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Rectilinear Motion

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Module________

RECTILINEAR MOTION

INTRODUCTION

How long does it take you to go home? This depends on how far you are from home (displacement), how fast (velocity) you can travel, and how often you must start and stop (acceleration).

This module treats kinematics, which is the part of physics concerned with the description of the motion of a body. The body may be an automobile, a baseball, a raindrop, a flower in the wind, or a running horse. The change in position of a body can be described in terms of the vector quantities: displacement, velocity, and acceleration. Calculus can be used to define the relationships among these quantities. It is therefore essential to know some basic techniques of calculus to understand the content of this module.

The applications in this module only consider motion in one dimension. A later module will treat the more general case of motion in two or three dimensions, but the fundamental concepts will be essentially the same.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Distinguish between vector and scalar quantities (needed for Objectives 1 and 2 of this module)	Dimensions and Vector Addition Module
*Differentiate and integrate simple polynomials (needed for Objectives 3 to 5 of this module)	Calculus Review
*Differentiate the sine and cosine functions (needed for Objectives 4 and 5 of this module)	Calculus Review Trigonometry Review

LEARNING OBJECTIVES

After you have mastered the content of this module you will be able to:

1. Displacement, velocity, acceleration - Write the mathematical definitions of displacement, instantaneous velocity, and acceleration, and define all terms.

2. Average-instantaneous, position-displacement, velocity-speed - Distinguish between average and instantaneous values of velocity and acceleration, and distinguish between position and displacement and between velocity and speed.

3. Graphical - (a) Given a graph of position as a function of time, for one-dimensional motion, determine either average or instantaneous velocity. (b) Given a graph of velocity as a function of time, determine acceleration and displacement.

4. Analytical - (a) Given a mathematical expression for position as a function of time, for one-dimensional motion, determine an equation for velocity as a function of time. (b) Given a mathematical expression for velocity as a function of time, determine the equations for acceleration and displacement as a function of time.

5. Constant acceleration problems - In the case of one-dimensional motion of a body with constant acceleration, determine the displacement, velocity, and/or acceleration of the body; e.g., a body falling freely near the surface of the earth.

GENERAL COMMENTS

Physics is an area of human knowledge based on accumulative learning. The concepts of rectilinear motion are the foundation for the study of physics of moving bodies. Mastery of the objectives of this module is essential to a successful understanding and completion of subsequent modules.

Rectilinear motion is the motion of a single particle along a straight line. Thus, we may suspend our use of vector notation in working problems, since in one dimension velocity and acceleration may be considered as their respective x components and treated algebraically. The vector notation will be kept when we are speaking rigorously of definitions, but otherwise the symbol a, for example, will represent both the vector acceleration and the absolute value of the acceleration, which is scalar. In later modules we shall have to be more particular and specify by subscripts, a_x, a_y, or a_z, for example, to which scalar component of the vector $\vec{a}$ we are referring. In one dimension, vector addition (subtraction) reduces and is equivalent to algebraic addition (subtraction). Note that careful observance must be made of signs.

ADDITIONAL LEARNING MATERIALS

There is a series of film loops available from John Wiley and Sons, Inc., that may assist in mastering Objectives 1, 2, and 3:

Velocity from Position,
Position from Velocity, I,
Position from Velocity, II,
One-Dimensional Motion.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read Chapter 4, Sections 4.1 to 4.4, 4.6, 4.7, and work at least Problems A through I of this module before attempting the Practice Test. Projectile Motion (Sec. 4.8) will be covered in a later module.

It is essential to work the problems related to Objective 4 (Analytical) because the book does not treat this subject in the reading. You may wish to refer to F. W. Sears and M. W. Zemansky [University Physics (Addison-Wesley, Reading, Mass., 1970) fourth edition], Sections 4-5 and 4-6.

Illustrations 4.4 and 4.5 are the most important part of the reading for Objective 5.

BUECHE

Objective Number	Readings	Problems with Solutions	Assigned Problems	
		Study Guide	Study Guide	Text
1	Secs. 4.1, 4.2	A		
2	Secs. 4.1, 4.2	B		
3	Secs. 4.1, 4.2, 4.4	C	F	Problem 10
4	Not covered in this text (see Sears and Zemansky, Secs. 4-5, 4-6)	D, G	H	Problems 27, 28
5	Secs. 4.4, 4.6, 4.7	E	I	Problems 13, 15

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read Chapter 3 and work at least Problems A through I before attempting the Practice Test.

It is essential to work the problems related to Objective 4 (Analytical) because the discussion in the book is limited to the one example for constant acceleration. You will be required to differentiate and integrate functions as shown in Problems D, G of this module and Problems 4 and 5 of the text.

Examples 7 and 8 on pp. 37, 38 are the most important part of the reading for Objective 5.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions	Assigned Problems	
		Study Guide	Study Guide	Text
1	Secs. 3-3, 3-4, 3-6	A		
2	Secs. 3-3, 3-4, 3-6	B		
3	Secs. 3-2, 3-5, 3-6, 3-8	C	F	10, 11, 12
4	Sec. 3-8 (in particular, Example 4)	D, G	H	4, 13
5	Secs. 3-8, 3-10	E	I	15, 16, 21, 30, 34

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read Chapter 4 and work at least Problems A through I before attempting the Practice Test. The section on Velocity Components (4-9) will be treated in a later module.

Examples 1 and 2 on pp. 48 and 49 are the most important part of the reading for Objective 5.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions	Assigned Problems	
		Study Guide	Study Guide	Text
1	Secs. 4-2 through 4-4	A		
2	Secs. 4-2 through 4-4	B		
3	Secs. 4-3, 4-5, 4-6	C	F	4-23, 4-24
4	Secs. 4-5, 4-6, 4-8	D, G	H	4-28, 4-29
5	Sec. 4-7	E	I	4-20, 4-25, 4-26

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Read Chapter 3 and work at least Problems A through I before attempting the Practice Test.

It is essential to work the problems related to Objective 4 (Analytical) because the discussion in the book is limited to the one example for constant acceleration. You will be required to differentiate and integrate functions as shown in Problems D and G of this module and Problems 3-12 and 3-13 of the text.

Examples 3-3, 3-4, and 3-5 on pp. 28-32 are the most important part of the reading for Objective 5.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions	Assigned Problems	
		Study Guide	Study Guide	Text
1	Secs. 3-2 through 3-4	A		
2	Secs. 3-2 through 3-4	B		
3	Secs. 3-2 through 3-5	C	F	3-1, 3-2
4	Sec. 3-5 [Middle of p. 27 to Eq. (3-7) on p. 28]	D, G	H	3-12, 3-13
5	Secs. 3-5, 3-6	E	I	3-20, 3-23, 3-24, 3-32

PROBLEM SET WITH SOLUTIONS

A(1). Write the mathematical definitions of displacement, instantaneous velocity, and acceleration, and define all terms.

Solution

Displacement:

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1,$$

where $\vec{r}_2$ is position at point 2 and $\vec{r}_1$ is position at point 1.

Instantaneous Velocity:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}.$$

Instantaneous Acceleration:

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}.$$

B(2). A woman swims the length of a 75-m pool and back again (one lap) in 60 s with a constant speed. (a) What is her displacement at the end of one lap? (b) What is her average velocity for one lap? (c) What is her average speed for one lap?

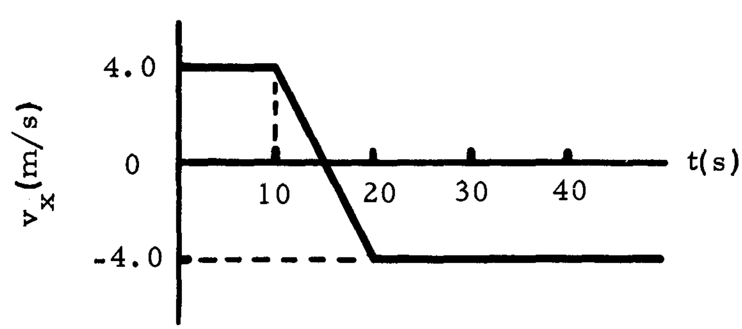
Solution

(a) $\vec{r}_2 = \vec{r}_1$; therefore, $\Delta\vec{r} = 0$.

(b) Since $\Delta\vec{r} = 0$, $\vec{v}_{av} = 0$.

(c) $v_{av} = 150\ \text{m}/60\ \text{s} = 2.5\ \text{m/s}$.

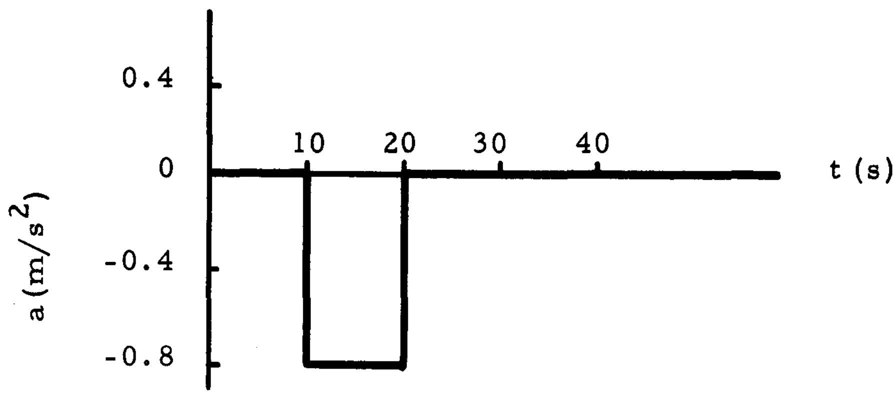
C(3). A particle's velocity is shown in the graph below. At t = 0, its displacement is x = 0.
(a) Sketch the acceleration vs. time graph corresponding to this velocity vs. time graph.



Solution

Acceleration is the slope of the v vs. t graph. For the first 10 s the acceleration is zero (v = 0). The acceleration is also 0 for t = 20 s. For the time between 10 and 20 s the slope is a constant. Therefore

$$a = a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{(-4.0)\text{m/s} - (4.0)\text{m/s}}{10\text{s}} = -0.80\ \text{m/s}^2.$$



C(3). (b) Sketch the graph of position (x) vs. time (t) corresponding to the velocity vs. time graph.

Solution

At t = 0, x = 0. The position at any time is the area under the v vs. t graph up to that time.

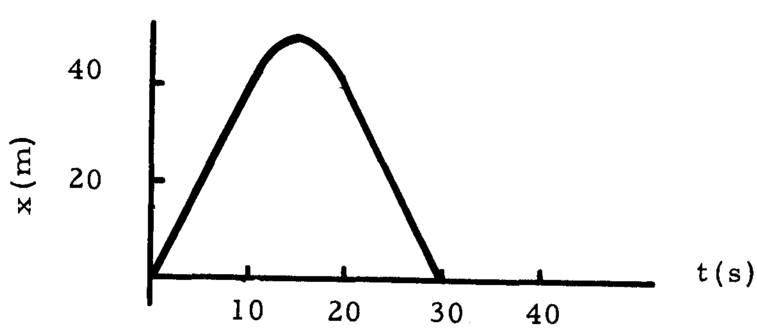
At t = 5 s; x = 4 m/s × 5 s = 20 m.

At t = 10 s; x = 4 m/s × 10 s = 40 m.

At t = 15 s; $x = 40\ \text{m} + (\frac{1}{2})4\ \text{m/s} \times 5\ \text{s} = 50\ \text{m}.$

At t = 20 s; $x = 50\ \text{m} + \frac{1}{2}(-4)\ \text{m/s} \times 5\ \text{s} = 40\ \text{m}.$

At t = 30 s; x = 40 m + (-4) m/s × 10 s = 0.



Note: The curve is not linear between t = 10 and 20 s. In fact, it is a parabola, which will be shown later.

C(3). (c) Determine the average acceleration between t = 0 and 20 s.

Solution

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{-4 - (+4)\ \text{m/s}}{20\ \text{s}} = \frac{-8\ \text{m/s}}{20\ \text{s}} = -0.4\ \text{m/s}^2.$$

D(4). The vertical position of a body under constant acceleration is given by

$$y = y_0 + v_{y0}t + \frac{1}{2}at^2,$$

where t = time,

y = vertical position at time t,

v_{y0} = initial velocity in vertical direction,

a = acceleration in vertical direction,

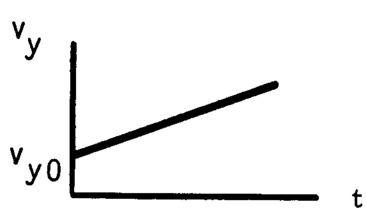
y_0 = initial position.

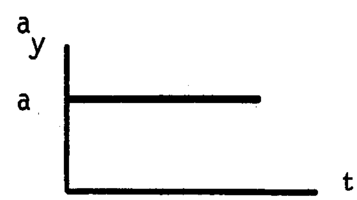
By the use of calculus, find the velocity and acceleration of the body as a function of time. Also construct a graph of velocity vs. time and acceleration vs. time for this case.

Solution

$$v_y = \frac{dy}{dt} = \frac{d}{dt}(y_0 + v_{y0}t + \frac{1}{2}at^2) = v_{y0} + at$$

$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}(v_{y0} + at) = a.$$





E(5). An object is thrown vertically upward in a uniform gravitational field that produces an acceleration of $a = -g = -9.8\ \text{m/s}^2$. It has a speed of 9.8 m/s when it has reached one-half its maximum height.

(a) How high does it rise?

(b) What is its speed 1 s after it is thrown?

(c) What is its acceleration when it reaches its maximum height?

Solution

This problem requires the solution of the folowing equation:

(a) $v^2 - v_0^2 = 2ay$ (constant acceleration).

When at one-half height, $y_{1/2} = y_{max}/2$; and since $v = 0$ at $y = y_{max}$,

$$-v_0^2 = -2gy_{max} \text{ or } v_0^2 = 2gy_{max}.$$

Using $v^2 - v_0^2 = 2ay$ (at $y = y_{max}/2$) gives

$$v^2 - 2gy_{max} = -2gy,$$

and using $y = y_{1/2} = y_{max}/2$ gives

$$v^2 = 2gy_{max} - gy_{max} = gy_{max}.$$

Solving for

$$y_{max} = \frac{v^2}{g} = \frac{(9.8\ m/s)^2}{9.8\ m/s^2} = 9.8\ m.$$

Also:

$$v_0 = \sqrt{2gy_{max}} = \sqrt{2(9.8\ m/s)^2} = 13.9\ m/s.$$

(b) $v - v_0 = at$; $v = -gt + v_0 = -9.8 + 13.9 = 4.1$ m/s.

(c) $a = -9.8\ m/s^2$ (acceleration is constant).

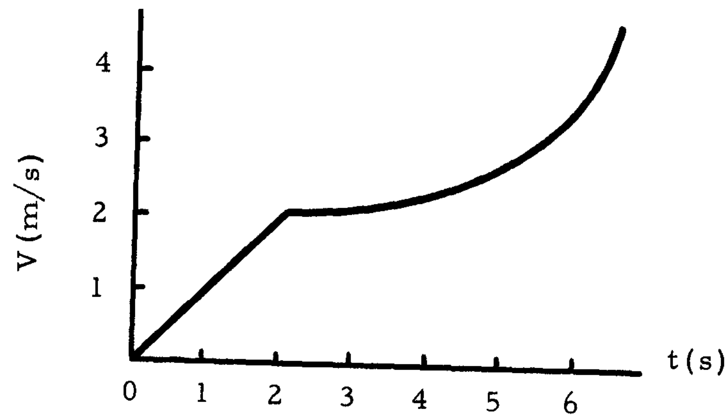
Problems

F(3). The velocity vs. time graph of a particle is shown in the figure below.

(a) During what time interval is the particle traveling with constant acceleration?

(b) Calculate graphically the displacement of the particle during the first 3 s.

(c) Estimate the particle's acceleration at t = 5 s.



G(4). A particle moves along the x axis in such a way that its x coordinate is given by $x = 3 + 20t - t^2$. x is in meters; t is in seconds. Find:

(a) Its velocity at t = 4 s.

(b) Its position at t = 4 s.

(c) Its acceleration at t = 4 s.

H(4). Let $x = A \cos \omega t$ where A and ω are constants. Find the expressions for velocity as a function of time and acceleration as a function of time.

I(5). A rocket is fired vertically and ascends with a constant vertical acceleration of $+20\ m/s^2$ for 80 s. Its fuel is then all used, and it continues upward with an acceleration of $-9.8\ m/s^2$. Air resistance can be neglected.

(a) What is its altitude 80 s after launching?

(b) How long does it take to reach its maximum altitude?

(c) What is this maximum altitude?

Solutions

F(3). (a) Acceleration is constant for t = 0 to 2 s.

(b) 4 m (area under curve).

(c) $0.6\ m/s^2$ (slope at t = 5 s).

G(4). (a) $v = \frac{dx}{dt} = \frac{d}{dt}(3 + 20t - t^2) = 20 - 2t$ at $t = 4$ s, $v = 12$ m/s.

(b) $x = 3 + 20t - t^2$ at $t = 4$ s, $x = 67$ m.

(c) $a = \frac{dv}{dt} = \frac{d}{dt}(20 - 2t) = -2\ m/s^2$.

H(4). $v = \frac{dx}{dt} = \frac{d}{dt}(A \cos \omega t) = A \frac{d}{dt}(\cos \omega t) = -A\omega \sin \omega t$.

$a = \frac{dv}{dt} = \frac{d}{dt}(-A\omega \sin \omega t) = -A\omega \frac{d}{dt} \sin \omega t = -A\omega^2 \cos \omega t$.

I(5). (This problem must be treated as two separate constant-acceleration problems.)
(a) 6.4×10^4 m. (b) 2.4×10^2 s. (c) 1.9×10^5 m.

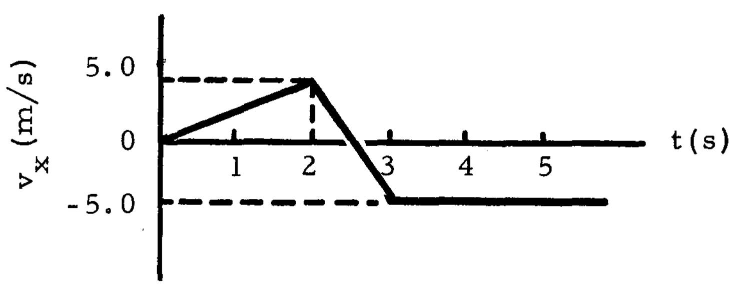
PRACTICE TEST

1. Write the mathematical definitions of displacement, instantaneous velocity, and acceleration.

2. A particle's velocity is shown on the graph. At t = 0, the displacement is x = 0.

(a) Sketch the displacement and acceleration as a function of the time.

(b) Determine the average acceleration during the time interval t = 0 to 2.5 s.



3. The position of a particle as a function of time is given by $y = \alpha t^3 - \beta t$, where α and β are constants. Find the expressions for velocity as a function of time and acceleration as a function of time.

4. A ball is thrown vertically upward from the ground with an initial speed of 24.5 m/s, under the influence of gravity ($g = 9.8\ m/s^2$).

(a) How long does the ball take to reach its highest point?

(b) How high does the ball rise?

(c) What is its velocity and acceleration at its maximum height?

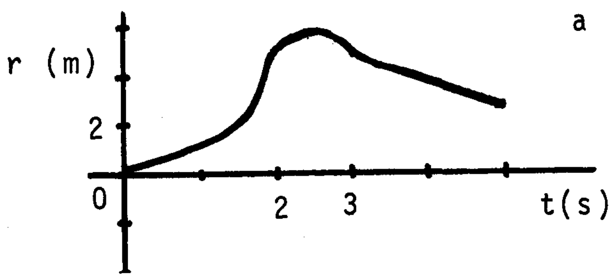
Answers to Practice Test

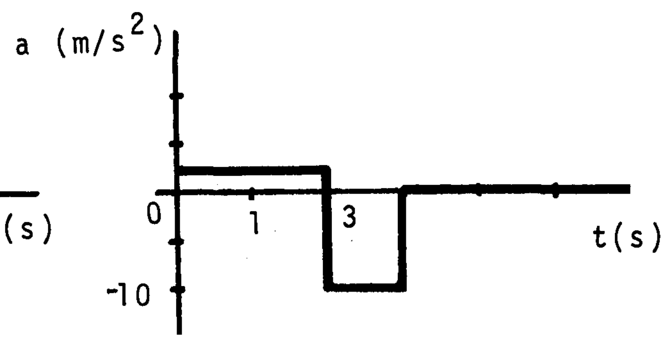
1. $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}.$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}.$$

2. (a)





(b) $a_{av} = (v_2 - v_1)/(t_2 - t_1) = 0/2.5 = 0$.

3. $v = dy/dt = 3\alpha t^2 - \beta$, $a = dv/dt = 6\alpha t$.

4. (a) At the highest point, $v = 0$. Therefore we can use the equation,

$v(t) = 0 = v_0 + at$ (and v_0 = initial velocity = 24.5 m/s),

$t = -v_0/a = (-24.5\ \text{m/s})/(-9.8\ \text{m/s}^2) = 2.5$ s.

(b) $x(t) = x_0 + v_0 t + (1/2)at^2$

$= 0 + (24.5\ \text{m/s})(2.5\ \text{s}) + (1/2)(-9.8\ \text{m/s}^2)(2.5\ \text{s})^2$

$= 61.2\ \text{m} - 30.6\ \text{m} = 30.6$ m.

(c) $v_{max} = v_0 + at = 24.5\ \text{m/s} + (-9.8\ \text{m/s}^2)(2.5\ \text{s})$

$= 24.5 - 24.5 = 0$ (which we already knew from the fact that at the maximum point the ball stops rising and begins falling.

$a = -g = \text{const} = -9.8\ \text{m/s}^2$.

Date________________

RECTILINEAR MOTION

Mastery Test   Form A

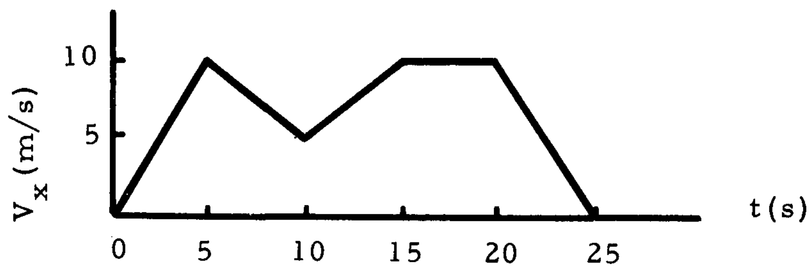
pass   recycle

1   2   3   4

Name___________________________________   Tutor______________________

1. Write the mathematical definitions of displacement, instantaneous velocity, and acceleration.

2. The graph below shows the straight-line velocity of an object as a function of time. What is the average acceleration during the time interval t = 5 to 15 s ? Plot a graph of the acceleration as a function of time. Indicate numerical values on the axes as accurately as freehand will permit.



3. The speed of a particle along the x axis is given as

$$v_x = \alpha t^2 \qquad (\alpha = \text{constant}).$$

(a) What is the position of the particle as a function of time if x = 0 at t = 0?

(b) What is the acceleration of the particle as a function of time?

4. On the moon, the acceleration due to gravity is 1/6 as large as on the earth. An object is given an initial upward velocity of 98 m/s at the surface of the moon.

(a) How long will it take for the object to reach maximum height?

(b) How high above the surface of the moon will the object rise?

Date___________________________

RECTILINEAR MOTION

Mastery Test   Form B

pass   recycle
1   2   3   4

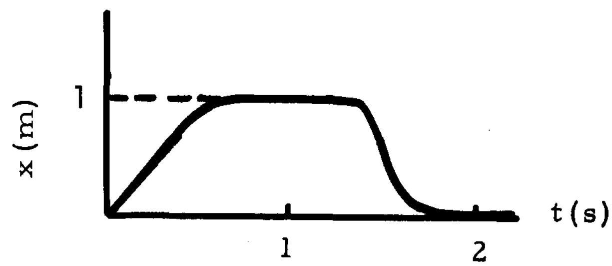
Name_________________________________ Tutor___________________________

1. Write the mathematical definitions of displacement, instantaneous velocity, and acceleration.

2. A graph of position (x) vs. time (t) for a particle is shown below.

   (a) Locate all regions where the instantaneous speed is zero.

   (b) What is the average speed for the time interval $t = 0$ to $t = 2$ s?



3. The position of a particle as a function of time is given by

$$x = A \sin \omega t,$$

where A and ω are constants. Find expressions for velocity and acceleration as a function of time.

4. A jet plane that is landing touches the ground at a speed of 100 m/s. It decelerates uniformly, coming to a stop after 40 s.

   (a) What is the plane's acceleration?

   (b) How far down the runway does the plane move before stopping?

Date____________________

RECTILINEAR MOTION

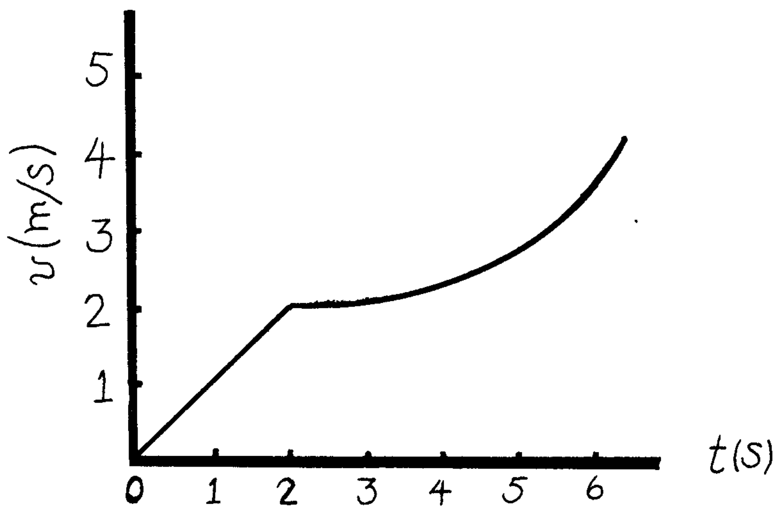
Mastery Test   Form C

pass   recycle
1   2   3   4

Name______________________________ Tutor______________________

1. Write the mathematical definitions of displacement, instantaneous velocity, and acceleration.

2. The velocity vs. time graph of a particle is shown in the following figure:



(a) During what portion of the motion is the particle traveling with constant acceleration?

(b) Estimate the displacement of the particle during the first 3 s.

(c) Estimate the particle's acceleration at t = 5 s.

3. A particle moves along the x axis according to the equation

$$x = 5t + t^2,$$

where x has units of meters and t has units of seconds. Calculate the instantaneous velocity and acceleration at t = 2 s.

4. A ball thrown vertically upwards returns to the starting point in 4 s. Find its initial speed.

INSTRUCTOR______________

RECTILINEAR MOTION

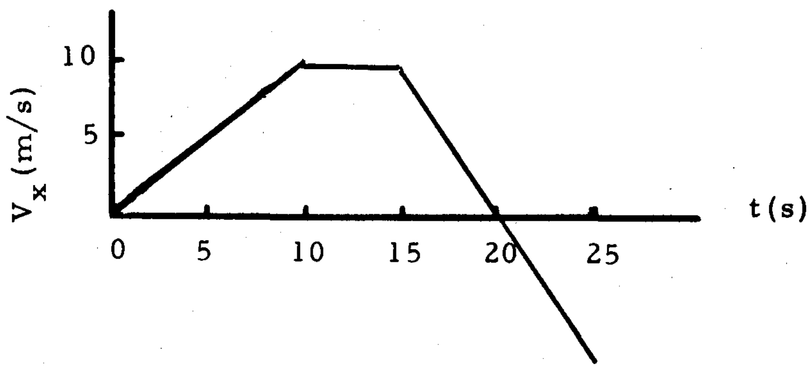
Mastery Test Form D

	pass		recycle	
	1	2	3	4

Name________________________________ Tutor____________________

1. Write the mathematical definitions of displacement, instantaneous velocity, and acceleration.

2. The graph below shows the straight-line velocity of an object as a function of time. What is the average acceleration during the time interval t = 5 to 15 s ? Plot a graph of the acceleration as a function of time. Indicate numerical values on the axes as accurately as freehand will permit.



3. The speed of a particle along the x axis is given as

$$v_x = 3\ m/_{sec} + 5\ m/_{sec^4}\ t^3$$

(a) What is the position of the particle as a function of time if x = 0 at t = 0?

(b) What is the acceleration of the particle as a function of time?

4. A rock dropped from a bridge takes 2 seconds to hit the water. How high is the bridge?

Courtesy of University of Missouri-Rolla

Date________________

INSTRUCTOR_____________

RECTILINEAR MOTION

Mastery Test   Form E

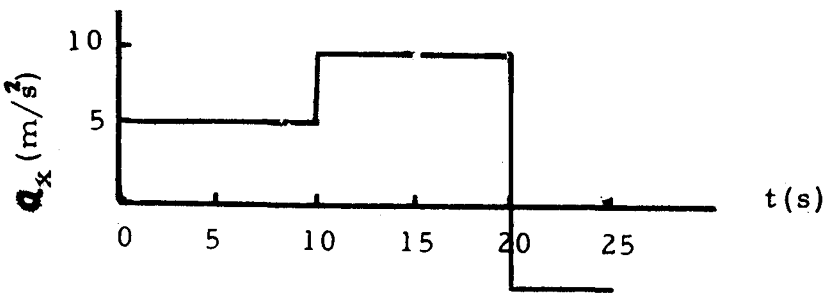
pass   recycle

1 2 3 4

Name________________________________   Tutor______________________

1. Write the mathematical definitions of displacement, instantaneous velocity, and acceleration.

2) The graph below shows the straight line acceleration of a particle as a function of time. What is the velocity at t=10 sec? Plot a graph of the velocity as a function of time. Indicate numerical values on the axes of your plot as accurately as free hand will permit.



3. A particle has a final velocity v = 3m/sec at x = 7m, if it was decelerating at a rate a = -1/2 m/sec² what was its velocity at x = 0.

4. An automobile travelling at 40 m/sec is braked to a stop in 80 m. What was its constant deceleration?

Courtesy of University of Missouri-Rolla

INSTRUCTOR________________

RECTILINEAR MOTION

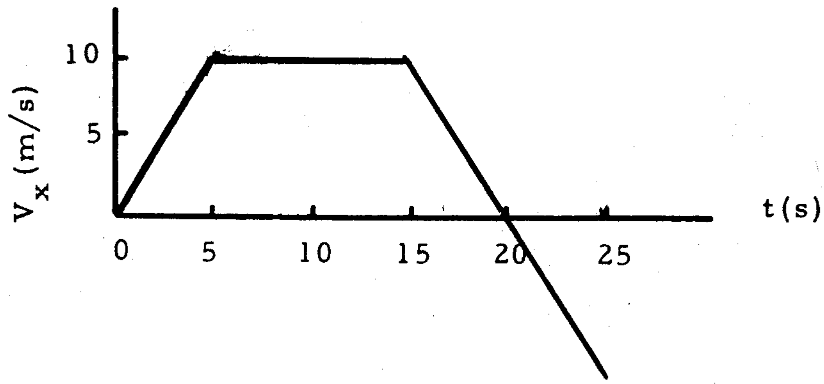
Mastery Test Form F

pass recycle
1 2 3 4

Name________________________ Tutor________________

1. Write the mathematical definitions of displacement, instantaneous velocity, and acceleration.

2. The graph below shows the straight-line velocity of an object as a function of time. What is the average acceleration during the time interval t = 5 to 15 s ? Plot a graph of the acceleration as a function of time. Also plot a graph of the position as a function of time. Indicate numerical values on the axes as accurately as freehand will permit.



3. The speed of a particle along the x axis is given as

$$v_x = A \cos \omega t \quad \text{(A and } \omega \text{ are constants)}$$

(a) What is the position of the particle as a function of time if x = 0 at t = 0?

(b) What is the acceleration of the particle as a function of time?

4. A bomb drops from a plane which is in level flight at 100 m/sec. If the pilot wants the bomb to automatically arm itself after it has fallen 1600 m, for how many seconds must the timer be set? (Neglect air resistance)

MASTERY TEST GRADING KEY - Form A

What To Look For

1. $\vec{r}$, $\vec{v}$, and $\vec{a}$ are written as vectors.

   $\lim_{\Delta t \to 0}$ and/or d/dt is used in definitions of $\vec{v}$ and $\vec{a}$.

2. (a) Average does not equal instantaneous.

   (b) Units for acceleration.

3. (a) Integration to obtain x from v_x and application of condition x = 0 at t = 0 to find integration constant equal to zero.

   (b) Differentiation to obtain a_x from v_x.

Solutions

1. (a) Displacement: $\vec{r} = \vec{r}_2 - \vec{r}_1$.

   (b) Instantaneous velocity:

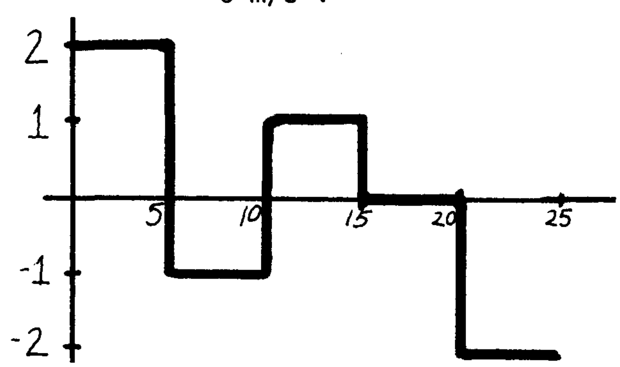
   $$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}.$$

   (c) Instantaneous acceleration:

   $$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}.$$

2. (a) $a_{av} = \frac{dv}{dt} = \frac{(10 - 10)\ \text{m/s}}{10\ \text{s}}$

   $= 0\ \text{m/s}^2$.



3. (a) $v_x = \alpha t^2 = dx/dt$,

   $$\int \frac{dx}{dt}\,dt = \int \alpha t^2\,dt = \alpha \int t^2\,dt,$$

   $$x = \alpha t^3/3.$$

   (b) $a_x = \frac{dv_x}{dt} = \frac{d}{dt}(\alpha t^2)$,

   $a_x = 2\alpha t$.

4. (a) $v_f = 0$ when $y = y_{max}$. Check units.

4. (a)

$$v_f - v_i = at,$$

$$t = (v_f - v_i)/a.$$

Since $a = (-98/6)\ m/s^2$,

$v_i = 98$ m/s, and

$v_f = 0$ at $y = y_{max}$,

$$t = \frac{(0 - 98)\ m/s}{-9.8/6\ m/s}$$

$$t = 60\ s.$$

(b)

$$v_f^2 - v_i^2 = 2ay,$$

$$y = \frac{v_f^2 - v_i^2}{2a}$$

$$= \frac{0 - (98\ m/s^2)^2}{-2(9.8/6)\ m/s^2} = 2940\ m.$$

MASTERY TEST GRADING KEY - Form B

What To Look For	Solutions
1. $\vec{r}$, $\vec{v}$, and $\vec{a}$ are written as vectors. $\lim_{\Delta t \to 0}$ and/or d/dt is used in definitions of $\vec{v}$ and $\vec{a}$.	1. (a) Displacement: $\vec{r} = \vec{r}_2 - \vec{r}_1$. (b) Instantaneous velocity: $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$. (c) Instantaneous acceleration: $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$.
2. (a) Slope of x vs. t graph. (b) Average <u>velocity</u> is zero because total displacement is zero.	2. (a) Instantaneous speed is zero in the interval $1/2 \le t \le 3/2$ s. (b) Average speed: $v = \frac{\text{total distance traveled}}{\text{total time}}$, $v = 2\text{ m}/2\text{ s} = 1\text{ m/s}$.
3. Differentiation of this function will be useful in the module on simple harmonic motion.	3. $x = A \sin \omega t$, $v = \frac{dx}{dt} = A\omega \cos \omega t$, $a = \frac{dv}{dt} = -A\omega^2 \sin \omega t$.
4. Since acceleration is constant, instantaneous acceleration is equal to average acceleration. Check units.	4. (a) $a = \frac{v_f - v_i}{t}$, $t = 40$ s, $v_i = 100$ m/s, $v_f = 0$, $a = (0 - 100\text{ m/s})/40\text{ s}$, $a = -2.5\text{ m/s}^2$. (b) $y = v_i t + \frac{1}{2} at^2$, $y = (100)(40) + \frac{1}{2}(-2.5)(40)^2$, $y = 4000 - 2000 = 2000$ m.

MASTERY TEST GRADING KEY - Form C

What To Look For	Solutions
1. $\vec{r}$, $\vec{v}$, and $\vec{a}$ are written as vectors. $\lim_{\Delta t \to 0}$ and/or d/dt is used in definitions of $\vec{v}$ and $\vec{a}$.	1. (a) Displacement: $\vec{r} = \vec{r}_2 - \vec{r}_1$. (b) Instantaneous velocity: $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$. (c) Instantaneous acceleration: $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$.
2. (a) Acceleration is slope of v vs. t graph. (b) Displacement is area under v vs. t graph, and the answer is approximately 4 m. (c) Slope of graph. Check units.	2. (a) Instantaneous acceleration for $0 \le t \le 2$ s. (b) Displacement graphically - area under curve: $x = \frac{1}{2}(2\text{ m/s} \times 2\text{ s}) + (2\text{ m/s} \times 1\text{ s}) = 4$ m. (c) Acceleration is slope of v vs. t: $a = \approx 0.5\text{ m/s}^2$
3. Check differentiation and note that acceleration is constant.	3. $x = 5t + t^2$ $v = dx/dt = 5 + 2t$; at $t = 2$ s, $v = 9$ m/s, $a = \frac{dv}{dt} = \frac{d}{dt}(5 + 2t) = 2\text{ m/s}^2$.
4. If student uses $t_{up} = t_{down}$, let him/her justify statement.	4. $y = y_0 + v_0 t + \frac{1}{2}at^2$, $y = y_0 = 0$, $0 = v_0 + \frac{1}{2}at$, $v_0 = -\frac{at}{2}$, $a = -9.8\text{ m/s}^2$, $t = 4$ s, $v_0 = -\frac{(-9.8)4}{2}$, $v_0 = 19.6$ m/s