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Simple Statements, Large Numbers

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Master of Arts in Teaching (MAT)
Masters Exam

Shana Streeks

In partial fulfillment of the requirements for the Master of Arts in Teaching with a Specialization in the Teaching of Middle Level Mathematics in the Department of Mathematics.
Gordon Woodward, Advisor

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Simple Statements, Large Numbers

Large numbers are numbers that are significantly larger than those ordinarily used in everyday life, as defined by Wikipedia (2007). Large numbers typically refer to large positive integers, or more generally, large positive real numbers, but may also be used in other contexts. Very large numbers often occur in fields such as mathematics, cosmology, and cryptography. Sometimes people refer to numbers as being "astronomically large". However, it is easy to mathematically define numbers that are much larger than those even in astronomy. We are familiar with the large magnitudes, such as million or billion. In mathematics, we may know a number as an approximation or as an exact amount; for example 531,441. This number could be called "half a million" but it is also the specific solution to the question "how many ways are there to color the 12 numerals on a clock face if you have three different colored markers?" (Contributed by Maria Pierce). You would compute this number as follows: you have 3 choices of color for each hour number 1,2,3,…,12. So there are 3 choices for 1; followed by 3 choices for 2; followed by 3 choices for 3; etc. This would give you 3^{12}= 531,441 number of choices.

A simple question involving small numbers can have a large number as its solution. I have been asked to find and explain a few other examples of simple statements that result in amazingly large numbers. The largest numbers we are likely to use in daily discourse are in the billions. For example, there are now 6.6 billion people on this planet; the solar system has been here about 4.5 billion years; the money spent by our government every year is in the billions; the number of stars we can see on a clear night is ‘billions and billions’. Exploring how large some numbers really are fascinates me. Saying, “I’ve told you a billion times,” has an all new meaning to me now.

Do numbers go on forever, or do they stop? What is the largest number? According to Dr. Math (2002), numbers are ideas, not anything you can see or touch. We use them to count and describe quantity or amount. We have to use our imaginations and picture what it would be like if we found a “larger number”. For example, if I say that quintillion is the largest number, someone could tell me that quintillion plus one is larger. One dictionary I looked in said the largest official group of numbers that has a name is, ‘centillion’ (a one followed by 303 zeroes). There are a couple of naming schemes that name large numbers. These schemes could never have a largest name. For example, two centillions is larger than one centillion. Suppose you want to know how to express 600,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000. The best way to do this is the scientific way: 6×10^{62}, read “six times ten to the power of sixty-two”. This is the most concise way and the most reproducible way to represent large numbers. (home.hetnet., 2007)

The English names for large numbers are coined from the Latin names for small numbers *n* by adding *–illion* suggested by the name ‘million’. The process of naming large numbers can be continued indefinitely, but one has to stop somewhere. Mathematicians have rules for naming numbers that allow them to use a small list of names and put them together to make names for more numbers than a dictionary could hold. Using those rules, mathematicians can make numbers like ‘a centillion centillion’. Back in the 1930’s, a man named Edward Kasner, a mathematician, asked his eight-year-old nephew Milton Sirotta to think of a name to give to a large number. Mr. Kasner then said that a “googol” was the number you would write as a one followed by 100 zeroes. The official naming scheme that has been around much longer would call this number ten duotrigintillion. Kasner just thought that ‘googol’ was cool and it caught on. But a googol isn’t even as large as a centillion. Mr. Kasner then used that new name to give a name to a number that is much larger, the “googolplex”. This is written as a one followed by a googol of zeroes. How big is that? A googol is so large that it is much larger than the current estimates for the total number of atoms in the whole universe or even a trillion universes; so even if you wrote a zero on every atom in the universe, you couldn’t even write the number googolplex, much less count it! Allow me to make another comparison: the number of seconds since the beginning of our solar system is only about 1 followed by 18 zeroes.

The word ‘infinity’ doesn’t represent an actual number that is bigger than all others, as some might think. Infinity just means “without end”, and is a way of describing something that never comes to an end. There are infinitely many numbers, because there is no last number (Dr. Math, para. 1-6). When talking about large numbers in every day life, many people throw around the terms million, billion, or trillion. How much is a million, really? What about a billion or trillion? What’s the difference between a million, billion, and trillion? Here are some examples from tysknews.com (2007) that puts these numbers in perspective. A million seconds is about 11.5 days. A billion seconds is 31 years. A trillion seconds is 31,688 years. A million minutes ago was 1 year, 329 days, 10 hours and 40 minutes ago. A billion minutes ago was just after the time of Christ. A million hours ago was in 1885. A billion hours ago man had not yet walked on earth. Trillion=1,000,000,000,000. The United States has not existed for a trillion seconds. Indeed a trillion seconds is over 31,000 years, but our national debt is estimated to be about 8 trillion dollars. To visualize a million, look closely at a strand of hair; if you pile a million of those on top of each other it would reach up to a seven story building (Peterson, 2003, para.6). In the book *How Much is a Million*? by David Schwartz, some of the pages are filled with tiny stars, 14,364 per page. It would take about (69.6) 70 of those pages to reach a million stars- amazing!

Let's talk money. How much is a million dollars? The largest U.S. bill in circulation is the hundred dollar bill, and it takes 10,000 of those to make one million dollars. Ten thousand bills! Tim, from fatwallet.com (2007), did a simulation of a million dollars. Real bills are 6.125" wide and 2.625" tall and one packet of them is exactly one centimeter thick (1 inch=2.54 cm, so 1 cm= .3937 inches). The piles had a volume of approximately 633 cubic inches (6.125 X 2.625 x .397). Tim bought six 500-sheet reams of paper and each sheet, when cut, would yield four bills. He then made a hundred one-centimeter paper piles, each centimeter represented $10,000. Tim wrapped the 'fake' money and had one hundred packets of hundred hundreds. The pile was about the size of a 15" television. It weighed almost 20 pounds. That's a lot of money! Think of having $5000, that's imaginable. Now, if you gave $5000 to each of your 200 closest friends, you just gave away a million dollars. That's a pretty concrete example. Bill Gates is worth roughly 40 billion dollars. If he puts his money in the bank at 5% interest, how much does he earn every day? Well, about 2 billion per year, so (roughly) 5 million dollars per day, or (roughly) 200,000 dollars per hour, or (roughly) 4000 dollars per minute, or about 63 dollars per second. That's amazing!

The solar system contains very large numbers. Now that I know how big a million really is, the distances from planets to the sun are unbelievable to me. This table shows the distances between planets (NTTI, 2000):

Planet	Mean Distance from Sun (millions of miles)	Mean Distance in AU	Mean Diameter (miles)
Mercury	36.0	0.39	3,031
Venus	67.1	0.72	7,521
Earth	92.9	1.00	7,926
Mars	141.5	1.52	4,221
Jupiter	483.4	5.20	88,734
Saturn	886.7	9.54	74,566
Uranus	1,782.7	19.14	31,566
Neptune	2,794.3	30.06	30,199
Pluto	3,666.1	39.53	1,450

It's very hard to understand just how large the solar system is. Scientists tell us that the largest number our minds can really comprehend, or grasp, is about a hundred thousand (100,000) (NTTI, 2000). Earth's average distance from the Sun is 93 million miles (by definition, that's one AU (astronomical unit)). If you got on a jet and flew at 600 miles per hour from the Sun to the Earth, it would take 18 years. If you could drive the same distance, it would take 152 years. The distance to fly from the Sun to the Earth can be computed as follows: Since distance= rate x time, and we know the distance is about 93 million miles and the rate is 600 miles per hour;

93,000,000=600t (divide by 600)

155,000=t, so it would take one hundred fifty-five thousand hours.

Since there are 24 hours in a day, 155000/24= approx. 6458.33 days.

Since there are approx. 365.25 days in 1 year, 6458.33/365.25=17.68, so that's why it takes approximately 18 years to travel from the Sun to Earth. Does this not amaze you?

Time is another thing that I can usually grasp, along with money. It takes a few seconds to type a sentence. In one day, there are 86,400 seconds. One month is 2,629,743.83 seconds. There are 31,556,926 seconds in a year. I can compute this as follows: 60 seconds in one minute, and 60 minutes in 1 hour, so 60 x 60= 3600 seconds in 1 hour. Since there are 24 hours in a day, multiply 3600 by 24= 86400 seconds in 1 day. There are approximately 365.25 days in 1 year, so 86,400 x 365.25= 31,557,600 seconds in one year. I am 30, so I have been alive roughly 946,707,779 seconds. I plan to live until I'm about 90, so that means I would have lived a total of about 2,840,123,340 seconds. This is two billion, eight hundred forty million, one hundred twenty-three thousand, three hundred forty seconds!

An anonymous person asked Dr. Math (Nov., 1996) what the number 9,600,000,000,000,000,000,000 meant. Dr. Daniel replied by saying, "That's a really big number!" First, he gave it a name…

We call 1,000,000 a million,

1,000,000,000 a billion,

1,000,000,000,000 a trillion,

1,000,000,000,000,000 a quadrillion,

1,000,000,000,000,000,000 a quintillion, and

1,000,000,000,000,000,000,000 a sextillion.

The number 9,600,000,000,000,000,000,000 is 9 sextillion, 600 quintillion. This is approximately 10 sextillion. I need to think of this in terms I can maybe imagine. Right now, there are about 5 billion (5,000,000,000) people on earth. If we divided 10 sextillion by 5 billion, we'd have 2 trillion as the quotient. So suppose there were something that each person had 2 trillion of. Then all people together would have roughly 10 sextillion of them. But what do we all have roughly 2 trillion of? We all have about that many nerve cells in our brains. So, the really big number 9,600,000,000,000,000,000,000 is something pretty close to the total number of nerve cells in people on earth. Wow!

Here's another big number: 8,200,000,000,000. The word for this is 8 trillion, 200 billion. If I divide this number by the number of people in the US (according to the U.S. Bureau of Census, the resident population of the US, projected on 8-6-07, is 302,537,371), then the result is roughly 27,000. That's about how much money the average person ages 18-25 makes in a year in the US. So, 8.2 trillion is then roughly the total amount of money from everyone in America's annual paycheck.

The number 1.2 million (1,200,000) sounds like a big number, right? This is the number of students who didn't graduate from U.S. high schools in 2006. What about 325 billion (325,000,000,000)? This is the amount of money in lost wages and taxes those 1.2 million students who didn't graduate cost the nation (all4ed.com, para. 5). Six million students throughout America are currently at risk of dropping out of high school. American businesses currently spend more than $60 billion each year on training, much of that on remedial reading, writing, and mathematics (Alliance for Excellent Education factsheet, n.d.). Current estimates put the number of youth who are not in school, do not hold a diploma, and not working at 3.8 million (urban.org, para. 12.). These numbers are huge, and they make my job as a teacher so much more difficult!

Let me throw out the number 17,576,000. In Lincoln, license plates contain 3 letters followed by 3 digits. If all letters and digits could be used and you allowed repeats, there would be 17,576,000 different plates available. Since there are 26 letters and 10 digits, this is computed as 26*26*26*10*10*10. I am assuming that they can repeat and I'm not taking into consideration the 'taboo' words that could be made by some combinations (ex. DAM). There may also be some letters that cannot be used. Think of something you use on a regular basis, say for example, the telephone. To find the total telephone numbers in the United States you would compute as follows: Area codes are three digits where the middle digit is either a 0 or 1, the first digit can be anything but 0 or 1, and no three exchange digits can be the same as the preceding area code. In the area code, there are 8 possible numbers to choose from (2-9) for the first digit, two choices (0 and 1) for the second digit, and 10 choices (0-9) for the third. To find the number of area codes, multiply 8x2x10, the result is 160. Each area code is followed by a three digit number that cannot equal the area code. Three digits from 0-9 gives a result of 10^3 or 1000 possible numbers and then subtract one for the number equal to the area code, leaving 999 possible combinations. To follow each of the 160 possible area codes, I multiply 160 by 999 and get 159,840. The last four numbers have no restrictions, so I can take 10^4 which gives me 10000 possibilities. 160*999*10000 will result in different numbers since each of the 159,840 combinations of area codes and three digit numbers will be followed by 10000 more possible numbers. The total number of telephone numbers in the United States is 1,598,400,000.

A cell is the structural and functional unit of all known living organisms, and is sometimes called the building block of life. Humans have an estimated 100 trillion, or 10^{14}, cells (wikipedia.org/wiki/cells, para 1). The human body contains a large number of bacteria. It is estimated that 500 to 1000 different species of bacteria live in the human body. Bacteria are much smaller than human cells, and there are about ten times as many bacteria as human cells in the body, 1000 trillion (10^{15}) versus 100 trillion (10^{14}) (wikipedia.org/wiki/bacteria, para. 2). Those are big numbers!

Let's talk ice cream! I love Cold Stone, but there are so many choices. They have 13 flavors of ice cream, 12 candy mix-ins, 6 types of fruit, 5 syrup choices, 3 dish sizes, 3 types of cones, you can have a sundae, shake, or cake. So when it takes me forever to decide, that's because there are at least 13*12*6*5*3*3*3= 126,360 different combinations to choose from!

What about the penny problem? Would you rather be given one million dollars today or one penny the first day, double that penny the next day, then double the previous day's pennies and so on for a month? After 30 days on the penny-doubling plan, you would have received a total of $2^{30} - 1 = 1{,}073{,}741{,}823$ pennies. That's more than a billion pennies! If I divide this number by 100 (100 pennies in a dollar), that would be

$10,737,418.23. That's almost 11 million dollars- wow! Here is how I arrived at the total number of pennies:

Day	# of pennies given	Total # of pennies
1	1	1
2	1x2=2	1+2=3
3	2x2=4	1+2+4=7
4	2x4=8	1+2+4+8=15
5	2x8=16	1+2+4+8+16=31

The series whose sum gives the total number of pennies follows a regular pattern: each new term added to it is a power of two. This is an example of a geometric series. In a geometric series there is a constant ratio between consecutive terms. The constant ratio between the terms of the geometric series that gives the number of pennies added from one day to the next in this problem is 2. Knowing this ratio, I can use the fact that the sum of a geometric series with n terms (denoted S_n) whose ratio is r is:

$$S_n = (\text{first term})(1-r^n)/(1-r).$$

For the penny problem the first term is 1 and the ratio is 2, so the sum after n days is given by:

$$S_n = 1(1-2^n)/(1-2) = -(1-2^n) = \mathbf{2^n - 1}.$$

This penny problem is very similar to the following rice problem: A man asked for 1 grain of rice for the first square on a chess board, 2 grains for the second, 4 for the third, 8 for the fourth, and so on for each of the 64 squares on the board. Unbelievably, the number of grains of rice on the chess board is 18,446,744,073,709,551,615. This number is eighteen quintillion, four hundred forty-six quadrillion, seven hundred forty-four trillion, seventy-three billion, seven hundred nine million, five hundred fifty-one thousand, six hundred fifteen. To find this, I again used the formula 2^n-1 because the first square has 2^0= 1 grain, the second has 2^1= 2, so that the nth square has twice as many grains of rice as the previous square. The sum of all grains of rice on the chess board is 1+2+4+8+…+2^{63}. Since this is a geometric sequence with a common ratio of 2, the sum is:

$$S= (2^{64}-1)/ (2-1)= 2^{64}-1$$

To gain an understanding of the volume of this much rice, let me assume that a single grain of rice has dimensions 2mm x 2mm x 5mm and that the average swimming pool is a rectangular prism of dimensions 15ftx30ftx6ft. We can compute the approximate number of swimming pools needed to hold all the rice as follows:

$$20mm^3/76.455m^3 \times 1.845 \times 1019= (20 \times 1.845 \times 10^{19})/(76.45 \times 10^9)= 4.825 \times 10^9$$

Thus it would take about 4 billion 8 million swimming pools to contain all of those grains of rice.

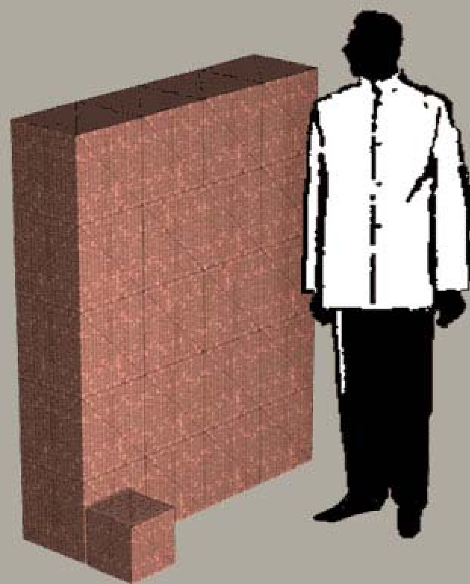
This next number is too big for me to grasp. How many ways are there to arrange 30 books on a bookshelf? The answer is 30!= 265,252,859,812,191,058,636,308,480,000,000. I've never encountered a number that is 33 digits long! 10^{33} is a decillion. There are about one fourth of a decillion ways to arrange 30 books on a bookshelf (Dr. Math, 2007)!

In conversation I often say million, billion, trillion to represent that I mean 'a lot'. For example, a phrase I have used many times is "I have a million things to do today".

Considering that it takes almost 12 days to count to a million saying one number per second, this is a bit of an exaggeration. The next time students say they want to make a million dollars, I will have a few examples to bring them back to reality. If the students each saved $50,000 per year, it would take them 20 years to have saved a million dollars.

Visualizing large numbers can be very difficult. People regularly talk about millions of miles, billions of bytes, or trillions of dollars, yet it's still hard to grasp just how much a "billion" really is. The MegaPenny Project (2001) aims to help by taking one small everyday item, the U.S. penny, and using this to answer the question "What would a billion (or a trillion) pennies look like?" Keeping in mind that a penny is about .75 inches in diameter, .0625 inches thick and weighs .1 ounces, look at these simulations by the MegaPenny Project to get an idea of how much a million, a billion, and a trillion really is:

One Million Pennies

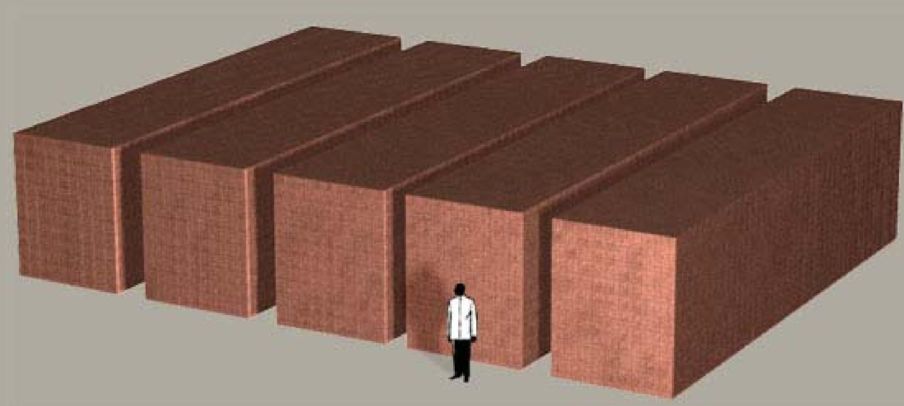


1,003,776
One million, three thousand, seven hundred and seventy-six Pennies
[A wall five by four by one feet thick with a 9-inch cube stepstool]

Say hello to our friend Graham. Now that the pennies have really begun to pile up, he'll be standing in for scale. Graham is about 5 feet, 10 inches tall, and he weighs about 180 pounds, or about 35 times less than the 1 million pennies stacked beside him.

value	**$10,037.76 (Ten thousand, thirty-seven dollars and seventy-six cents)**
width	**Four feet**
height	**Five feet**
thickness	**12 inches, (one foot)**
weight	**6273.6 pounds (3.14 tons)**
height stacked	**5,228 feet (0.99 Miles)**
area (laid flat)	**3,921 square feet**

One Billion Pennies



1,000,018,176

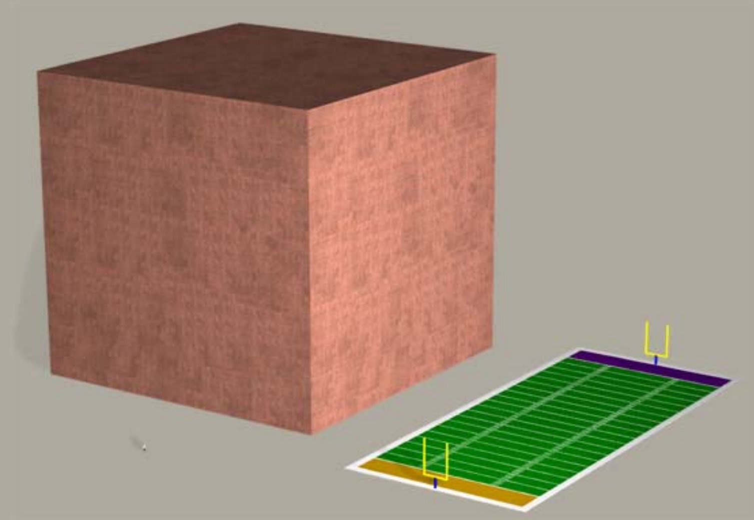
One billion, eighteen thousand, one hundred and seventy-six Pennies
[Five school buses.]

Each of these blocks represents one 9x11x41 foot school bus - as seen below. If you were to stack all these pennies in a single pile, one atop the other, the stack would reach nearly one
thousand miles high. For comparison, note that the Space Shuttle typically orbits only 225 miles
above the Earth's surface.

Only in North America and the general scientific community is this number (1,000,000,000) called
a "billion". Most European countries call this number either "one thousand million" or,
in some cases, a "milliard".

value	**$10,000,181.76 (Ten million, one hundred eighty-one dollars and seventy-six cents)**
width	**45 feet**
height	**11 feet**
thickness	**41 feet**
total weight	**3,125 tons**
height stacked	**987 Miles**
area (laid flat)	**3,906,321 square feet (89.7 acres)**

One Trillion Pennies



1,000,000,016,640

One trillion, sixteen thousand six hundred and forty Pennies

[One cube measuring 273 x 273 x 273 feet]

Notice our friend Graham, still barely visible as a speck at lower left.

value	**$10,000,000,166.40 (Ten billion, one hundred and sixty-six dollars and forty cents)**
width	**273 feet**
height	**273 feet**
thickness	**273 feet**
total weight	**3,125,000 tons**
height stacked	**986,426 Miles**
area (laid flat)	**89,675.2 acres**

So, why study large numbers? In the 70's, the world's population was over 4 billion people. On October 12, 1999, the world population reached the 6 billion mark. The United Nations predicts that by 2025, there will be 8.5 billion people in the world. This exponential growth puts a massive global strain on our capability to provide resources and services to a starving world. On May 22, 2006 the United State's national debt totaled $8.3 trillion. The current national debt is $7.9 trillion. In Third World countries, 50,000 people die every day. People in Niger are dying of starvation in feeding centers. There are 3.6 million people facing food shortages (askquestions.org, 2007). Close to home, according to Risk Management Solutions, Hurricane Katrina's losses exceeded $100 billion. In 1972, the US spent $24 million each hour on the Vietnam War. It is important for us to understand our history in order to choose the best path for the future, and comprehending large numbers plays a significant role in that understanding.

Resources

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