2 \times 10^{-21}\ \text{J}.$

(b) $v_{rms} = [2(KE)_{av}/m]^{1/2} = [2(6.2 \times 10^{-21}\ \text{J})/(1.67 \times 10^{-27}\ \text{kg})]^{1/2} = 2.70 \times 10^3\ \text{m/s}.$

5. Ammonia has three translational degrees of freedom and three rotational ones:

$K_{trans} = (3/2)N_0kT$; $K_{rotat} = (3/2)N_0kT$;

$U = K_{trans} + K_{rotat} = 3N_0kT$; $C_v = dU/dt = 3N_0k = 3R.$

Date ____________________

KINETIC THEORY OF GASES — pass recycle

Mastery Test Form A — 1 2 3 4 5

Name ____________________ Tutor ____________________

1. (a) Give a macroscopic definition of an ideal gas.
   (b) How many molecules per cubic meter are there in a radio-tube vacuum? The pressure is about 10^{-9} atm (1 atm = 1.01×10^5 Pa). Assume T = 300 K.

2. List three of the six postulates that define an ideal gas in the microscopic model.

3. Professor M.V. Square handed to his secretary the following kinetic-theory derivation of the pressure on the wall of a cubical box of side ℓ, containing N molecules in random motion. The secretary left blanks when she could not decipher his handwriting. Fill in the blanks.

"Suppose a particle strikes the wall. The change in __________/collision = __________. The change in __________ of the wall (area = ℓ^2) = __________ by conservation of __________. After a time t, the particle should again strike the same wall. This time t = __________, so that the number of collisions per second with the wall is __________. The average force is given by the rate of change of __________ per unit __________. Therefore, for one particle F = __________. We now add up the forces for N molecules and divide by __________ to get

$$p = (\_\_\_\_\_\_\_\_\_\_) \sum_{i}^{N} \_\_\_\_\_\_\_\_\_\_ .$$

However, the general definition of the average is

$$\sum_{i=1}^{N} \frac{x_i}{N} = x_{av}.$$

Therefore,

$$p = (\_\_\_\_\_\_\_\_\_\_)(v_x^2)_{av}.$$

Now

$$v^2 = v_x^2 + v_y^2 + v_z^2,$$

and since the velocities are assumed random

$$(v_x^2)_{av} = (\_\_\_\_\_\_\_\_\_\_)(v^2)_{av} \quad \text{and} \quad p = \_\_\_\_\_\_\_\_\_\_ ."$$

(Continued on next page)

KINETIC THEORY OF GASES

Mastery Test   Form A

4. The kinetic theory model of an ideal gas contains many predictions. For instance:
(a) Show what happens to the pressure when the speed of every molecule is doubled, and the volume is kept unchanged.
(b) What does this doubling do to the internal energy of the gas?
(c) What does this indicate about the relationship between the pressure and the temperature, in general, when the volume is kept constant? (You should come up with what is known as Gay-Lussac's or Charles' law. Recall that in the kinetic-theory model, the temperature is defined in terms of the internal energy - what is that definition?)

5. (a) State the equipartition theorem.
(b) Equal masses of hydrogen and oxygen are mixed. What is the ratio of numbers of molecules? What is the ratio of average kinetic energy per molecule? What is the ratio of average speeds? What is the ratio of pressures exerted on the walls? ($M_{H_2} = 2$, $M_{O_2} = 32$.)

Date ______________________

KINETIC THEORY OF GASES

pass recycle

Mastery Test Form B

1 2 3 4 5

Name ______________________ Tutor ______________________

1. (a) Give a macroscopic definition of an ideal gas.
(b) An automobile tire has a volume of 1.60×10^{-2} m^3 and contains air at a pressure of 2.70×10^5 Pa (approximately 24 lb/in.^{2} gauge pressure) when the temperature is 0.00°C. What is the pressure of the air in the tube when its temperature rises to 27.0°C and its volume increases to 1.63×10^{-2} m^3?

2. Give three of the six postulates that define a microscopic ideal gas.

3. Suppose that one molecule occupies a cubical box all by itself. Assume the walls of the box are perfectly smooth, so that the molecule bounces away from any wall at the same angle at which it struck the wall. Originally, the molecule has the speed v_0, and exerts the <u>average</u> pressure p_0 on the right-hand wall. But now, by some cunning device, we manage to accelerate this molecule to twice its original speed (i.e., to $2v_0$) without changing its direction of motion. How large is the average force it now exerts on the right-hand wall?

4. Find the rms speed of helium atoms at 40°C given that the rms speed of oxygen molecules at 0.00°C is 460 m/s. The molecular weights are O_2 = 32.0 g/mol; He = 4.0 g/mol.

5. (a) State the equipartition theorem.
(b) Show that the kinetic-theory model of an ideal gas obeys Dalton's law of partial pressures: When several different gases occupy the same container, the pressure they exert on the container is the sum of the pressures that each one would exert if it alone occupied the container.

Date ____________________

KINETIC THEORY OF GASES

pass recycle

Mastery Test Form C

1 2 3 4 5

Name ____________________ Tutor ____________________

1. (a) Give a macroscopic definition of an ideal gas.
(b) While preparing a sealed-off 2.00×10^{-5} m^3 tube at low temperatures, one nitrogen drop of mass 5.6×10^{-5} kg is accidentally sealed off in the tube. Find the nitrogen (N_2) pressure within the tube when the tube warms to 300 K (M_{N_2} = 14.0 g/mol).

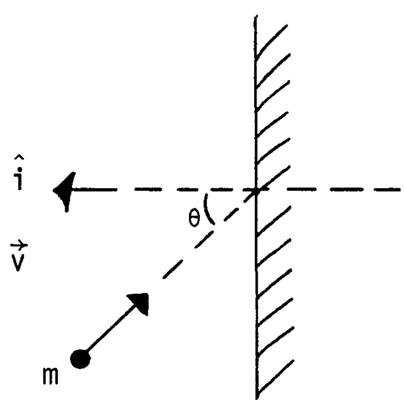
2. List three of the six postulates that define an ideal gas in the microscopic model.

3. A stream of gas molecules (mass m) is incident upon a wall as in Figure 1. If the collisions are elastic, the velocity of the molecules is $\vec{v}$, and the normal to the wall is $\hat{i}$, what pressure is exerted on the wall by the molecules if N molecules strike the wall per unit time per unit area?

4. (a) Determine the average value of the kinetic energy of the molecules of an ideal gas at 0.00°C.
(b) What is the kinetic energy per mole of an ideal gas at these temperatures?

5. (a) State the equipartition theorem.
(b) The mass of a gas molecule can be computed from the specific heat at constant volume. Take C_V = 0.314 J/kg K for argon and calculate (i) the mass of an argon atom, and (ii) the atomic weight of argon.

Figure 1



M 5 - KINETIC THEORY OF GASES　　　Date____________

Mastery Test Form (D)　　　pass ______　　recyle______
Instructor______________________　　　1 2 3 4 5
Name______________________　　Tutor______________

1. a) What does the formula PV=nRT have to do with a perfect gas?

   b) The following assumptions are used to define the microscopic model of an ideal gas; true or false.

      i) The particles are at zero temperature (T, F).

      ii) There are no forces except during collisions (T, F).

      iii) The collisions between molecules are inelastic and therefore conserve momentum but not mechanical energy (T, F).

2. A beam of 6×10^{17} particles/second, each of mass 5×10^{-26} kg, and velocity $v = 2 \times 10^5$ m/s strikes a wall at normal incidence and the particles bounce back elastically. The beam is uniformly distributed over a 0.01 m^2 area. What is the pressure on the wall?

3. A certain diatomic molecule can rotate about its X-axis and Y-axis, but not the Z-axis which joins the 2 atoms. However, it can vibrate along this Z-axis only.

   a) How many degrees of freedom does this molecules have as it moves through space?

   b) Calculate the molar specific heat C_V for a gas of these molecules.

M 5 - KINETIC THEORY OF GASES Date________________________

Mastery Test Form (E) pass ________ recyle________
Instructor____________________________ 1 2 3 4 5
Name___________________________________ Tutor____________________________

1. a) Give a macroscopic definition of an ideal gas.

   b) One mole of an ideal gas at an absolute pressure of $1x10^5 N/m^2$ is contained in a $1m^3$ tank at 300°K. A second mole of the same gas is added at 300°K. What is the final pressure in the tank?

2. Which of the following are assumptions used to define the microscopic model of an ideal gas?

   a) Model works as well for few particles as for many (T, F).

   b) Particles are in random motion (T, F).

   c) The sum of the volumes of the individual particles is equal to the volume of the container (T, F).

3. Helium atoms do not have rotational or vibrational energy.

   a) What is the total random kinetic energy of 1 mole of helium at a temperature of 300°K (R=8.31 J/mol°K; $k=1.38x10^{-23}$ J/°K ($m_{He}=6.7x10^{-27}$ kg, $N_o=6x10^{23}$)?

   b) What is the rms speed of the atoms?

   c) State the equipartition of energy theorem.

   d) Find the molar specific heat at constant volume of helium.

M 5 - KINETIC THEORY OF GASES

Date____________________

Mastery Test Form (F)

pass ________ recyle________

1 2 3 4 5

Instructor________________________

Name________________________ Tutor________________________

1. a) Define an ideal gas.

   b) The kinetic theory of gases assumes: (true - false)

      i) inelastic collisions between particles (T, F)
      ii) random motion of particles (T, F)
      iii) a small number of particles (T, F).

2. The 10^{25} atoms in a gas which fills a 1 m^3 volume have total kinetic energy of 10^5 J.

   a) State the equipartition theorem.

   b) Find the pressure on the walls of the container.

   c) Find the temperature of the gas.
   ($k=1.38x10^{-23}$ J/K).

3. A tank having a volume of ½ m^3 contains helium at 300^oK and absolute pressure of $3x10^6$ N/m^2. Several Mickey Mouse balloons are filled from the tank reducing the pressure to $2x10^6$ N/m^2. How many mols of gas were used? (The temperature did not change.)

MASTERY TEST GRADING KEY - Form A

1. Solution: (a) An ideal gas is one that exactly obeys the equation of state $pV = nRT$. (b) $pV = nRT$. We want to transform to a molecular basis since we need molecules per cubic meter = N/V:

$$n = N/N_0 \quad \text{and} \quad R = N_0 k,$$

where N is the number of molecules, N_0 is the number of molecules per mole, and k is Boltzmann's constant. Therefore, $pV = NkT$ or

$$\frac{N}{V} = \frac{p}{kT} = \frac{(1.01 \times 10^5 \text{ Pa}) \times 10^{-9}}{(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})} = 2.44 \times 10^{16} \text{ molecules/m}^3.$$

2. What To Look For: Student should have any three of the following.

   Solution: (1) Large number of particles; (2) Negligible volume; (3) Random motion; (4) No forces except during collision; (5) Elastic collisions; (6) Newtonian mechanics applies.

3. Solution:

   "Suppose a particle strikes the wall. The change in MOMENTUM/collision = $-2mv_x$. The change in MOMENTUM of the wall (area = ℓ^2) = $+2mv_x$ by conservation of MOMENTUM. After a time t, the particle should again strike the same wall. This time $t = 2\ell/v_x$, so that the number of collisions per second with the wall is $v_x/2\ell$. The average force is given by the rate of change of MOMENTUM per unit TIME. Therefore, for one particle

$$F = dp/dt = (2mv_x)(v_x/2\ell) = mv_x^2/\ell.$$

We now add up the forces for N molecules and divide by AREA to get

$$p = \frac{m}{A\ell}\sum_i^N v_{x_i}^2.$$

However, the general definition of the average is

$$\sum_{i=1}^{N} \frac{x_i}{N} = x_{av}.$$

Therefore,

$$p = (mN/A\ell)(v_x^2)_{av}.$$

Now,

$$v^2 = v_x^2 + v_y^2 + v_z^2,$$

and since the velocities are assumed random

$$(v_x^2)_{av} = \underline{(1/3)}(v^2)av \quad \text{and} \quad p = \underline{(mN/3A\ell)(v^2)_{av} = (mN/3V)v_{rms}^2}.$$

4. Solution: (a) $p = (mN/3A\ell)v_{rms}^2$.

If we double v_{rms}^2, p is quadrupled.

(b) $U = N[(1/2)mv_{rms}^2]$. If we double v_{rms}^2, U is quadrupled.

(c) The temperature is proportional to the internal energy U. In fact, to fit the macroscopic and microscopic pV expressions $pV = nRT = NkT$ and

$$pV = (mN/3)v_{rms}^2 = (2/3)N(\tfrac{1}{2}mv_{rms}^2),$$

T must equal $(2/3k)[(1/2)mv_{rms}^2]$ or $(3/2)kT = (1/2)mv_{rms}^2$. If V = constant and $p = NkT/V$ = const T, therefore p/T = const, which is Gay-Lussac's law.

5. Solution: (a) The available energy depends only on temperature and distributes itself in equal shares to each of the independent ways in which the molecule can absorb energy.

(b) If the total masses of H_2 and O_2 are equal:

$$m_{H_2}N_{H_2} = m_{O_2}N_{O_2};$$

where m is the mass of one atom or

$$(M_{H_2}/N_0)N_{H_2} = (M_{O_2}/N_0)N_{O_2}$$

since $m = M/N_0$. Therefore

$$N_{H_2}/N_0 = M_{O_2}/M_{H_2} = 32/2 = 16.$$

By the equipartition theorem

$$(\tfrac{1}{2}m_{H_2}v_{H_2}^2) = (\tfrac{1}{2}m_{O_2}v_{O_2}^2.$$

Therefore the ratio of kinetic energies is 1 and the ratio of the speeds

$$v_{H_2}/v_{O_2} = \sqrt{M_{O_2}/M_{H_2}} = \sqrt{16} = 4.$$

The pressures are

$$p_{O_2} = (m_{O_2}N_{O_2}/3V)v^2_{O_2}, \qquad p_{H_2} = (m_{H_2}N_{H_2}/3V)v^2_{H_2},$$

and dividing we find

$$\frac{p_{H_2}}{p_{O_2}} = \frac{M_{H_2}}{M_{O_2}}(16)(4) = (\frac{1}{16})(16)(4) = 4.$$

MASTERY TEST GRADING KEY - Form B

1. Solution: (a) An ideal gas is one that exactly follows the equation of state $pV = nRT$.
(b) When the tire is cold (condition 1) $p_1V_1 = nRT_1$. When hot (condition 2) $p_2V_2 = nRT_2$. Dividing we get

$$p_1V_1/p_2V_2 = T_1/T_2$$

or

$$p_2 = \frac{p_1V_1T_2}{V_2T_1} = \frac{(2.70 \times 10^5\ \text{Pa})(1.60 \times 10^{-2}\ \text{m}^3)(273 + 27)\ \text{K}}{(1.63 \times 10^{-2}\ \text{m}^3)(273\ \text{K})} = 2.91 \times 10^5\ \text{Pa}$$

or

$$\frac{2.90 \times 10^5\ \text{Pa}}{6.9 \times 10^3\ \text{Pa lb}^{-1}\ \text{in.}^2} = 42\ \text{lb/in.}^2\ \text{abs.} = 42 - 15 = 27\ \text{psi gauge.}$$

2. What To Look For: Student should have any three of the following.

Solution: (1) Large number of particles; (2) Negligible volume; (3) Random motion; (4) No forces except during collisions; (5) Elastic collisions; (6) Newtonian mechanics applies.

3. Solution: Originally

$$p_0 = \frac{\text{av. force}}{\text{area}} = \left(\frac{\text{number collisions}}{\text{unit time}}\right)\frac{(-\Delta p)}{\text{area}} = \left(\frac{v_0}{2\ell}\right)\left(\frac{2mv_0}{A}\right) = \frac{mv_0^2}{\ell A} = \frac{mv_0^2}{V}.$$

If v_0 is doubled,

$$p_1 = (2v_0/2\ell)(2m2v_0/A) = 4mv_0^2/V.$$

Therefore

$$p_1/p_0 = 4.$$

4. Solution: Here is how we will proceed:

$$mv_{rms}^2/2 = 3kT/2$$

represents the kinetic energy of a molecule of an ideal gas. From the information given on oxygen, i.e., T and v_{rms}, knowing k (Boltzmann's constant), we can calculate $m(O_2)$, which is the mass of an O_2 molecule. We know that a mole of a gas contains N_0 (Avagadro's number) molecules. Relative to carbon this mole is assigned M (molecular weight) grams. Therefore a molecule has a mass of M/N_0; so $m(O_2) = M(O_2)/N_0$:

N_0 (molecules per mole) × m (mass of a molecule) = M (mass of a mole of gas).

We have $m(He) = m[m(O_2)/M(O_2)] = M(He)$.

Knowing the mass of the argon and helium molecules; we can then calculate v_{rms}:

$$v_{rms} = [3kT/m]^{1/2},$$

so

$$m(O_2) = \frac{3kT}{v_{rms}^2(O_2)} = \frac{(3)(1.381 \times 10^{-23})}{(460\ m/s)^2} \frac{J}{molecule\ K}(273.15\ K)$$

$$= \frac{(3)(1.381)(2.7315)}{(4.6)(4.6)\ m^2/s^2} \times \frac{10^{-25}\ kg\ m^2/s^2}{molecule} = 5.348 \times 10^{-26}\ kg/molecule.$$

$$M(He) = 5.348 \times 10^{-26}\ kg/molecule\ \frac{4}{32}\frac{g/mole}{g/mole} = 0.6685 \times 10^{-26}\ kg/molecule,$$

$$v_{rms}(He) = [(3)(1.381 \times 10^{-23}\ J/molecule\ K)\frac{313.15\ K}{0.6685 \times 10^{-26}\ kg/molecule}]^{1/2}$$

$$= [1.9407 \times 10^6\ m^2/s^2]^{1/2} = 1393\ m/s.$$

5. Solution: (a) The available energy depends only on temperature and distributes itself in equal shares to each of the independent ways in which the molecules can absorb energy.
(b) By the equipartition theorem the energy will be equally distributed over all molecules. Therefore each mass m_i has a kinetic energy of (1/2)kT and

$$K_{av} = \frac{1}{2}m_i v_{i(rms)}^2.$$

The pressure from each mass species is

$$p_i = \frac{2N_i}{3V}\left(\frac{1}{2}m_i v_{i(rms)}^2\right) = \frac{3}{2}V \sum_{i=1}^{\ell} p_i.$$

But the equation of state is

$$p_{tot}V = (2/3)U.$$

Therefore

$$p_{tot} = \frac{2}{3}\frac{U}{V} = \sum_{i=1}^{\ell} p_i.$$

MASTERY TEST GRADING KEY - Form C

1. Solution: (a) An ideal gas is one that exactly obeys the equation of state pV = nRT, where p = pressure, V = volume, n = number of moles, R = gas constant, and T = absolute temperature.
(b) We use the equation of state pV = nRT. From the initial conditions we can find n:

$$n = \text{number of moles} = \frac{m}{M} = \frac{5.6 \times 10^{-5}\ \text{kg} \times 10^{3}\ \text{g/kg}}{28\ \text{g/mole}} = 2.00 \times 10^{-3}\ \text{mole.}$$

Therefore

$$p = \frac{nRT}{V} = \frac{(2.00 \times 10^{-3}\ \text{mole})(8.31\ \text{J/kg mole K})(300\ \text{K})}{(2.00 \times 10^{-5}\ \text{m}^3)} = 2.50 \times 10^{5}\ \text{Pa.}$$

2. What To Look For: Ask about one of the answers: for example, random motion - student should tell you it is random motion of the molecules. They move in all directions with equal probability and a variety of speeds.

Solution: (1) Large number of particles; (2) Negligible volume; (3) Random motion; (4) No forces except during collision; (5) Elastic collisions; (6) Newtonian mechanics applies.

3. What To Look For: Look for sign error. Ask student to work out units for final equation. Ask student what θ will give maximum pressure (θ = 0).

Solution: A single molecule hitting the wall changes it momentum by

$$\Delta p\hat{i} = mv_f\hat{i} - mv_i\hat{i} = m(v_f - v_i)\cos\theta = m(-v - v)\cos\theta = -2mv\cos\theta.$$

The change of momentum of the wall is +2mv cos θ by conservation of momentum.

The force on the wall is $-\Delta p\hat{i}/\Delta t = 2mv\cos\theta\ (\text{number/s})$.

The pressure is force per unit area = 2mv cos θ (number/s × area) = 2mv cos θ M.

4. Solution: (a) The kinetic energy per molecule is

$$K_{av} = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT = \frac{3}{2}kT = \left(\frac{3}{2}\right)(1.38 \times 10^{-23}\ \text{J/mole K})(273\ \text{K}) = 5.6 \times 10^{-21}\ \text{J.}$$

(b) The kinetic energy per mole is

$$N_0K_{av} = \frac{1}{2}Nmv_{rms}^2 = \frac{3}{2}N_0kT = (6.02 \times 10^{23})(5.6 \times 10^{-21}\ \text{J}) = 3.40 \times 10^{3}\ \text{J.}$$

5. Solution: (a) The available energy depends only on temperature and distributes itself in equal shares to each of the independent ways in which the molecules can absorb energy.
(b)(i) The specific heat per molecule is $C_V = dU/dt$. For a monatomic gas we can have only three translational degrees of freedom and no rotation or vibrations. Therefore, U = (3/2)kT per molecule, and

$$C(\text{per molecule}) = (d/dt)(3kT/2) = 3k/2.$$

The specific heat per unit mass is

$$C(\text{per mass}) = \frac{C_V(\text{per molecule})}{m(\text{kg/molecule})}.$$

Therefore,

$$m = \frac{(3/2)k}{C_V(\text{per mass})} = \frac{(3/2)(1.38 \times 10^{-23}\ \text{J/K}}{0.314\ \text{J/kg K}} = 6.6 \times 10^{-26}\ \text{kg}.$$

(ii) One atomic weight M is the <u>mass</u> of one mole measured in <u>grams</u>. Therefore

$$M = mN_0(10^3\ \text{g/kg}) = (6.6 \times 10^{-26}\ \text{kg/atom})(6.02 \times 10^{23}\ \text{atoms/mol})(10^3\ \text{g/kg})$$
$$= 40\ \text{g/mol}.$$

<u>Alternate solution</u>: We can use the specific heat per mole in a parallel solution:

$$C_V(\text{per mole}) = d(U)/dt.$$

From kinetic theory U(per mole) = (3/2)RT. Therefore C_V(per mole) = (3/2)R. But

$$C_V(\text{per mass}) = \frac{C_V(\text{per mole})}{M(\text{g/mole})}.$$

Therefore

$$M = \frac{3}{2}\left(\frac{8.368\ \text{J/mol K}}{0.314\ \text{J/kg K}}\right) = 40\ \text{g/mol}$$

$$m = M/N_0 = \frac{40}{6.02 \times 10^{23}} = 6.6 \times 10^{-23}\ \text{g} = 6.6 \times 10^{-26}\ \text{kg}.\ .$$