1975

Traveling Waves

TRAVELING WAVES

INTRODUCTION

"For many people - perhaps for most - the word 'wave' conjures up a picture of an ocean, with the rollers sweeping onto the beach from the open sea. If you have stood and watched this phenomenon, you may have felt that for all its grandeur it contains an element of anticlimax. You see the crests racing in, you get a sense of the massive assault by the water on the land - and indeed the waves can do great damage, which means that they are carriers of energy - but yet when it is all over, when the wave has reared and broken, the water is scarcely any further up the beach than it was before. That onward rush was not to any significant extent a bodily motion of the water. The long waves of the open sea (known as the swell) travel fast and far. Waves reaching the California coast have been traced to origins in South Pacific storms more than 7000 miles away, and have traversed this distance at a speed of 40 mph or more. Clearly the sea itself has not traveled in this spectacular way; it has simply played the role of the agent by which a certain effect is transmitted. And here we see the essential feature of what is called wave motion. A condition of some kind is transmitted from one place to another by means of a medium, but the medium itself is not transported. A local effect can be linked to a distant cause, and there is a time lag between cause and effect that depends on the properties of the medium and finds its expression in the velocity of the wave. All material media - solids, liquids, and gases - can carry energy and information by means of waves....

"Although waves on water are the most familiar type of wave, they are also among the most complicated to analyze in terms of underlying physical processes. We shall, therefore, not have very much to say about them. Instead, we shall turn to our old standby - the stretched string - about which we have learned a good deal that can now be applied to the present discussion."*

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Provide a mathematical and pictorial description of a particle undergoing sinusoidal motion (needed for Objectives 1 through 4 of this module)	Simple Harmonic Motion Module
*Find the partial derivative of a simple function of two variables (needed for Objectives 1 through 4 of this module)	Partial Derivatives Review

*A. P. French, Vibrations and Waves (Norton, New York, 1971), pp. 201, 202.

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Description of wave - Describe and interpret descriptions of traveling transverse waves on a string, using both pictorial and mathematical formulations.
2. Wave velocity - Relate the wave speed to the physical properties of string.
3. Superposition - Apply the superposition principle to (a) reflections at a boundary, (b) waves moving in the same direction, and (c) waves moving in opposite directions (standing waves, resonance).
4. Power - Discuss the dependence of transmission of power in a wave in a string on the physical variables.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

This module is limited to transverse waves on a string. Your text's treatment of waves on a string is combined with other wave phenomena, thus to satisfy the stated objectives in this module you will have to skip around a bit, reading some material that does not apply specifically to waves on a string. That should not hurt, and you may pick up some related ideas that are interesting.

Read the following material in Bueche: Appendix 9, Section 29.4 to 29.6, 31.1 to 31.3, and 34.1 to 34.3. Work at least Problems A through K before attempting the Practice Test.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	
1	Appendix 9, Sec. 29.4	A		F, G	Chap. 29, Probs. 1, 3
2	Sec. 29.5	B	Illus.[a] 29.2	H	
3	Secs. 31.1 to 31.3, 34.1 to 34.3	C, D	Illus. 34.1	I, J	Chap. 31, Quest. 2, Probs. 7, 8; Chap. 34, Probs. 1, 3
4	Sec. 29.6	E	Illus. 29.3	K	Chap. 29, Prob. 4

[a]Illus. = Illustration(s).

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4	Sec. 29.6	E	Illus. 29.3	K	Chap. 29, Prob. 4

[a]Illus. = Illustration(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read Chapters 21 and 22. Since this module is limited to waves on a string, you can skip Sections 21-4, 21-5, 21-6, 22-6, 22-7, 22-8, and 22-9 and still achieve the objectives of this module. Work at least Problems A through K before attempting the Practice Test.

For Objective 4, look at the situation pictured in Figure 21-5. Not only is the transverse force F providing a transverse impulse, but it is also doing work on the string. This work goes into increasing the kinetic energy of the string, since more and more of the string is moving. We also see that energy is being transmitted along the string since to the left of the point P the string has kinetic energy and to the right of that point it has none, and point P is moving along the string with the wave speed c.

We can compute the instantaneous power furnished by F in the following way. The instantaneous power is $\vec{F} \cdot \vec{v}$. We see from Figure 21-5 that the negative of the slope of the string is given by F/S, so we have a value for F in terms of the slope of the string and of S.

Power = S × (negative of the slope of the string) × v.

Suppose we have a force that is producing a sinusoidal wave of the form in Eq. (21-3), the force being applied at x = 0. Now,

$$v = \partial y/\partial t = \omega Y \cos(\omega t - kx),$$

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems
1	Secs. 21-1, 21-2	A	Ex.[a] 2 (Sec. 21-3)	F, G	21-3, 21-7
2	Sec. 21-3	B	Ex. 1 (Sec. 21-3)	H	21-1
3	Secs. 22-1, 22-2, 22-3, 22-5	C, D		I, J	22-3
4	This module	E		K	

[a]Ex. = Example(s).

and the slope

$$\partial y/\partial x = -kY \cos(\omega t - kx),$$

so that the equation for the power gives, at $x = 0$,

$$\text{Power} = k\omega Y^2 S \cos^2 \omega t.$$

Using the facts that $c^2 = S/\mu$, $c = \omega/k$, and the fact that the time-average value of $\cos^2 \omega t = 1/2$ (which you can see by noting that $\cos^2 \omega t$ spends equal amounts of time at equal distances above and below 1/2), we find

$$\text{Power}_{av} = (1/2)\omega^2 Y^2 \rho c.$$

This is the time-average power that is put into the wave and the power that the wave carries off to the right. Note that the power is proportional to the square of the angular frequency ω, the square of the amplitude Y (reminiscent of the energy in a simple harmonic oscillator), and is directly proportional to the velocity of the wave and to the mass of the string.

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Read Chapter 16 through Section 16-18. Note the following points: (a) In Eq. (16-2a), note that F(x - ct) stands for "function of the quantity x - ct." Don't confuse this F with the F_t that represents the tension in the string. (b) In Figure 16-4, and in most of the figures following, the displacement of the string away from its equilibrium line has been greatly exaggerated. The superposition principle for waves on a string will only hold for waveforms with very small slopes. Work at least Problems A through K before attempting the Practice Test.

WEIDNER AND SELLS

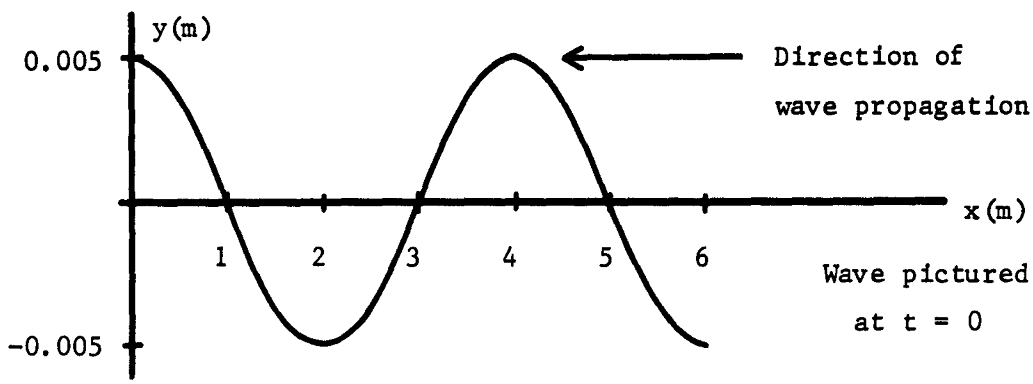
Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text (Ex.[a])	Study Guide	
1	Secs. 16-1, 16-4	A	16-2	F, G	16-3, 16-9, 16-10
2	Sec. 16-1	B	16-1, 16-2	H	16-1, 16-8
3	Secs. 16-3, 16-6, 16-7, 16-8	C, D	16-3	I, J	16-22
4	Sec. 16-5	E		K	16-11, 16-22, 16-14

[a]Ex. = Example(s).

PROBLEM SET WITH SOLUTIONS

A(1). A "snapshot" of a traveling sinusoidal wave at t = 0 traveling to the left is shown in Figure 1 along with some other information. (Note that the vertical scale is exaggerated.) At the position x = 0, the wave reaches a positive maximum 30 times each second.
(a) Determine the frequency, wavelength, wave speed, angular frequency, period, propagation constant (or wave number) and amplitude of this traveling wave.
(b) Sketch the "snapshot" of the wave that you would obtain at t = 1/120 s.
(c) Write the mathematical expression that represents the transverse displacement as a function of x and t.
(d) Determine an expression for the displacement of the point P that moves with the string at x = 2.50 m, and compute the velocity of P at t = 1/60 s.

Figure 1



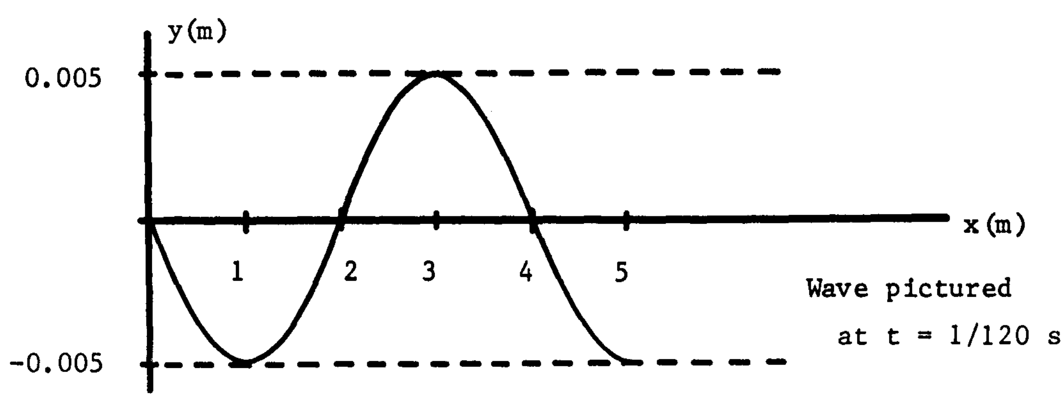
Solution

(a) From the text of the problem, the frequency = 30.0 Hz. Thus angular frequency = $2\pi f$ = 188 s^{-1} and the period = 1/f = 1/30.0 s. From the figure we see that the wavelength λ = 4.0 m. Thus the wave speed = frequency × wavelength = 120 m/s. We also know that

wave speed = (angular frequency)/(propagation constant) = ω/k, $k = 2\pi/\lambda = 1.57\ m^{-1}$.

From the figure we see that the amplitude of the wave is 0.0050 m.
(b) We could work part (c) first and then plug in t = (1/120) s to see what we get, but let's try a graphical approach. We know that since the wave is moving to the left at 120 m/s, in 1/120 s it has moved 1.00 m to the left. We can immediately sketch the wave shifted 1.00 m to the left as in Figure 2.

Figure 2



(c) We recall that a sinusoidal wave traveling in the negative x direction is written in the following form (or one equivalent to it):

$$y = A \sin(kx + \omega t + \phi).$$

How do we recall this without resorting to rote memory? Here's how: The amplitude factor A is the number (whose dimension is a length for this kind of wave) that multiplies a sinusoidal function (which oscillates between +1 and -1). What's left is the argument of the sine function - the phase. It has to be dimensionless (radians) and has to depend independently on x and t. Therefore we multiply x by k and t by ω. To get a wave moving in the negative x direction we add: $kx + \omega t$. This guarantees that the phase (corresponding to, say, the maximum of the sine curve) remains constant as x decreases while t increases. (Here you should check to see that you can argue that $kx - \omega t$ is what you want for a wave traveling in the +x direction.) Finally, we need a phase constant ϕ to fix up the sine curve to agree with the picture at t = 0. If we write $y = A \sin(kx + \omega t + \phi)$, how do we determine ϕ? Well, if $\phi = 0$ we don't get the right picture at t = 0, thus we have to give ϕ a nonzero value. If we let ϕ go negative, this has the effect of shifting the sine curve to the right at t = 0. Since we want to shift the sine curve a quarter wavelength to the left, we need to make $\phi = +\pi/2$. Now we have it:

$$y = A \sin(kx + \omega t + \pi/2),$$

where A, k, and ω have already been determined in part (a). We could write it as $y = A \cos(kx + \omega t)$, since $\sin(\theta + \pi/2) = \cos \theta$.

(d) Before putting any numbers into this part, let's get symbolic expressions first. Use x_0 for the x coordinate of P. Then the transverse displacement of P is given by the expression for the wave with x_0 plugged in for x:

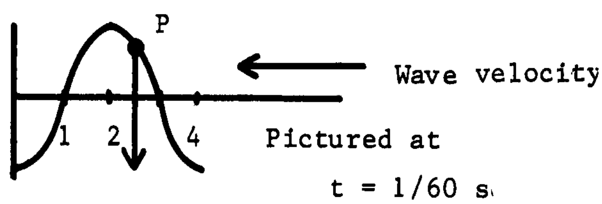
$$y = A \sin(kx_0 + \omega t + \pi/2).$$

This is the expression for the displacement of P. Note that the only variable is the time and that P is executing simple harmonic motion. The velocity of P is given by

$$\partial y_P/\partial t = \omega A \cos(kx_0 + \omega t + \pi/2).$$

Plugging in all the other numbers and t = (1/60) s we get -0.666 m/s. (Take care that you look up the cosine for an argument in radians.) Note that we can check the sign by looking at a sketch of the wave for which the wave has been shifted another quarter wavelength to the left beyond the situation in part (b): Figure 3. In this figure we see that P is moving in the negative y direction, in agreement with our calculation.

Figure 3



B(2). A string with linear density 15.0 g/m propagates waves at a speed of 20.0 m/s. The string is driven transversely by an oscillating arm with an angular frequency 20.0 s^{-1} and an amplitude 0.0300 m.
(a) Determine the tension in the string.
(b) Determine the maximum velocity of a point on the string 2.00 m from the driver.

Solution

(a) We know the wave speed from the derivation in the text:

$$v = \left(\frac{\text{tension in string}}{\text{mass/unit length of string}}\right)^{1/2} = \left(\frac{F}{\rho}\right)^{1/2}$$

Before proceeding, we can check this formula to see it it is dimensionally correct (If it isn't, we have the wrong formula!):

$$[v] = [L/T]$$

$$\left(\frac{F}{\rho}\right)^{1/2} = \left[\left(\frac{ML}{T^2}\frac{L}{M}\right)^{1/2}\right] = \frac{L}{T}.$$

The formula is dimensionally correct. Thus we find $F = \rho v^2 = 6.0$ N.

(b) Since the maximum velocity of any point on the string is the same as any other (although the maximum velocity occurs at different times at different points) the 2.00 m is not relevant. We can, for example, consider the point at which the driver is attached to the string. You should get $(\partial y/\partial t)_{max} = 0.60$ m/s.

C(3). A pulse moves along a string with wave speed v as shown in Figure 4. The right end of the string is fixed to a wall. The situation is shown at t = 0. Sketch the vertical displacement of the point P as a function of time.

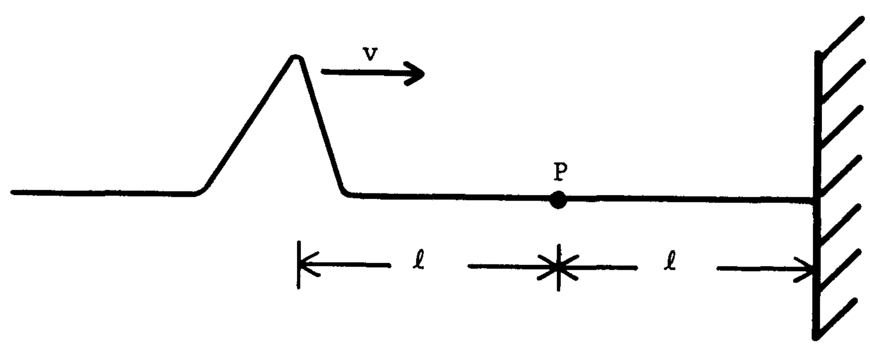


Figure 4

Solution

The boundary condition at the wall is that the string has no transverse displacement (i.e., it is fixed). We can guarantee this boundary condition by imagining that the string does not stop at the wall but extends off to the right where a symmetrically located wave is propagating to the left. See Figure 5. This new wave is shaped just like the other wave except that it is inverted and is reflected front to back. Now, the principle of superposition states that the displacement of the string is just the algebraic sum of the displacements in the two waves. The new wave has been set up so that, at the former position of the wall, the displacement of the string is always zero, and as far as the left half of the string is concerned, it is just as if there were a wall at that position still. The motion of point P thus is first affected by the passage of the wave pulse going to the right and, later, by the reflected wave going to the left.

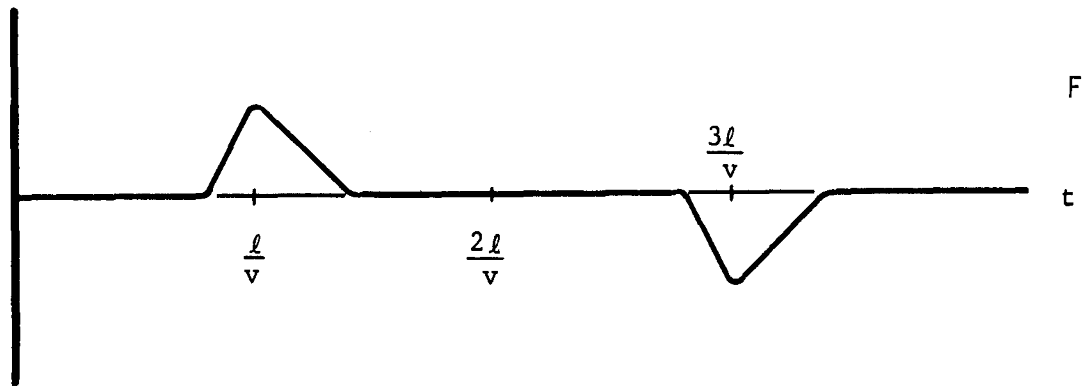


Figure 5

D(3). A string vibrates according to the equation

$$y = (0.0040\ \text{m}) \sin[(25.0\ \text{m}^{-1})x] \cos[(400\ \text{s}^{-1})t].$$

(a) What are the amplitude and velocity of the component waves whose superposition gives rise to this vibration?
(b) Determine the distance between nodes.
(c) Sketch the shape of the string at several different times to provide a "motion picture" of the motion of the string.

Solution

We hope that you recognized this to be the form of a standing wave, produced by two traveling waves of equal amplitude and frequency traveling in opposite directions. You can consult your textbook for a mathematical plug for the answer to part (a), but it is important that you also be able to reason out the answer pictorially. Let us answer part (c) first to get Figure 6. Before doing arithmetic, write the formula you were given in algebraic quantities:

$$y = A \sin kx \cos \omega t.$$

By inspection of the formula and Figure 6 we see that both the standing and the traveling waves take a distance $2\pi/k$ to go through one cycle along x, thus $\lambda = 2\pi/k$. It is also easy to see that the period of the standing wave is the same as that of the traveling waves, thus $f = \omega/2\pi$. The wave speeds are thus $v = \lambda f = \omega/k$, and for this example

$$v = (400\ s^{-1})/(25.0\ m^{-1}) = 16.0\ m/s.$$

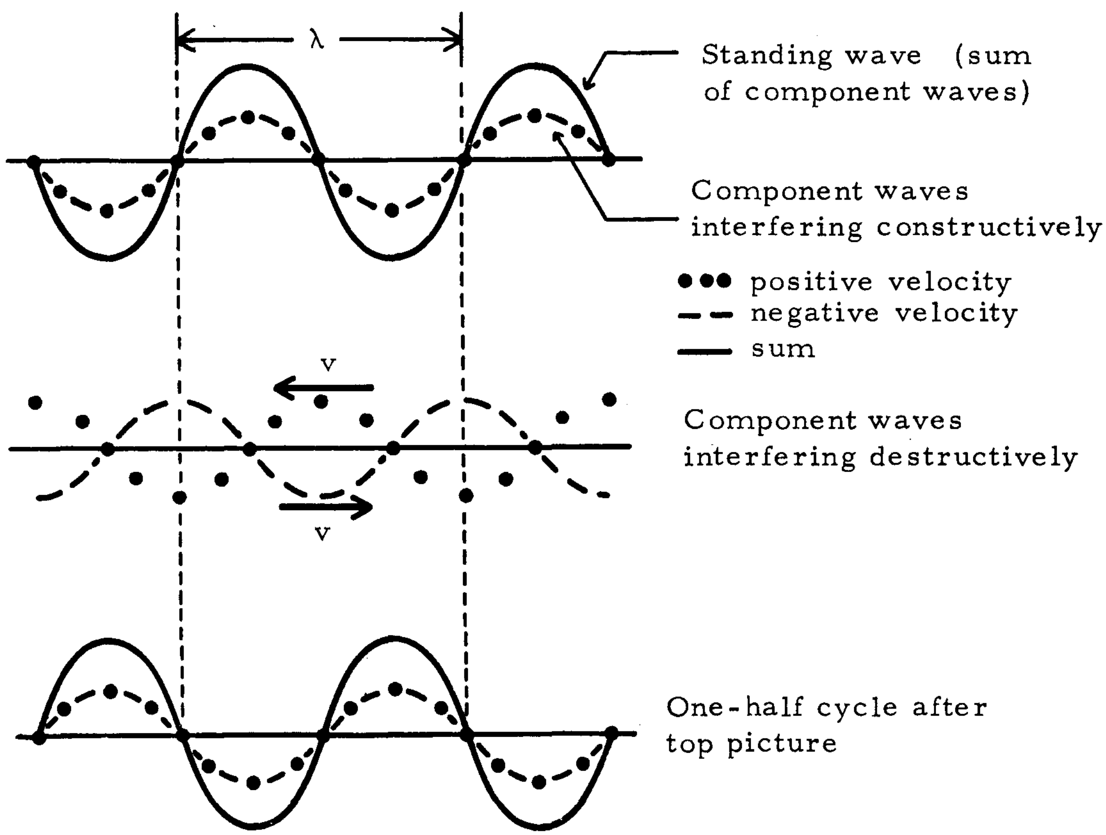
The amplitude of both traveling waves is one-half of the maximum amplitude of the standing wave, as we see from the figure, therefore the

amplitude of traveling waves = $A/2 = 0.0020$ m.

Again, as we can see from the figure, the distance between nodes equals

$$\frac{\lambda}{2} = \frac{1}{2}\frac{2\pi}{k} = \frac{\pi}{k} = \frac{\pi}{25.0\ m^{-1}} = 0.126\ m.$$

Figure 6



E(4). Give a physical argument (give an analogy to some other, possibly simpler, physical situation) to justify that the power carried in a sinusoidal traveling wave depends on the square of the frequency. Does your analogy also provide an explanation for why the energy transmission also depends on the other variables the way it does, or do you have to dredge up something else? (After answering this question, you should not have to claim that you have to "memorize" the formula for power transmitted by a wave on a string.)

Solution

Suppose you start to wiggle, with a transverse motion, the end of a string that is initially at rest. What does the string look like at successive instants? Consider a small segment of the string. It is moving up and down with simple harmonic motion, pulling the next segment up or down and thus doing work on it. Power is work over time, and thus we find that the power for a sinusoidal wave is similar to that for simple harmonic motion, with the square of the frequency (which arose from the derivative) taking the place of angular velocity. How does the kinetic energy of the piece of string depend on the amplitude and frequency? How does it depend on the mass per unit length of the string?

Problems

F(1). A sinusoidal traveling wave has the form

$$y = \underset{\text{m}}{0.0300}[\sin(\underset{\text{m}^{-1}}{0.50}\underset{\text{m}}{x} - \underset{\text{s}^{-1}}{20.0\pi}\underset{\text{s}}{t} - \underset{\text{rad}}{\pi/4})],$$

where the dimensions of each quantity are shown below the equation.
(a) Determine the frequency, wavelength, wave speed, angular frequency, period, amplitude, and direction of travel of this wave.
(b) Sketch "snapshots" of this wave at $t = 0$ and at $t = 0.0250$ s.

G(1). A pulse travels along a string with a speed of 5.0 m/s. Its shape and direction of motion at $t = 0$ are shown in Figure 7.

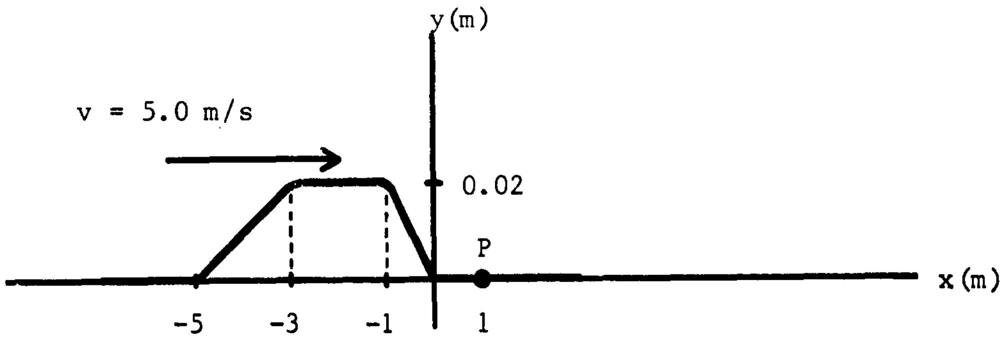


Figure 7

(a) Sketch the pulse at t = 0.80 s.
(b) The point P is fixed to the string at x = +1.00 m. Sketch the displacement and velocity of P versus time. Label the axes of your sketch carefully, showing the scales on each of the axes.

H(2). A rope under 80 N tension carries a sinusoidal wave of wave constant (wave number) $k = 4.0\ m^{-1}$ at a wave speed of 25.0 m/s. Determine the frequency of the waves and the mass per unit length of the rope.

I(3). Show that the superposition of two sinusoidal waves of equal amplitude, frequency, and wave speed that are traveling in the same direction gives a sinusoidal traveling wave regardless of the phase difference between them. Determine the amplitude of this traveling wave in terms of the amplitude of the two waves and the phase difference between them.

J(3). A string 2.00 m long is attached to the prong of an electrically driven tuning fork that vibrates perpendicularly to the length of the string at a frequency of 80 Hz. The mass of the string is 12.0 g. Determine the tension that must be applied to the string to make it resonate in three loops.

K(4). Two long wires of linear densities 2.00×10^{-2} kg/m and 5.0×10^{-2} kg/m, respectively, are joined at one end. The two free ends are pulled apart with a tension of 10.0 N. At the point at which the two wires are joined, an oscillating arm starts to shake the wires transversely with a sinusoidal oscillation of 25.0 Hz, 0.00300 m in amplitude. Determine the energy supplied to the wires after 3.00 s.

Solutions

F(1). (a) The equation is of the form of a wave moving in the +x direction:

$$y = A \sin(kx - \omega t - \phi),$$

from which we immediately note the substitutions

$k = 0.50\ m^{-1}$, angular frequency $\omega = 20\pi\ s^{-1} = 62.8\ s^{-1}$, amplitude $A = 0.0300$ m,

from which we can derive the other quantities:

wave speed = ω/k = 126 m/s, frequency = $\omega/2\pi$ = 10.0 Hz,

wavelength = $\dfrac{\text{wave speed}}{\text{frequency}} = = \dfrac{\omega}{k}\dfrac{2\pi}{\omega} = \dfrac{2\pi}{k}$ = 12.6 m, period = $\dfrac{1}{f}$ = 0.100 s.

(b) We note that this is a wave of amplitude 0.0300 m and wavelength 12.6 m. It is shifted at t = 0 from sin kx by the phase shift ϕ. For $\phi = \pi/4$, the wave slope is shifted $(\pi/4)\ (2\pi)^{-1}$ wavelengths to the right, and thus the snapshot looks like Figure 8. At t = 0.0250 s we have

$$y = 0.030 \sin(0.50x - \pi/2 - \pi/4),$$

which is a sine function shifted $(3\pi/4)\ (2\pi)^{-1}$ wavelengths to the right, as in Figure 9.

G(1). (a) See Figure 10.
(b) See Figure 11.

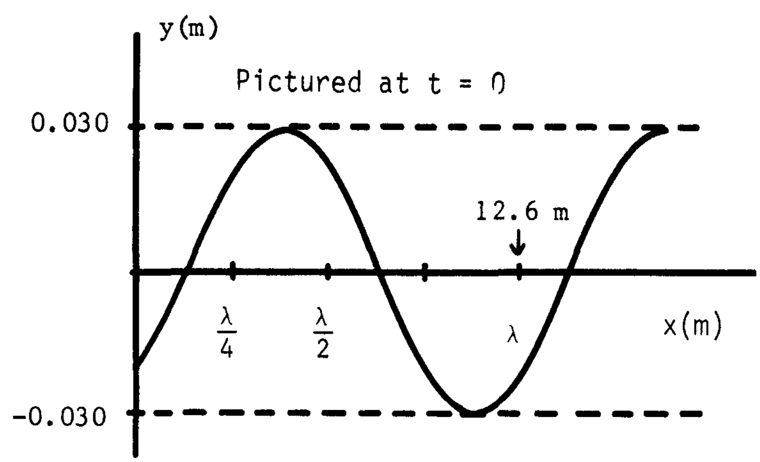
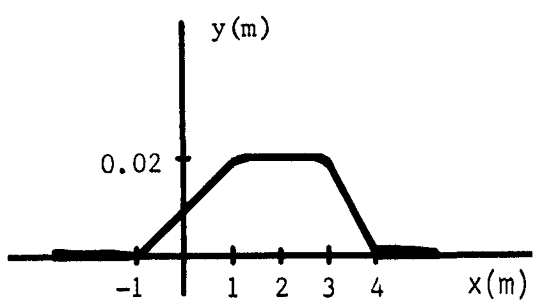


Figure 8

Figure 10



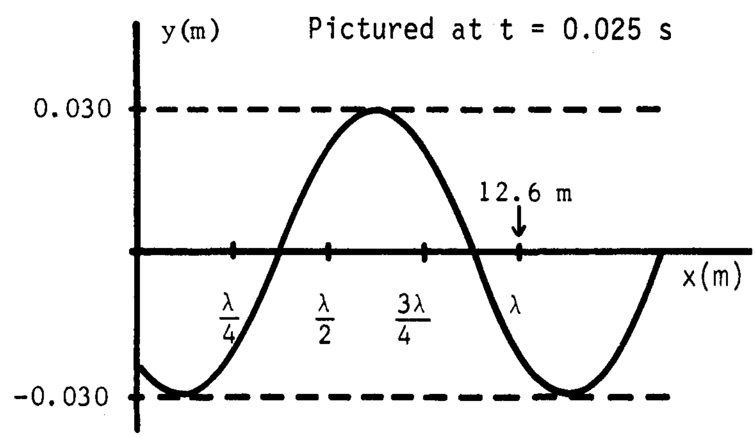


Figure 9

H(2). The frequency is 15.9 Hz, the density 0.128 kg/m.

I(3). This problem gives a little practice in applied trigonometry. Let us write expressions for the two waves:

$$y_1 = A \sin(kx - \omega t), \quad y_2 = A \sin(kx - \omega t - \phi),$$

that have the same amplitude, frequency, wave speed, and direction of propagation. The wave y_2 leads y_1 by a distance ϕ/k, which you should verify for yourself. We wish to calculate $y = y_1 + y_2$ but first we can reexpress y_1 and y_2, using the identity (which you should know)

$$\sin(\alpha + \beta) = (\sin \alpha)(\cos \beta) + (\cos \alpha)(\sin \beta)$$

to write

$$\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin \alpha \cos \beta.$$

We can put $y_1 + y_2$ in this form by letting

$$\alpha + \beta = kx - \omega t, \quad \alpha - \beta = kx - \omega t - \phi,$$

which gives

$$\alpha = kx - \omega t - \phi/2, \quad \beta = \phi/2,$$

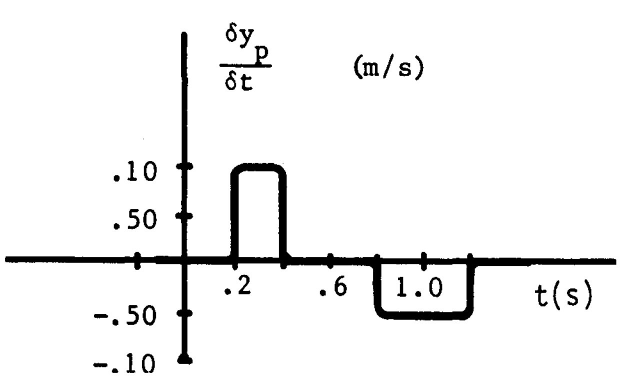
and

$$y = 2A \sin(kx - \omega t - \phi/2) \cos(\phi/2).$$

This is a traveling sinusoidal wave, regardless of the value of ϕ. The amplitude of the resultant wave is $2A \cos(\phi/2)$. If $\phi = 0, 2\pi, 4\pi$, etc., the waves are "in phase," and the amplitude is 2A - an example of constructive interference between the two waves. If $\phi = \pi, 3\pi, 5\pi$, etc., the amplitude is zero - destructive interference.

J(3). 68 N.

K(4). 0.39 J.



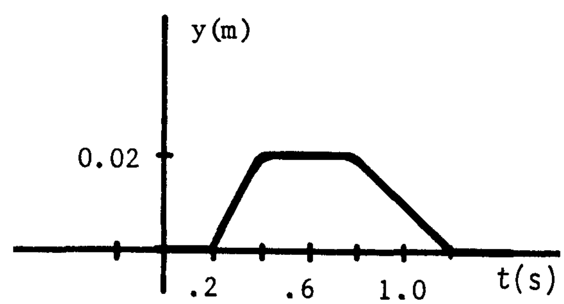
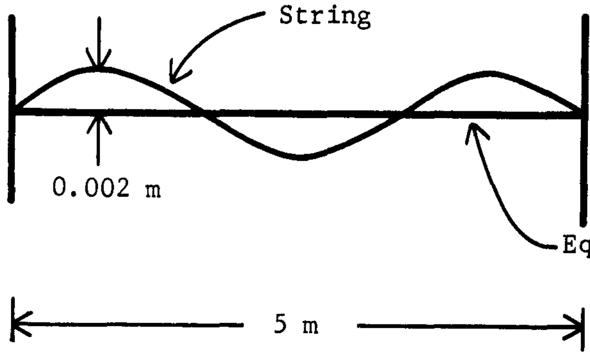


Figure 11

PRACTICE TEST

1. A standing wave on a string is shown at t = 0 in Figure 12. The frequency of its motion is 25.0 Hz, and the displacement of the string is a maximum.
(a) Write down symbolic expressions for the traveling waves whose superposition produces this standing wave, and give the numerical values for the symbols in your expressions.
(b) Determine the tension in the string if its mass per unit length is 2.00 g/m.

Figure 12



2. The motion of a string is given by

$y = A \sin(\omega t - kx)$,

where A = 0.00200 m, $\omega = 65\ s^{-1}$, and $k = 2.00\ m^{-1}$
(a) Determine the maximum speed of a point on the string at x = 4.0 m.
(b) In what direction is power being transmitted along the wave?
(c) If the amplitude and frequency are kept the same, but the tension in the string is increased by a factor of 2, how is the power transmitted by the string affected?

Practice Test Answers

1. (a) $y_1 = A \sin(kx - \omega t)$ wave traveling to right, $y_2 = A \sin(kx + \omega t)$ wave traveling to left, where A = 0.00200 m, $k = 1.89\ m^{-1}$ and $\omega = 157\ s^{-1}$

(b) $\mu = 2.00$ g/m, f = 25.0 Hz, $\lambda = 3.30$ m.
$f = (1/\lambda)\sqrt{T/\mu}$, $f^2 = (1/\lambda^2)(T/\mu)$, $T = f^2\lambda^2\mu = (25.0)^2(3.30)^2(2.00 \times 10^{-3}) = 13.6$ N

2. (a) y = 0.00200 sin(65t - 2.00x), v = dy/dt = (0.00200)(65) cos(65t - 2x),
$v_{x=4} = (0.00200)(65) = 0.130$ m/s.
(b) In the -x direction. (c) Power increases by $\sqrt{2}$.

TRAVELING WAVES   Date ____________

Mastery Test   Form A   pass   recycle

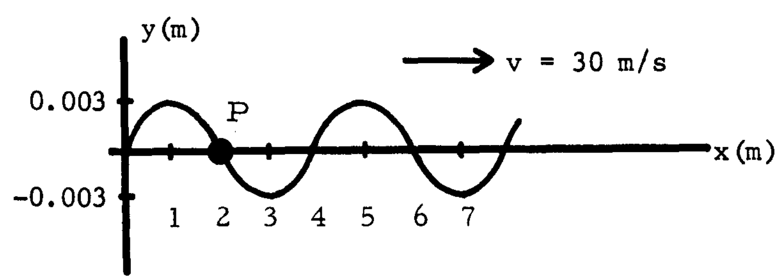
1   2   3   4

Name ____________________   Tutor ____________________

1. A standing wave of 5 loops (or antinodes) is set up on a flexible wire. The wire is 1.20 m long, and its total mass is 10.0 g. The maximum amplitude of oscillation of the wire is 0.50 mm, and its frequency of oscillation is 400 Hz.
(a) Sketch the shape of the wire at several instants, each separated by a quarter cycle from the previous instant.
(b) Determine the tension in the wire.
(d) Let one end of the wire be at x = 0 and the other end at x = 1.20 m. At t = 0 the wire is straight, but its tranverse velocity at x = 0.050 m is in the +y direction. Write the mathematical expression for the displacement y of the wire as a function of the variables x and t.

2. A sinusoidal traveling wave on a string moving to the right with a velocity 30.0 m/s is shown in Figure 1.
(a) Sketch the vertical velocity of the point P, which is at x = 2.00 m, as a function of time, labeling the axes of your sketch carefully.
(b) If the wavelength of this wave is increased by a factor of 3, keeping the amplitude the same (and presuming the tension and mass of the string are unchanged), determine the change in the rate of energy propagation on the string.

Figure 1



TRAVELING WAVES

Date ____________________

Mastery Test Form B

pass recycle

1 2 3 4

Name ______________________________ Tutor ______________________________

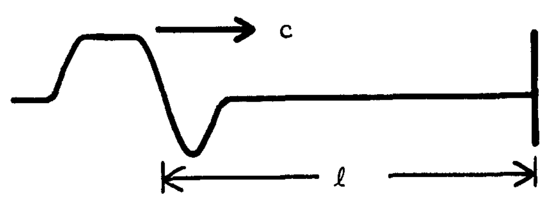
1. A wave propagates along a string whose mass per unit length is 0.0040 m kg/m. The transverse displacement of the wave is given by

$$y = A \sin(kx - \omega t - \phi),$$

where A = 0.00300 m, $k = 6.0\ m^{-1}$, $\omega = 80\ s^{-1}$, and $\phi = \pi/2$.
(a) Determine the tension in the string.
(b) Sketch a "snapshot" of the string at $t = \pi/320$ s, showing the horizontal and vertical scales.
(c) Determine (i) the direction and (ii) the magnitude of the wave velocity.
(d) If the amplitude of the wave is halved, what effect does this have on the power transmitted by the wave? (Give your answer in a complete sentence.)

2. A pulse propagates with speed c down a long string that is fixed to a wall at one end. Figure 1 shows the string at t = 0. Sketch the shape of the string at $t = 3\ell/2c$ showing its location with respect to the wall.

Figure 1



TRAVELING WAVES   Date ____________

Mastery Test   Form C

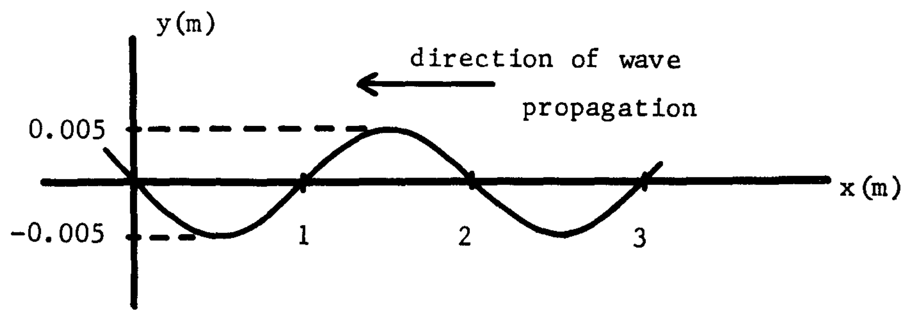
pass   recycle

1   2   3   4

Name ____________________   Tutor ____________________

1. A wave propagating to the left is shown at $t = 0$ in Figure 1. The frequency of the wave is 60 Hz.
   (a) Sketch the wave at $t = 1/30$ s.
   (b) Sketch as a function of time the vertical position of the point on the string at $x = 0$. Show the scale clearly on the axes of your sketch.
   (c) Determine the mass per unit length of the string if its tension is 50 N.
   (d) If the appropriate traveling wave is superposed on the wave in the figure to produce a standing wave, determine the amplitude of the standing wave and determine the distance between nodes.

Figure 1



2. A wave given by

$$y_1 = A \sin(\omega t - kx)$$

transmits power P. Determine the power transmitted by a wave that is the superposition of y_1 with the wave

$$y_2 = A \cos(\omega t - kx).$$

TRAVELING WAVES

Mastery Test Form D

Date ______________

pass recycle

1 2 3 4 5

Name ______________________________ Tutor ____________________

1) Write the equation for a wave traveling to the left with velocity 200 m/s, frequency 50 hz and a maximum amplitude of .10 m.

2) A piano tuner wishes to change the frequency of a wire from 330 hz to 300 hz. What is the required fractional change in the tension.

3) A sinusoidal traveling wave is moving on a wire .01 kg/m with a frequency of 10 hz and a wavelength of 2.0 meters. If the maximum amplitude of the wave is .05 m calculate the average power carried by the wave.

TRAVELING WAVES

MASTERY TEST FORM E

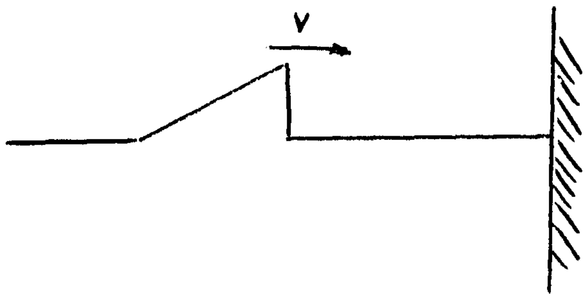
Date ____________

pass recycle

1 2 3 4 5

Name ______________________________ Tutor ____________________

1) Sketch the wave shown below after reflection from the fixed end.



2) A traveling sinusoidal wave with velocity 30 m/s is in a wire of mass/unit length = 0.03 kg/m. The amplitude of the wave is .5m and the wavelength is 1.5 m.
   a) Write the equation of the wave.
   b) What is the tension on the wire?

TRAVELING WAVES

Mastery Test Form F

Date ______________

pass recycle

1 2 3 4 5

Name ________________________________ Tutor ____________________

1) $y = 10\text{ m} \sin(3\pi t+20\pi x)$ (a) Give the velocity (including direction) (b) the amplitude (c) the frequency and (d) the wavelength of this traveling wave. (e) If the mass/length is 0.05 kg/m what is the average power input into the above wave.

2) A guitar string vibrates at 400 hz. Calculate the fractional increase on tension required to change the frequency to 450 hz. What is the change in wave velocity?

TRAVELING WAVES

Mastery Test   Form G

Date ____________

pass   recycle

1 2 3 4 5

Name ______________________________   Tutor ____________________

1) Sketch to approximate scale the sinusoidal wave given by
$y = 5 \sin (5\pi t - 0.1\pi x + \pi/2)$ at t = o sec and at t = sec. All constants are given in MKS units.

2) If the mass per unit length of the wire carrying the wave is 0.05 kg/m. What is the average power put into the above sinusoidal wave?

TRAVELING WAVES

Mastery Test Form H

Date ______________

pass recycle

1 2 3 4 5

Name ________________________________ Tutor ______________________

A traveling wave on a wire of 0.02 kg/m, can be expressed as
$y = 520 \text{ m} \sin (4\pi t + \frac{\pi x}{20} + \phi)$ with all constants in MKS units.

a) What is the amplitude of the wave?
b) What is the frequency of the wave?
c) What is the wavelength of the wave?
d) What is the power carried by the wave?
e) Which way is the wave traveling?
f) What is the tension in the wire?
g) Find ϕ, if $y = 16$ at $x = o$ and $t = o$.

MASTERY TEST GRADING KEY - Form A

1. What To Look For: (a) Clear picture. (b) Encourage algebraic manipulation before plugging in numbers. (c) Expression for standing wave.

Solution: (a) See Figure 16, in which ℓ = 1.20 m, f = 400 Hz, m = 0.0100 kg, and the amplitude is 5.0 10^{-4} m.

(b) Since $\ell = 5(\lambda/2)$, $\lambda = (2/5)\ell$. $v = (F/\rho)^{112}$,

$$F = \rho v^2 = \frac{m}{\ell}(f\lambda)^2 = \frac{m}{\ell}f^2\left(\frac{4}{25}\right)\ell^2 = \frac{4mf^2\ell}{25} = 307 \text{ N}.$$

(c) See Figure 17.

$y = A \sin \omega t \sin kx$,

where $A = 5.0 \times 10^{-4}$ m from the problem. $\omega = 2\pi f = 2.51 \times 10^3\ s^{-1}$,

$k = 2\pi/\lambda = 2\pi(2/5)\ell = 3.02\ m^{-1}$. Note that this form provides that $y = 0$ everywhere at $t = 0$ and $\partial y/\partial t > 0$ in the first loop.

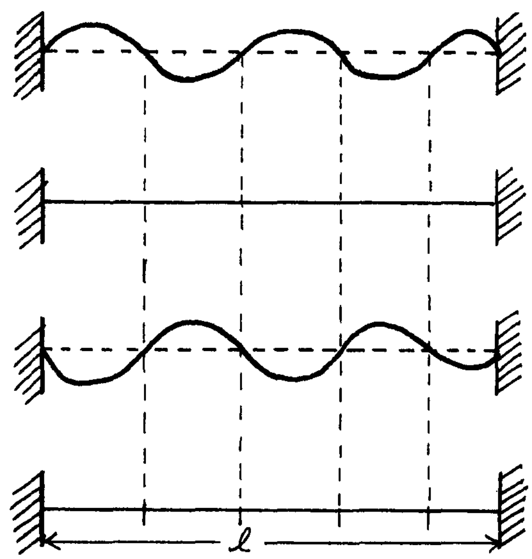


Figure 16

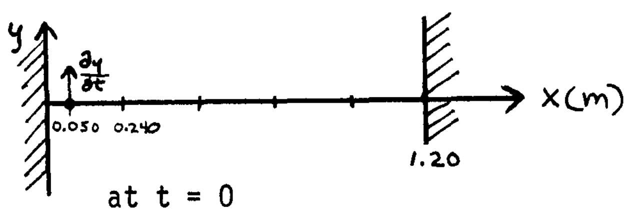


Figure 17

2. What To Look For: (a) Clear picture, appropriate mathematical justification. (b) Clear statement.

Solution: (a) See Figure 18. Wave is written as $y = A \sin(kx - \omega t)$, where $A = 0.00300$ m, $k = 2\pi/\lambda = (2\pi/4)\ m^{-1} = (\pi/2)\ m^{-1}$,

$\omega = 2\pi f = 2\pi(v/\lambda) = 2\pi(30/4) = 15\pi\ s^{-1}$.

$(\partial y/\partial t)|_{x=2} = -\omega A \cos(-\omega t + \pi) = \omega A \cos \omega t$, where $\omega A = 0.141$ m/s. By inspection of the figure we see that at $t = 0$, P has its maximum positive-y velocity.

(b) Power transmission is proportional to $(\omega A)^2 \rho v$. We keep A, ρ, and v the same but change λ:

$\lambda = v/f = v2\pi/\omega$, thus $P \propto 1/\lambda^2$

if A, ρ, and v remain constant. Therefore power, or rate of energy propagation on string, is reduced by a factor of 9 (is multiplied by 1/9) if λ is increased by a factor of 3 (is multiplied by 3).

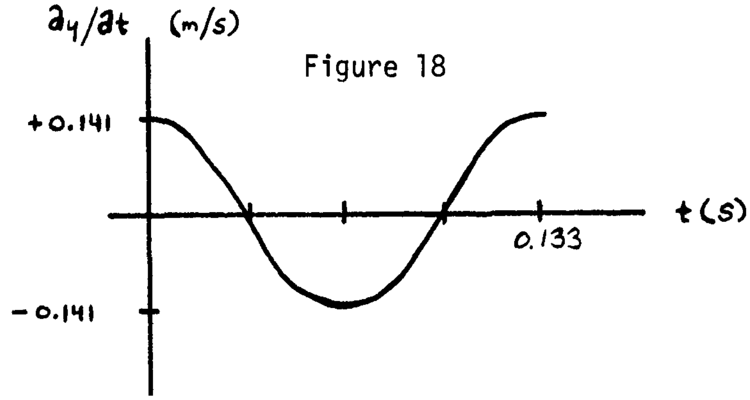


Figure 18

MASTERY TEST GRADING KEY - Form B

1. What To Look For: Correct scales on x and y axes; unambiguous statements.

Solution: (a) $y = A \sin(kx - \omega t - \phi)$, where $A = 0.00300$ m, $k = 6.0\ m^{-1}$, $\rho = 0.0040$ kg/m, $\lambda = 1.050$ m, $\omega = 80\ s^{-1}$, period $= 2\pi/\omega = \pi/40$ s, $\phi = \pi/2$, wave speed $c = \omega/k = (F/\rho)^{1/2}$, and $F = \rho\omega^2/k^2 = 0.71$ N.

(b) See Figure 19. At $t = \pi/320$ s, we write the equation of the displacement of the string as

$y = A \sin(kx - \pi/4 - \pi/2)$, a sine curve shifted $3\pi/4k$ toward positive x.

(c) (i) The direction is along +x. (ii) The magnitude of the wave velocity is $c = \omega/k = 80/6.0 = 13.3$ m/s.

(d) $P \propto A^2$. If the amplitude of the wave is halved, the power transmitted by the wave is decreased by a factor of 4 (i.e., the power transmitted is multiplied by 1/4 if the amplitude is multiplied by 1/2).

2. Solution: See Figure 20. The string travels a distance ct = c(3 /2c) = (3/2) and is reflected at its fixed end.

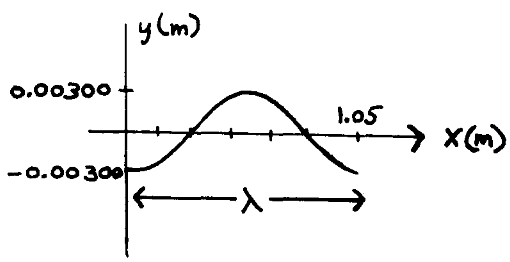


Figure 19

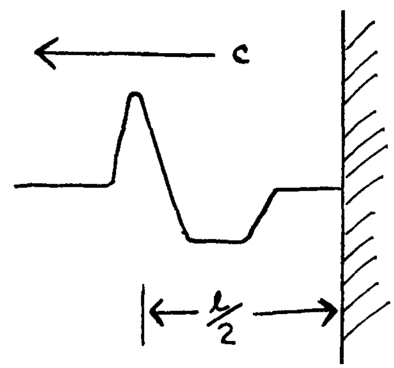


Figure 20

MASTERY TEST GRADING KEY - Form C

1. Solution: (a) See Figure 21. $t = 1/30$ s is two cycles later than $t = 0$. The figures look exactly the same.
(b) See Figure 22.
(c) $c = (F/\rho)^{1/2}$, thus $\rho = F/c^2 = F/(\lambda f)^2 = 50\ \text{N}/(2.00\ \text{m})^2(60\ \text{Hz})^2 = 3.47$ g/m.
(d) To get a standing wave, we must superpose a traveling wave of the same amplitude, traveling in the opposite direction. Thus the amplitude is 2.00×0.0050 m $= 0.0100$ m, and the distance between nodes is $\lambda/2 = 1.00$ m.

2. Solution: $y_1 = A \sin(\omega t - kx)$, given $P_1 \propto A^2$.
$y_2 = A \cos(\omega t - kx) = A \sin(\omega t - kx + \pi/2)$. If we do the trigonometry correctly, using $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$, we get
$y_1 + y_2 = 2A \cos(\pi/4) \sin(\omega t - kx + \pi/4)$,* where $2A \cos(\pi/4)$ is the new amplitude. The new power is thus
$P' \propto [2A \cos(\pi/4)]^2 = 4(1/\sqrt{2})^2 A^2$ and $P' = 2P$.
(*This can be checked for reasonableness for the cases when y_1 and y_2 are exactly in phase and out of phase.)

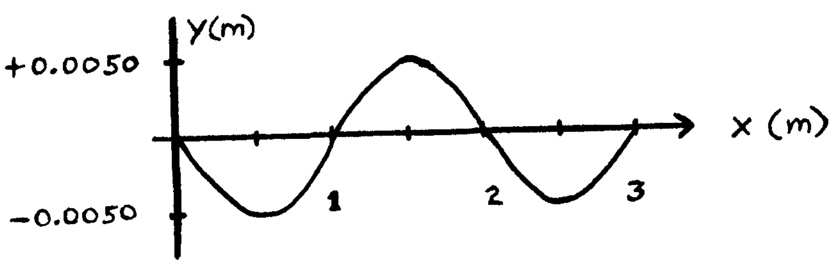


Figure 21

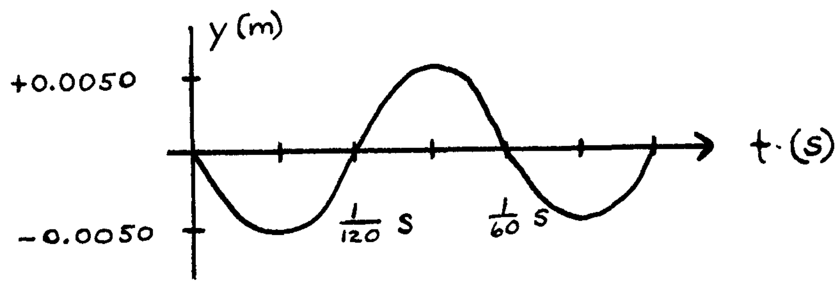


Figure 22