1975

Newton's Laws

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NEWTON'S LAWS

INTRODUCTION

When a body is at rest, we know from experience that it will remain at rest unless something is done to change that state. A heavy box on the floor will stay in place unless it is pushed or pulled. We walk without fear beside a massive rock on level ground because we know it won't suddenly move and crush us.

Undoubtedly you have leaned against a chair only to have it move and send you scurrying for your balance. Did you then question the relationship of the interaction between you and the chair to the ensuing motion of the chair?

It was Isaac Newton who first clearly made the connection between the interactions on a body and its motion. In Newton's theory, the acceleration of every object has to be explained in terms of the interactions with other objects. Newton's laws of motion cover an enormous range of experience. At one stroke they convert what in retrospect had previously seemed chaos into a beautifully organized universe. There have been few achievements to rank with this in the history of science.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Check the units of a given mathematical expression and show that it is dimensionally correct (needed for Objectives 2 and 4 of this module)	Dimensions and Vector Addition Module
*Add or subtract two, three, or four two-dimensional vectors given in unit-vector notation, finding the resultant (needed for Objectives 2 and 4 of this module)	Dimensions and Vector Addition Module
*Describe the position, velocity, and acceleration of an object moving in one dimension with constant acceleration (needed for Objectives 2 and 4 of this module)	Rectilinear Motion Module
*Describe the position, velocity, and acceleration of a single body moving in a plane or moving in projectile motion (needed for Objectives 2 and 4 of this module)	Planar Motion Module

LEARNING OBJECTIVES

After you have mastered the contents of this module you will be able to:

1. Free-body diagram - Draw a diagram of a particle representation of a body isolated from its environment in an inertial reference frame; and
    - (a) illustrate, with vectors, all forces that act upon it; and
    - (b) identify, by name, the source of each force illustrated.
2. $\vec{F} = m\vec{a}$ - Write Newton's first and second laws in mathematical form; and
    - (a) choosing an appropriate coordinate system, apply the second law to a given problem involving a single massive body, solving for either a specified force or the acceleration of the body; and
    - (b) use the second law to distinguish between weight and mass.
3. Action-reaction - Apply Newton's third law to a problem to relate the forces exerted and experienced by a body.
4. Motion of a particle - Solve a problem concerning the motion of a body (acceleration, velocity, and displacement) given sufficient information concerning the external forces acting on the body. (These external forces may be gravitational forces or contact forces exerted by another particle, by friction, by nonstretchable ropes, or by rigid rods.)

GENERAL COMMENTS

It should be emphasized that a particle model is used throughout this module, i.e., each body is considered as if it were concentrated at a point and had no extension in space. This should be remembered in your study of examples in the text and in working the Problem Set. All objects (blocks, cars, passengers in elevators, etc.) are to be treated as particles.

Objective 2 is stated as $\vec{F} = m\vec{a}$ because Newton's first law is implicit in the second. That is, if $\vec{F} = 0$, $\vec{a} = 0$: no force acting on the body, no acceleration (change in motion). We should also stress that the $\vec{F}$ in Newton's second law is the resultant force (vector sum) of all the forces acting on a body.

In solving problems in this module, here are some suggested rules to follow, step by step:

1. Identify the particular body to be considered.
2. Identify all interactions (forces) between the body and its environment.
3. Choose a suitable inertial coordinate system (be judicious in your choice, and you will save yourself a lot of effort).
4. Draw a diagram of the object representing it by a point; show all forces acting on the body, and show the coordinate system (free-body diagram).
5. Resolve those forces not lying along a coordinate axis into their rectangular components.
6. Apply Newton's second law.

Following these rules, especially rule 4, will save you much time in mastering Objective 4, where it is easy to confuse the various interactions, forces, and bodies.

Regarding Objective 3 (action-reaction), a more complete treatment will be given in the later module: Applications of Newton's Laws.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read Section 2.3 in Chapter 2 and all of Chapter 5 (excluding Sec. 5.5). Objective 1 is discussed in the initial part of Section 5.3 in relation to the application of Newton's second law. Newton's first and second laws are stated in Section 5.1 (p. 54) in connection with Objective 2, and Newton's second law ($\vec{F} = m\vec{a}$) is further discussed in Section 5.2. The relation between weight and mass is given in Eq. (5.4).

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 2.3, 5.3	A, B		I, J	Chap. 2, Prob. 7	Chap. 2, Prob. 4
2	Secs. 5.1, 5.2	C, D		K, L	Chap. 5, Prob. 7	Chap. 5, Prob. 9
3	Sec. 5.1	E, F		M, N	Chap. 5 Ques. 1	
4	Secs. 5.3, 5.4, 5.6, 5.7	G, H	Illus. 5.1, 5.2, 5.3, 5.4	O, P, Q	Chap. 5 Prob. 13, 23	R, S; Chap. 5, Probs. 10, 11, 12

Objective 4 is covered in Sections 5.3, 5.4, 5.6, and 5.7 with several good illustrations.

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read Chapter 5 of your text, excluding Section 5-13 and Examples 5 and 6 (which will be covered in the module: Applications of Newton's Laws). Objective 1 is spread over four sections (5-1, 5-3, 5-8, 5-11). Table 5-1 on p. 60 gives a very helpful overview of interactions between a body and its environment that are described by forces; compare this with Table 5-4 on p. 70 for a mathematical description of the forces in each case.

Buried in the first paragraph of Section 5-11 are five rules. These rules are indispensable in working out the problems of Objective 4. Note the explicit and excellent free-body diagrams (and the particle representation) drawn for the examples, i.e., Figures 5-4(b), 5-5(b), 5-6(b), 5-8(b), and 5-11.

Newton's first law is stated and discussed in Section 5-3. Newton's second law is stated mathematically in Eq. (5-1) and discussed at length in Sections 5-2 to 5-5 and 5-8. The relationship between weight and mass is given in Eq. (5-5) and explained in Section 5-9.

Objective 3 is thoroughly explained in an excellent discussion in Section 5-6, and two very good examples are given (Examples 1 and 2). Objective 4 is presented in Sections 5-11 and 5-12 with many excellent examples.

After completing this reading, study the Problem Set and solve the assigned problems. Finally, check your understanding by taking the Practice Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 5-1, 5-3, 5-8, 5-11	A, B		I, J		
2	Secs. 5-2 to 5-5, 5-7 to 5-9	C, D	Examples 3, 4(a)	K, L	Chap. 5, Probs. 4, 11	Chap. 5, Probs. 5, 7, 14
3	Sec. 5-6	E, F	Examples 1, 2	M, N	Chap. 5, Prob. 2	
4	Secs. 5-11, 5-12	G, H	Examples 4(b), 7, 8	O, P, Q	Chap. 5 Probs. 25, 50	R, S; Chap. 5, Probs. 8, 9, 12, 13, 16, 18, 21, 35

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

This particular module is spread over Chapters 1, 2, and 5. Interspersed in Chapters 2 and 5 is introductory material on Gravitation and Statics, topics that will be taken up in more depth in the modules Gravitation and Equilibrium of Rigid Bodies. We suggest that you read Sections 1-4 and 1-5 of Chapter 1, Chapters 2 and 5 in order, but skip Sections 2-4 and 5-4.

Objective 1 is discussed quite thoroughly over parts of five sections. In Section 1-5, the concept of force is introduced; however, it is introduced independent of acceleration. When particles interact (do something to each other), one measure of their interaction strength is provided by their acceleration, and this measure leads to the force concept. On p. 14 the technique for determining the resultant force from the individual forces is introduced. On p. 15 an inertial coordinate system is defined, and on p. 18 there appears an excellent discussion of the particle model and rules for constructing a free-body diagram and for solving problems. [Note an example of a free-body diagram in Figure 5-8(b) on p. 66.]

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 1-4, 1-5, 2-2, 2-3, 2-6	A, B		I, J		
2	Secs. 2-2, 2-3, 5-2, 5-3, 5-5	C, D	Sec. 2-6, Ex.* 1-4 Sec. 5-3, Ex. 2, 3	K, L	5-1	
3	Sec. 2-5	E, F	Sec. 2-6, Ex. 1, 2 Sec. 5-6, Ex. 4	M, L	5-3	2-1(a)-(g), 2-2
4	Secs. 2-6, 2-7, 5-6	G, H	Sec. 2-7, Ex. 1, 2, 3 Sec. 5-6, Ex. 1, 2, 3, 6, 7	O, P, Q	2-22, 5-24	2-17, 5-13, 5-17, 5-21, 5-22

*Ex. = Example

Objective 2 is spread over five sections. You will find Newton's first law stated on p. 15 with a discussion of it preceding and following this statement. Newton's second law is stated verbally and mathematically [Eq. (5-3)] on p. 58. The relation between mass and weight is given in Eq. (5-6). Disregard Eq. (5-5) since you will take this up in the module Gravitation.

Objective 4 is handled in Sections 2-7 and 5-6 with several examples. Disregard Examples 4 and 5 (Sec. 5-6) for now.

Rule 4 on p. 18 explicitly applies to the case where the resultant force is zero (Newton's first law), but it can be extended to the more general case (Newton's second law) by setting the algebraic sum of all the x components of the forces equal to the mass times the x component of acceleration ($\Sigma F_x = ma_x$) and the algebraic sum of all the y components of the forces equal to the mass times the y component of acceleration ($\Sigma F_y = ma_y$).

Study Problems A through H before working Problems I through Q. When you feel prepared, take the Practice Test.

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Your text approaches mechanics from a momentum point of view (a concept to be covered in a later module). Momentum is defined and experiments are used to establish the conservation of momentum as the empirical starting point for the development of some of the concepts to be developed in this module.

To make your text compatible with the sequence of ideas as developed in these modules, we suggest the following reading sequence: Sections 5-1 through 5-6 (excluding all the examples in these sections), 7-1 through 7-3, 7-5 through 7-6, and 8-1 through 8-3 (excluding Example 8-6). You may wish to read through Sections 5-5 and 5-6 quickly since the material in these sections is not included in any of the learning objectives of this module but is needed to facilitate the understanding of the material in this module.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 5-1, 7-1, 7-3, 8-2	A, B		I, J		
2	Secs. 5-1, 5-2, 7-2, 7-3, 7-6, 8-1	C, D	Ex.* 7-1, 8-2	K, L		
3	Secs. 7-5, 8-1	E, F		M, N	8-5	
4	Secs. 8-2, 8-3	G, H	Ex. 7-2, 8-3, 8-4, 8-5, 8-7, 8-9, 8-10	O, P, Q, R, S	8-11, 8-12	8-21

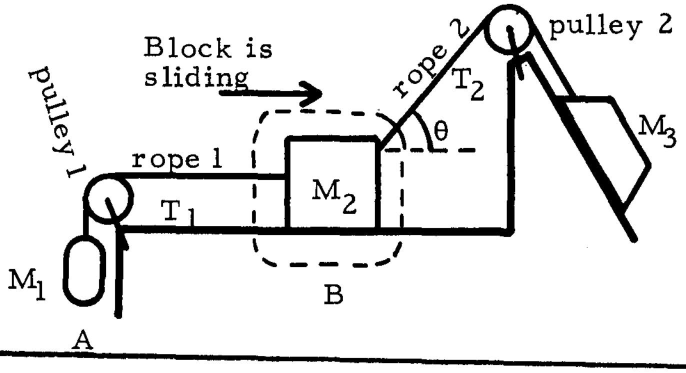
*Ex. = Example(s)

Objective 1 is spread over four sections of reading. An inertial reference frame is defined in Section 5-1. A general discussion is given in Section 7-1 of the concept that interacting particles can manifest this interaction by altering their motion, and this change in motion can be used to measure the strength of the interaction or the force. An explanation of the combination of the forces of interaction to obtain a resultant force is given in Section 7-3. The rules given on the bottom of p. 108 pertain to drawing a free-body diagram and solving problems. Carefully study some of the excellent free-body diagrams in Figures 8-5(b) and 8-6(b) and (c).

Objective 2 is covered in several sections. Newton's first law is stated on pp. 56 and 106. Newton's second law is stated verbally in Section 8-1 and mathematically in Eqs. (7-4) (for this module you will only need the simpler form $\vec{F} = m\vec{a}$) and (8-1). The relationship between mass and weight is given in Eq. (7-10). Objective 3 is developed in Section 7-5.

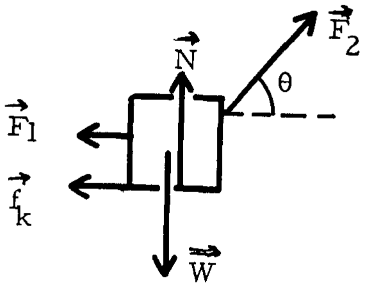
PROBLEM SET WITH SOLUTIONS*

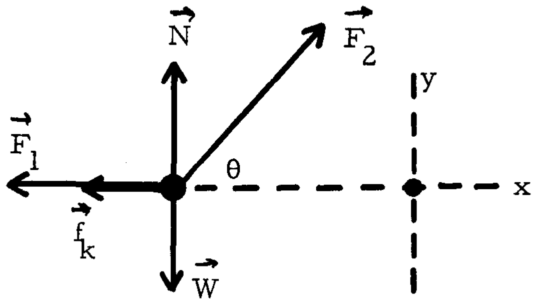
A(1). Given the situation pictured at the right, specify all the forces acting on the body B (mass M_2) and draw a free-body diagram for B.



Solution

We are interested in body B (contained in the dotted lines). $\vec{F}_1$ represents the force that rope 1 exerts on B. $\vec{F}_2$ represents the force that rope 2 exerts on B. $\vec{f}_k$ represents the frictional force on B that opposes the motion. $\vec{W}$ represents the weight or the attraction of the earth for B, $\vec{W} = m\vec{g}$. $\vec{N}$ represents the force that the supporting plane exerts on B.





Free-Body Diagram

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*Each of the problems satisfying Objective 4 should also satisfy Objective 1 if worked properly.

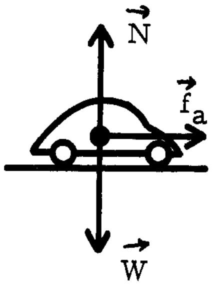
B(1). A small car has a mass of 800 kg. It can accelerate uniformly from rest to 30.0 m/s in 8.0 s, and its brakes slow it down from 30.0 m/s to 15.0 m/s in 5.0 s.

(a) Draw free-body diagrams for the car while it is accelerating and while it is decelerating (brakes applied).

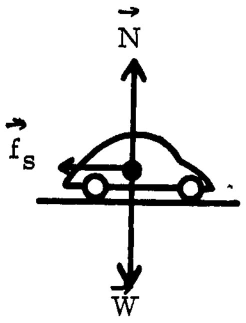
(b) Express the force exerted by the road in terms of the resultant force and the weight.

Solution

(a)



Acceleration



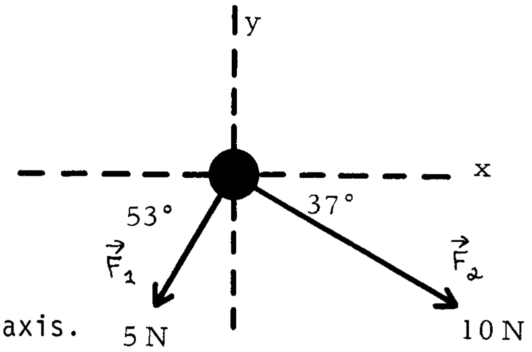
Deceleration - Brakes Applied

(b) The supporting road exerts a normal force $\vec{N}$ perpendicular to the plane of the road, and a frictional force $\vec{f}_a$ or $\vec{f}_s$ parallel to the plane of the road.

$$\vec{F}_c = \vec{F} - \vec{W}$$

where $\vec{F}_c = \vec{N} + (\vec{f}_a \text{ or } \vec{f}_s)$.
$\vec{F}$ is the resultant force which is parallel to the road, and $\vec{F} = \vec{F}_c + \vec{W}$.

C(2). Find the acceleration of the object (weight 5.0 N) under the action of the forces shown in the diagram. The object shown is constrained to move in the x-y plane under the influence of $\vec{F}_1$ and $\vec{F}_2$. The weight of the object acts straight into the paper along the z-axis.



Solution

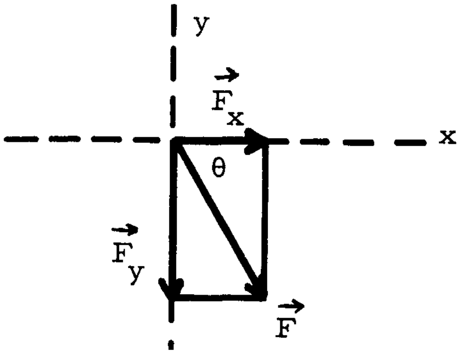
From Newton's second law $\vec{F} = m\vec{a}$, where $\vec{F}$ is the net force. Therefore $\vec{F} = \vec{F}_1 + \vec{F}_2$. Vectorially adding $\vec{F}_1$ and $\vec{F}_2$:

$F_x = F_{1x} + F_{2x} = -F_1 \cos 53° + F_2 \cos 37° = -3.0\ \text{N} + 8.0\ \text{N},\ F_x = 5.0\ \text{N},$

$F_y = F_{1y} + F_{2y} = -F_1 \sin 53° - F_2 \sin 37° = -4.0\ \text{N} - 6.0\ \text{N},\ F_y = -10.0\ \text{N},$

$F = [F_x^2 + F_y^2]^{1/2} = [125\ \text{N}^2]^{1/2} = 11.2\ \text{N}.$

$\tan\theta = \dfrac{10\ \text{N}}{5\ \text{N}} = 2.0,\quad \theta = 63.4°$ below x axis.



$\vec{F} = m\vec{a} = (W/g)\vec{a}$ or $|\vec{a}| = Fg/W.$

$$a = \frac{(11.2\ \text{N})(9.8\ \text{m/s}^2)}{5.0\ \text{N}} = 22\ \text{m/s}^2$$

in the same direction as $\vec{F}$, or 63.4° below the +x axis. Using unit-vector notation:

$$\vec{a} = a_x\hat{i} + a_y\hat{j} = \frac{F_x}{m}\hat{i} + \frac{F_y}{m}\hat{j} = \vec{a} = (9.8\hat{i} - 19.6\hat{j})\ \text{m/s}^2.$$

D(2). A particle of weight 19.6 N is subject to forces of 3.00 N east and 2.00 N north but travels south at a constant acceleration of 1.00 m/s^2. What third force must act on the particle?

Solution

The net force $\vec{F}$ must be south. (You should know why.) From Newton's second law:

$\vec{F} = m\vec{a},$

$\vec{F} = \dfrac{W}{g}\vec{a} = \left(\dfrac{19.6\ \text{N}}{9.8\ \text{m/s}^2}\right) 1.00\ \text{m/s}^2 = 2.00\ \text{N}$ (net force).

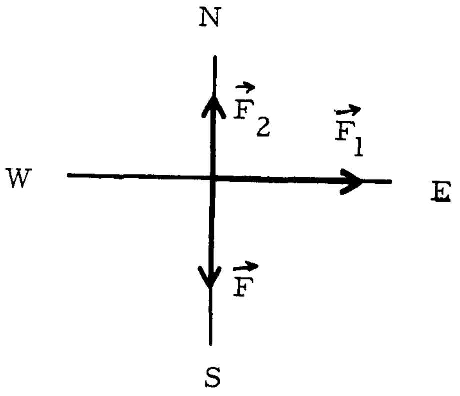
$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3,$

$F_x = F_{1x} + F_{2x} + F_{3x},$

$F_y = F_{1y} + F_{2y} + F_{3y},$

$F_{3x} = -3.00\ \text{N},$

$F_{3y} = -4.00\ \text{N},$



so $\vec{F}_3 = 5.00\ \text{N}$, 53° south of west, or

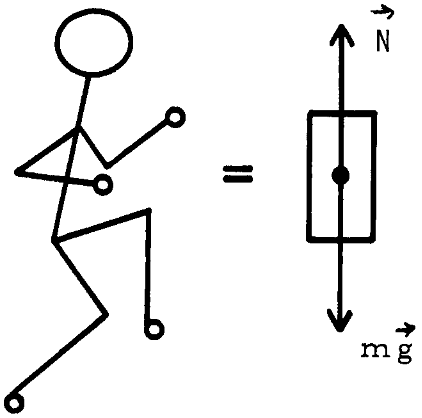
$\vec{F}_3 = (-3.0\ \hat{i} - 4.0\ \hat{j})\ \text{N}.$

E(3). When a high jumper leaves the ground, what object exerts the force that accelerates him upward?

Solution

Let's draw a free-body diagram and see. Considering the man as a unit, we find that the only external forces acting are his weight $m\vec{g}$ and the normal force $\vec{N}$ of the ground. Where does $\vec{N}$ come from? It's the reaction force to the downward force of magnitude N exerted by the man's foot against the ground. As long as $\vec{N} = -m\vec{g}$, the man's center of mass can accelerate neither up nor down.

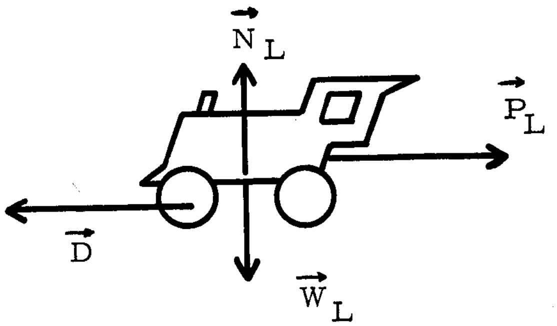
By suddenly straightening his leg, however, the high jumper can increase the force exerted against the ground. $\vec{N}$ will increase correspondingly and his center of mass will accelerate upward. If this acceleration is large enough, he can actually leave the ground. So in one sense, the force that accelerates him (as a unit) upward is exerted by the ground. However, bear in mind that part of $\vec{N}$ is simply a reaction force to forces generated inside his body - forces that allowed him to push his foot down against the ground with a force considerably greater than his own weight.

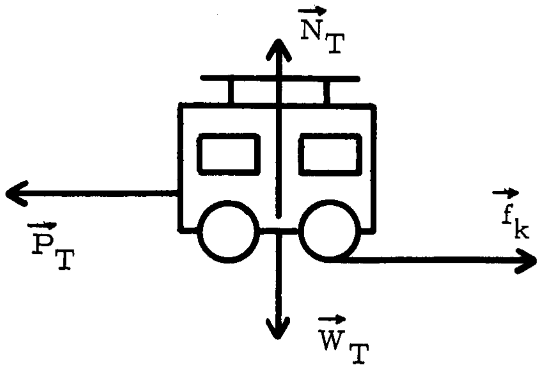


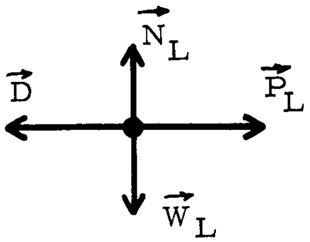
F(3). A locomotive engineer reads an excerpt from a freshman physics text and then decides to quit his job. His reason is that, according to Newton's third law, the train always pulls backward on the locomotive with a force just as great as that which the locomotive exerts on the train, and therefore the train can never move. As personnel supervisor, you are assigned the task of explaining the situation. Explain it.

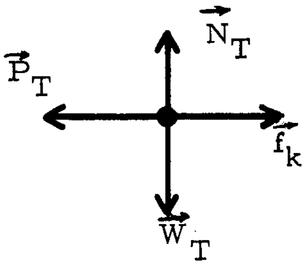
Solution

These two forces form an action-reaction pair and must act on different bodies. Let's draw a free-body diagram of the locomotive and the train.









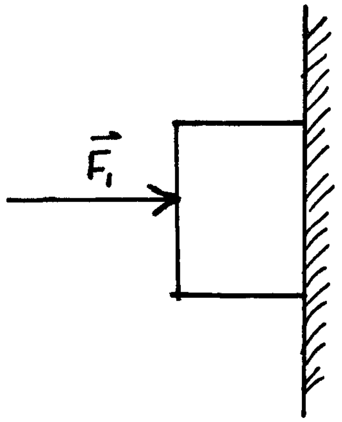
Since $\vec{P}_L$ and $\vec{P}_T$ form the action-reaction pair,

$$\vec{P}_L = -\vec{P}_T,$$

$\vec{D} + \vec{P}_L = M_L\vec{a}$, Locomotive where $D - P_L > 0$;

$\vec{P}_T + \vec{f}_k = M_T\vec{a}$, Train where $P_T - f_k > 0$.

G(4). A constant horizontal force $\vec{F}_1$ pushes a block of mass 2.00 kg against a vertical wall. The coefficient of static friction between the block and the wall is $\mu_s = 0.60$, and the coefficient of kinetic friction is $\mu_k = 0.40$.



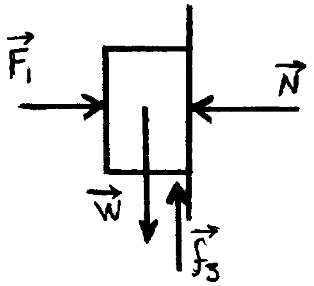
(a) Draw a free-body diagram for the block.

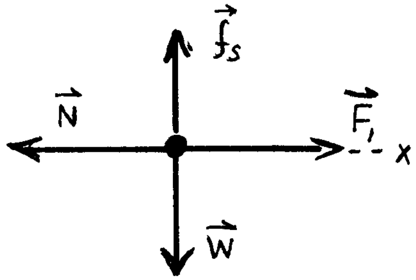
(b) What is the minimum value of $\vec{F}_1$ for which the block will not slip, if it is at rest initially?

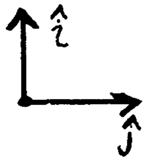
(c) Suppose that $\vec{F}_1$ has this minimum value. The block is given a short downward push, just to start it moving. What acceleration will the block have after this push?

Solution

(a)







$\vec{N}$ is the normal force that the wall exerts on the block;

$\vec{f}_s$ is the static frictional force that the wall exerts on the block;

$\vec{W}$ is the weight of the block;

$\vec{F}_1$ is the applied force.

(b) Assume the block is stationary, i.e., zero accelerations in the x and y directions. Apply Newton's second law to the block:

$$\Sigma F_x = ma_x, \quad \vec{F}_1 + \vec{N} = 0 \tag{1}$$

$$\Sigma F_y = ma_y, \quad \vec{f}_s + \vec{W} = 0, \quad \vec{W} = m\vec{g}. \tag{2}$$

The frictional force is given by

$$\vec{f}_s \le \mu_s \vec{N}.$$

Note: The frictional force depends on the normal, but since $\vec{N} = \vec{F}_1$, the normal depends on the applied force. Therefore the frictional force that "holds" the body up is controlled by the applied force (i.e., we are "pressing" the surfaces in closer contact and making them "bind" more).

From Eq. (2) $\vec{f}_s = \vec{W} = m\vec{g} \le \mu_s \vec{N} = \mu_s \vec{F}_1$, so $\vec{F}_1 \le m\vec{g}/\mu_s$.

The minimum value of $\vec{F}_1$ is $m\vec{g}/\mu_s$:

$$(\vec{F}_1)_{min} = \frac{(2.0\ \text{kg})(9.8\ \text{m/s}^2)}{(0.60)} = 33\ \text{N}.$$

(c) Now $f_k - mg = ma_y$ and $_1 = N$:

$$f_k = \mu_k N = \mu_k F_1,$$

so

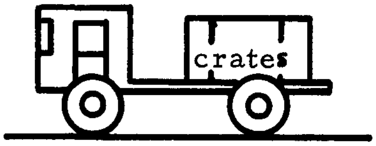
$$a_y = \frac{f_k - mg}{m} = \frac{\mu_k F_1 - mg}{m} = \frac{(\mu_k mg/\mu_s) - mg}{m},$$

$$a_y = g(\mu_k/\mu_s - 1) = (9.8\ \mathrm{m/s^2})\ (0.40/0.60 - 1),$$

$$a_y = -3.3\ \mathrm{m/s^2}.$$

The minus sign designates the acceleration is downward.

H(4). A truck driver has a load of crates on a flat-bed truck and is traveling along a highway at 26 m/s. If the coefficient of friction between crates and truck is 0.80, what is the shortest distance in which he can stop without letting his load slide?



Solution

Assume the deceleration is constant and the final speed is zero. Take the origin of our coordinate system to be a point where the truck begins to decelerate and the x axis along the roadway.

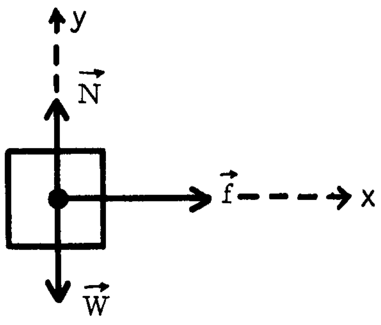
In order for the crate not to slide, what must its speed be relative to the truck bed? Therefore what must be its deceleration relative to our coordinate system?

Free-body diagram of the crate

$\vec{N}$ = normal force,

$\vec{W}$ = weight of crate = $m\vec{g}$,

$\vec{f}$ = frictional force.



Applying Newton's second law:

$$\Sigma F_x = ma_x \quad \text{and} \quad \Sigma F_y = ma_y,$$

$$f = ma_x, \quad N = W = mg,$$

but $f \le \mu_s N$. Assume the maximum frictional force to allow the greatest deceleration before sliding begins: $f = \mu_s N$.

Combining equations leads to $a_x = \mu_s g$ or $a_x = 7.8\ \mathrm{m/s^2}$.

Now we have the initial speed, final speed, and acceleration, which allows us to determine the distance:

$$x = \frac{v_x^2 - v_{x0}^2}{2a_x} = -43 \text{ m}.$$

Problems

I(1). A youngster coasts down a hill of angle θ on his sled.

(a) Identify all forces acting on the youngster.
(b) Draw a free-body diagram of the youngster.

J(1). A tractor climbs a steep grade of angle θ.

(a) Identify all forces acting on the tractor.
(b) Draw a free-body diagram of the tractor.

K(2). A body whose mass is 3.00 kg lies on a smooth horizontal plane. Introduce a coordinate system for answering the following questions:

(a) A horizontal force of 12.0 N is applied in a certain direction. Find the vector acceleration.
(b) A horizontal force of 5.0 N is applied, at right angles (clockwise, looking down) to the force in (a), which is removed. Find the vector acceleration.
(c) Find the vector acceleration when both forces are applied (polar and rectangular descriptions).

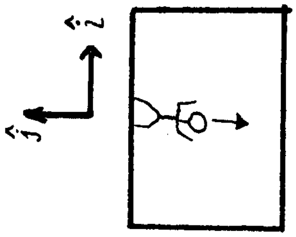
L(2). A skier of mass M gathers speed down a slope (angle θ to the horizontal) even though experiencing a force of friction $\vec{f}$.

(a) Make a free-body diagram for the skier.

(b) Find the acceleration of the skier.

(c) What information about the force $\vec{f}$ is implied by this problem?

M(3). A 72-kg astronaut pushes away from the side of his 700-kg space capsule with a force of 20.0 N. What happens to the space capsule?



N(3). A book with a rock lying on top of it lies in turn on a table top.

(a) Draw a free-body diagram of the book.

(b) Identify the reaction force to each force acting on the book.

O(4). The coefficient of dynamic friction of a block on an inclined plane may be determined by raising the plane just enough so the block slides with uniform speed, and measuring the angle of inclination of the plane:

(a) Make a free-body diagram for the block.
(b) Derive a relationship between the angle and the coefficient of friction for these conditions.

P(4). A block slides down an inclined plane of angle ϕ with constant velocity. It is then projected up the same plane with an initial speed v_0.

(a) How far up the incline will it move before coming to rest?
(b) Will it slide down again?

Q(4). A 29.0-kg block is pushed up a 37° inclined plane by a horizontal force of 440 N. The coefficient of friction is 0.250. Find:

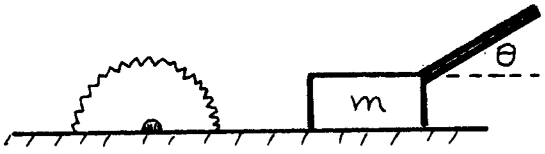
(a) the acceleration;
(b) the velocity of the block after it has moved a distance of 6.0 m along the plane, assuming it started at rest; and
(c) the normal force exerted by the plane.

R(4). A hockey puck having a mass of 0.110 kg slides on the ice for 15.2 m before it stops.

(a) Draw a free-body diagram of the puck including friction.
(b) If its initial speed was 6.1 m/s, what is the force of friction between the puck and ice?
(c) What is the coefficient of kinetic friction?

S(4). A man pushes a board of mass m through a circular saw by pushing down on it with a stick as shown:

(a) Find the force $\vec{F}$ that must be exerted to make the board slide with constant speed, in terms of the angle θ, the coefficient of friction, and the mass of the board.
(b) Show that if the angle is too steep, the board cannot be made to slide, no matter how great a force is applied. Find this critical angle.

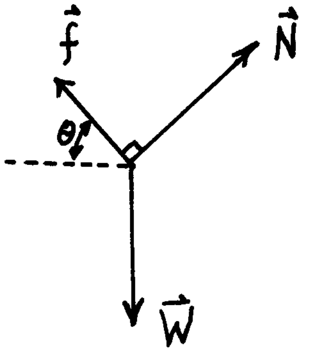


Solutions

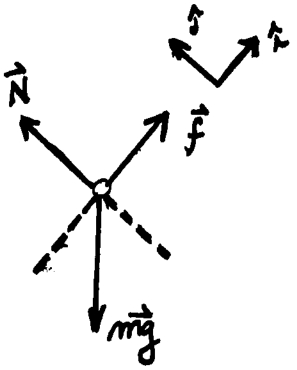
I(1). (a), (b) $\vec{N}$ = normal force that the sled exerts on the youngster (to sled),

$\vec{f}$ = frictional force that the sled exerts on the youngster (to sled),

$\vec{W}$ = weight of the body.



J(1). (a), (b) The surface of the ground exerts both $\vec{N}$ and $\vec{f}$ whereas the earth as a whole exerts $\vec{W}$.

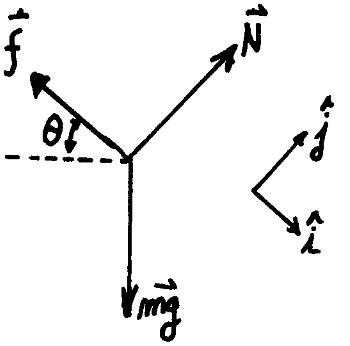


K(2). (a) $4.0\hat{j}$ m/s^2,

(b) $1.70\hat{i}$ m/s^2,

(c) $(1.70\hat{i} + 4.0\hat{j})$ m/s^2, 4.3 m/s^2, $\theta = 1.2$ rad.

L(2). (a)

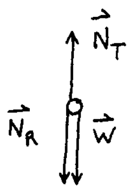


(b) $g \sin\theta - f/M$.

(c) $\vec{f} < M\vec{g} \sin\theta$.

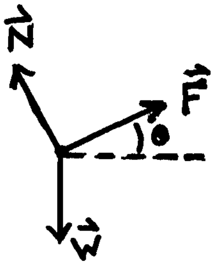
M(3). Accelerates at 0.0300 m/s^2 in j direction until the astronaut hits the other side.

N(3).



Action	Reaction
$\vec{N}_T$	Book exerts downward force on the table $(-\vec{N}_T)$
$\vec{N}_R$	Book exerts upward force on the rock $(-\vec{N}_R)$
$\vec{W}$	Book attracts the earth with a force $-\vec{W}$

O(4). (a)



(b) $\mu = \tan\theta$.

P(4). (a) $v_0^2/4g \sin\phi$.

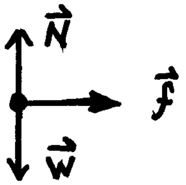
(b) No.

Q(4). (a) 2.00 m/s^2.

(b) 4.9 m/s

(c) 490 N.

R(4). (a)



(b) 0.130 N.
(c) 0.120.

S(4). (a) $\mu mg/[\cos\theta - \mu\sin\theta]$.

(b) $\theta = \text{arc cot}\ \mu$.

PRACTICE TEST

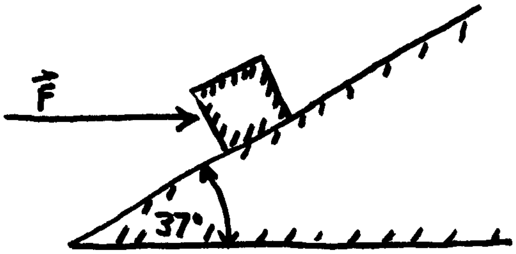
1. A 50-kg block on an inclined plane is acted upon by a horizontal force of 50 N. The coefficient of friction between block and plane is 0.300.

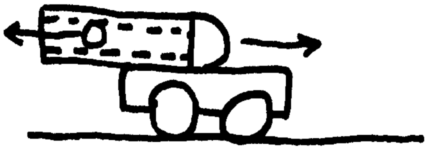
   (a) Draw a free-body diagram of the block, assuming it is moving up the plane.

   (b) What is the reaction force to the force $\vec{F}$ acting on the block?

   (c) What is the acceleration of the block if it is moving up the plane?

   (d) How far up the plane will the block go if it has an initial upward speed of 4.0 m/s?

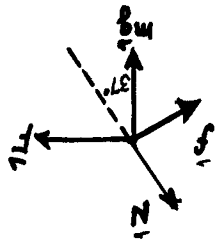




2. A 730-kg cannon fires a 4.0-kg cannon ball. While it is in the barrel, the cannon ball has an acceleration of 30 000 m/s^2. What is the acceleration of the cannon?

Practice Test Answers

1. (a)



   (b) The reaction is a force $-\vec{F}$ pushing on the agent.

   (c) -7.2 m/s^2.

   (d) 1.10 m.

2. 164 m/s^2 to the right.

DATE______________________________

NEWTON'S LAWS

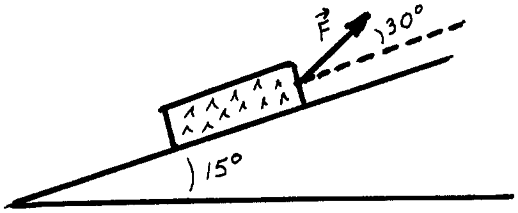
pass recycle

Mastery Test Form A

pass		recycle	
1	2	3	4

Name______________________________ Tutor______________________________

An elephant is being used to pull a large log up a hill having a slope of 15°. The log has a mass of 500 kg, and the coefficient of friction between the log and the ground is 0.200. The pulling rope makes an angle of 30° with respect to the ground as shown in the figure.



1. Draw a free-body diagram of the log.

2. What is the reaction force to each of the forces acting on the log?

3. What force must the elephant exert ($\vec{F}$) for the log to be pulled at a constant speed?

4. If the rope breaks and the log slides down the hill, what is its acceleration?

<u>Note</u>: $\sin 15^\circ = 0.258$; $\sin 30^\circ = 0.50$;

$\cos 15^\circ = 0.97$; $\cos 30^\circ = 0.87$.

Date____________________

NEWTON'S LAWS

pass recycle

Mastery Test Form B

1 2 3

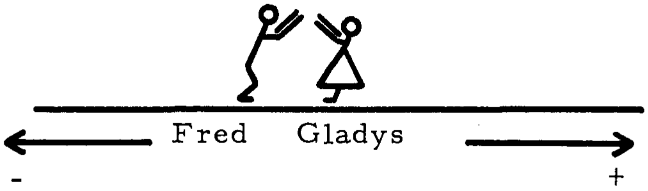
Name____________________ Tutor____________________

A horizontal force of 16.0 N will drag an 8.0-kg block along a certain horizontal surface with constant velocity. This same surface is then inclined at an angle of 37^{o} with the horizontal, and a force parallel to the plane draws the block up the plane with an acceleration of 2.00 m/s^2.

1. Draw free-body diagrams for the block when it is on the horizontal surface and for when it is on the inclined surface.

2. What is the magnitude of the force drawing the block up the plane?

3. Fred (100 kg) and Gladys (72 kg) are standing on a smooth surface (ice). Gladys pushes Fred with a force of -260 N. What is Gladys' acceleration?



Date____________________

NEWTON'S LAWS

pass recycle

Mastery Test Form C

1 2 3 4

Name ____________________ Tutor____________________

A student couple are filling their new waterbed for the first time on a lawn that slopes 6.0°. <u>NOTE</u>: $\sin 6.0^{\circ} = 0.104$; $\cos 6.0^{\circ} = 0.99$.

1. Draw a free-body diagram for the waterbed including friction with the lawn.

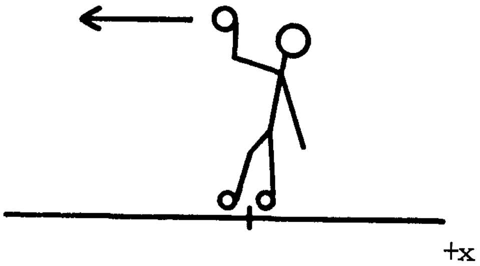
2. What is the minimum coefficient of static friction needed to keep the 680-kg waterbed from sliding away?

3. The coefficient of sliding friction is 0.03 on the wet grass. If the bed starts to move, how fast is it moving after sliding 46 m?

4. A 100-kg ice skater (no friction) throws a 0.50-kg snowball with an acceleration of 25.0 m/s^2 in the horizontal direction.

   (a) What agent exerts the force that moves the skater?

   (b) What are the magnitude and direction of the acceleration of the skater?



DATE____________________

Instructor____________________

NEWTON'S LAWS

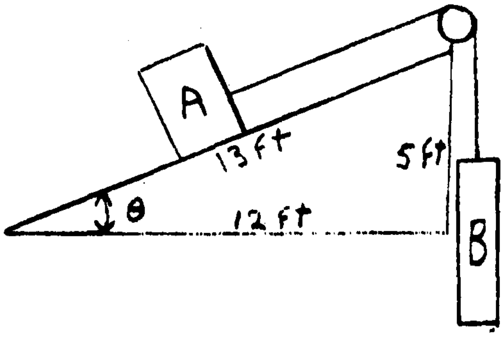
pass recycle

Mastery Test Form D

1 2 3 4

Name____________________________ Tutor____________________________

In the accompanying figure, block A is sliding up the inclined plane with an acceleration of 2.45 m/sec^2. The coefficient of friction between the plane and block A is 0.25, and the hanging block, B, weighs 19.6N. Assume the pulley to be massless and frictionless.



1. a) Draw a free-body diagram for Block A.

   b) List action-reaction pairs of forces for Block A.

2. Distinguish clearly between mass and weight.

3. What is the tension in the cord above block B?

4. What is the weight of Block A?

Courtesy of University of Missouri-Rolla

Instructor____________________ Date____________________

NEWTON'S LAWS

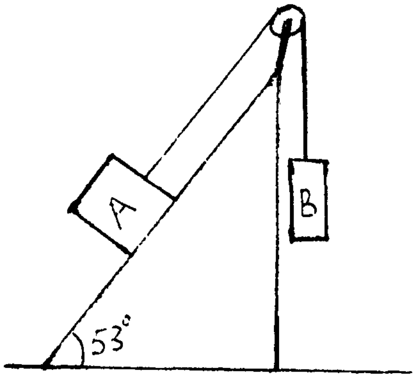
pass recycle

Mastery Test Form E

1 2 3 4

Name____________________ Tutor____________________

In the accompanying figure, block A is sliding down the inclined plane and accelerating downwards at a rate of 2 m/sec^2. The plane is inclined at an angle of 53° to the horizontal and has a coefficient of friction 0.50. The hanging block B weighs 5 newtons. Assume the pulley to be massless and frictionless.



1. a) Make a free-body diagram of block A.
   b) Are there any action-reaction pairs on block B? If so, what are they?
2. State Newton's Second Law of Motion.
3. What is the tension in the cord?
4. What is the mass of block A?

Courtesy of University of Missouri-Rolla

Instructor____________________ Date____________________

NEWTON'S LAWS

1 2 3 4

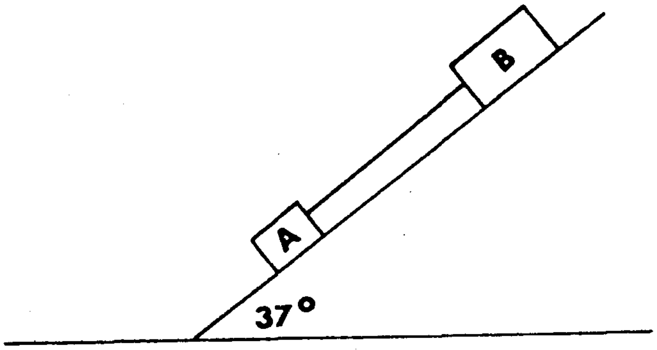
Mastery Test Form F

pass recycle

Name________________________

Tutor________________________

In the accompanying figure, object A weighs 50 newtons, and object B weighs 100 newtons. The coefficient of friction between A and the plane is 0.5, and the coefficient of friction between B and the plane is 0.2. A and B are connected by a rigid rod (neglect its mass).



1. Make a free body diagram for Block B.

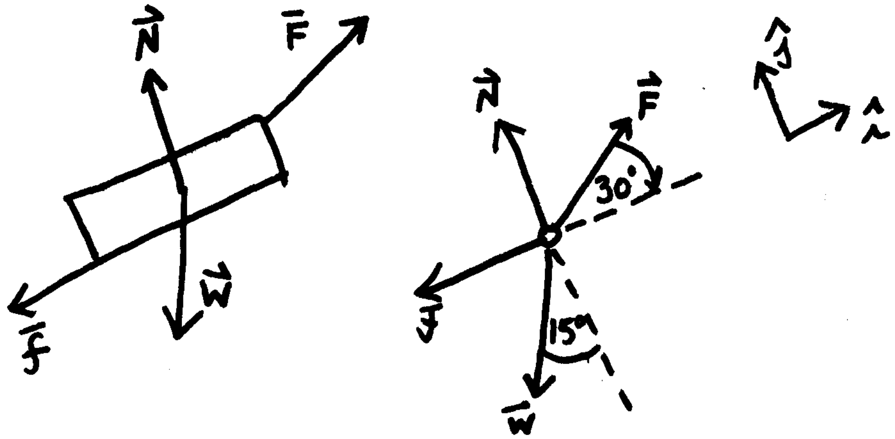
2. Distinguish clearly between mass and weight.

3. A 1500 kgm automobile climbs a hill at a speed of 12 m/sec. If the hill rises 1 m in 10 m and there is a frictional resisting force of 500 newtons, how much force must the driving wheels exert?

4. What is the acceleration of the system shown above, and the compression in the rod connecting A and B?

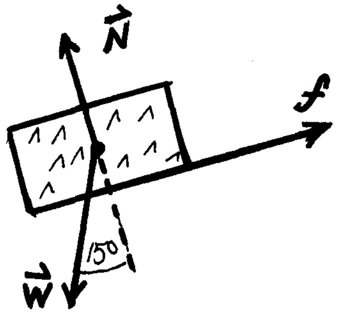
Courtesy of University of Missouri-Rolla

MASTERY TEST GRADING KEY - Form A

What To Look For	Solutions
1. Make sure all forces are present!	1. 
2. Each force should be given.	2. Action Force — Reaction Force $\vec{N}$ — Log exerts normal force on the ground ($-\vec{N}$). $\vec{F}$ — Log exerts a pull on the rope ($-\vec{F}$). $\vec{W}$ — Log attracts the earth ($-\vec{W}$). $\vec{f}$ — Log exerts frictional force on the ground ($-\vec{f}$).
3. Constant speed up the plane requires $a_x = 0$.	3. Applying Newton's second law to the log: $\Sigma F_x = ma_x$: $F_x \cos 30° - f - W \sin 15° = 0$, (1) $\Sigma F_y = ma_y$: $N + F_y \sin 30° - W \cos 15° = 0$. (2) Solve for N from Eq. (2) and substitute into $\vec{f} = \mu\vec{N}$: $f = \mu W \cos 15° - \mu F \sin 30°$. Substitute this into Eq. (1): $F \cos 30° - \mu W \cos 15° + \mu F \sin 30° - W \sin 15° = 0$, $F(\cos 30° + \mu \sin 30°) = W \sin 15° + \mu W \cos 15°$, $\vec{F} = m\vec{g}\,\frac{\sin 15° + \mu \cos 15°}{\cos 30° + \mu \sin 30°}$ $= \frac{(500\text{ kg})(9.8\text{ m/s}^2)[(0.258) + (0.200)(0.97)]}{0.87 + (0.200)(0.50)}$ $\vec{F} = 2300$ N.

4. Free-body diagram of the log. Log is sliding down the slope, thus frictional force has changed direction.

4.



Coefficient of friction is the same as in the first part of the problem, but the normal force has changed. Minus sign means the log is accelerating in the -x (down the slope) direction.

Apply Newton's second law to the log:

$$\Sigma F_x = ma_x: \quad f - W \sin 15° = ma_x, \tag{3}$$

$$\Sigma F_y = ma_y: \quad N - W \cos 15° = 0,$$

$$f = \mu N = \mu W \cos 15°$$

Substitute into Eq. (3), we find

$$\mu W \cos 15° - W \sin 15° = ma_x,$$

$$\mu g \cos 15° - g \sin 15° = a_x,$$

$$a_x = (0.200)(9.8\ \text{m/s}^2)(0.97) - (9.8\ \text{m/s}^2)(0.258),$$

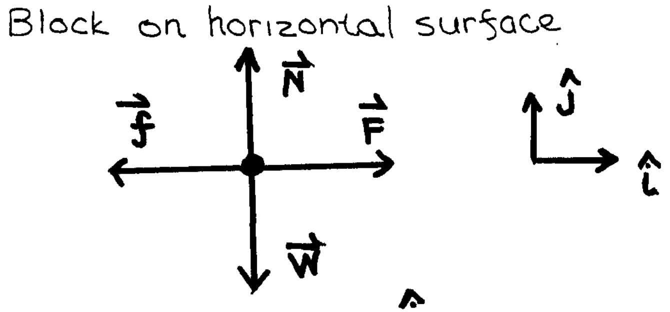
$$a_x = -\ 0.65\ \text{m/s}^2.$$

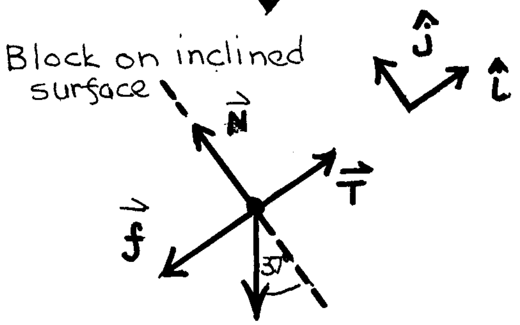
MASTERY TEST GRADING KEY - Form B

What To Look For | **Solutions**

1\.

1\.





2\. Solve using information from the horizontal situation.

2\. For the block on a horizontal surface, applying Newton's second law:

$$\Sigma F_x = ma_x: \quad F - f = ma = 0,$$

$$\Sigma F_y = ma_y: \quad N - W = 0,$$

$$N = W = mg, \quad \vec{f} = \mu\vec{N} = \mu m\vec{g}.$$

Equation (1) becomes

$$\vec{F} - \mu m\vec{g} = 0$$

$$\mu = \frac{\vec{f}}{m\vec{g}} = \frac{16.0\ \text{N}}{(8.0\ \text{kg})(9.8\ \text{m/s}^2)},$$

$$\mu = 0.20.$$

For the block on the inclined surface, we apply Newton's second law:

$$\Sigma F_x = ma_x: \quad T - f - W \sin 37° = ma_x, \qquad (2)$$

$$\Sigma F_y = ma_y: \quad N - W \cos 37° = 0$$

Coefficient of friction doesn't change because the surfaces are the same.

$$f = \mu N = \mu W \cos 37°,$$

$$f = \mu mg \cos 37°.$$

Substitute this value for f into Eq. (2), we find

$$T - \mu mg \cos 37° - mg \sin 37° = ma_x,$$

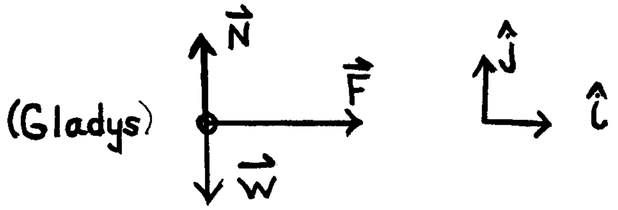
$$T = ma_x + \mu mg \cos 37° + mg \sin 37°$$

$$= (80\ \text{kg})(2.00\ \text{m/s}^2) + (0.200)(8.0\ \text{kg}) \times (9.8\ \text{m/s}^2)(0.80) + (8.0\ \text{kg})(9.8\ \text{m/s}^2)(0.60)$$

$$T = 76\ \text{N}.$$

3. Make sure free-body diagram is correct.

3. From Newton's third law, Fred pushes Gladys with a 260-N force:



Using Newton's second law, we find

$$\Sigma F_x = ma_x: \quad F = ma,$$

$$\Sigma F_y = ma_y: \quad N - W = 0,$$

$$a = \frac{F}{m} = \frac{260\ \text{N}}{72\ \text{kg}},$$

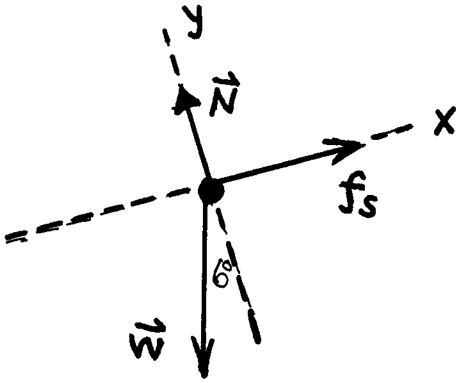
$$\vec{a} = 3.6\ \text{m/s}^2.$$

MASTERY TEST GRADING KEY - Form C

What To Look For

Solutions

1. Static Case:



2. Static case, the bed is not moving:
$a_x = a_y = 0.$

$f_s = \mu_s N$, but we want to know about this situation just before the bed slides, so $f_s = \mu_s N$.

μ_s is unitless

2. Applying Newton's second law in component form:

$$\Sigma F_x = ma_x: \quad f_s - W \sin 6.0° = 0, \qquad (1)$$

$$\Sigma F_y = ma_y: \quad N - W \cos 6.0° = 0, \qquad (2)$$

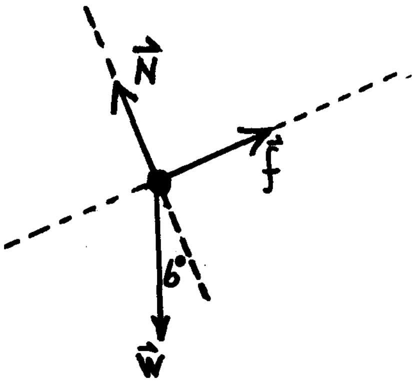
$f_s = \mu_s N,$

$f_s = \mu_s W \cos 6.0°$. Substitute this into Eq. (1):

$$\mu_s W \cos 6.0° - W \sin 6.0° = 0$$

$$\mu_s = \frac{\sin 6.0°}{\cos 6.0°} = \tan 6.0° = 0.11$$

3. Sliding Case:



3. a was explicitly assigned a minus sign since the bed is moving in -x direction.

Using Newton's second law in component form

$\Sigma F_x = ma_x$: $f_k - W \sin 6.0° = - ma$, (3)

$\Sigma F_y = ma_y$: $N - W \cos 6.0° = 0$,

$f_k = \mu_k N = \mu_k W \cos 6.0°$.

Substitute this into Eq. (3):

$\mu_k W \cos 6.0° - W \sin 6.0° = - ma$,

$W = mg$,

$\mu_k mg \cos 6.0° - mg \sin 6.0° = - ma$,

$a = g \sin 6.0° - \mu_k g \cos 6.0°$,

$a = g(\sin 6.0° - \mu_k \cos 6.0°)$,

$a = (9.8 \text{ m/s}^2)(0.104 - 0.03)$,

$a = 0.69 \text{ m/s}^2$.

Because of the explicit sign choice above, a should come out positive.

v_f = final speed,

v_i = initial speed,

x = distance traveled.

Referring back to the <u>Rectilinear Motion</u> module for one-dimensional uniform motion:

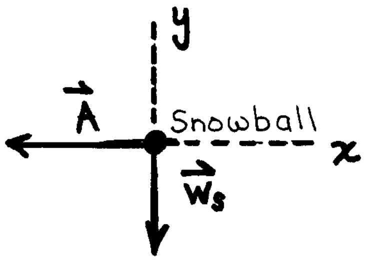
$v_f^2 = v_i^2 + 2ax$,

$v_f^2 = 0^2 + 2(0.69 \text{ m/s}^2)(46 \text{ m})$,

$v_f = 8.0 \text{ m/s}$.

4. (a) Acceleration of snowball is in direction of $\vec{A}$, which is in the -x direction; hence the negative sign for a.

4. (a)



$\vec{A}$ is force man exerts on snowball;

$\vec{W}$ is weight of snowball.

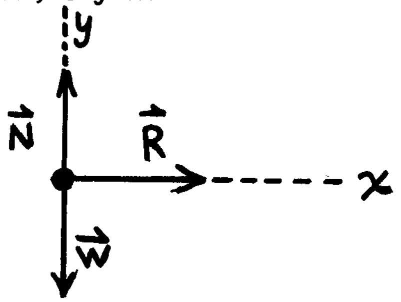
Newton's second law:

$F_x = ma_x$,

$A = ma = (0.50\ \text{kg})(-25.0\ \text{m/s}^2)$,

$A = -12.5\ \text{N}$.

From Newton's third law: If $\vec{A}$ is action force then an equal but opposite force is exerted on the man, say $\vec{R}$:



$\vec{N}$ is normal force that ice exerts on skater;

$\vec{W}$ is weight of the skater $W = \mu g$;

$\vec{R}$ is force that snowball exerts on the man.

(b) Applying Newton's second law:

$F_x = ma_x$,

$R = Ma'$, $a' = \frac{R}{M} = \frac{12.5\ \text{N}}{100\ \text{kg}}$,

$a' = 0.12\ \text{m/s}^2$.

a' will be in the direction of $\vec{R}$ or in +x direction.