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Methods of Applied Mathematics: Honors Mathematics 450 and 451

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Methods of Applied Mathematics

Honors Mathematics 450 and 451
Each 3 Credit Hours

BRUCE BUKIET AND ROY GOODMAN
NEW JERSEY INSTITUTE OF TECHNOLOGY, ALBERT DORMAN HONORS COLLEGE
JOEL S. BLOOM, DEAN

GENERAL DESCRIPTION

In this course, students perform and analyze physical experiments in the context of an advanced mathematics course. This capstone course integrates the students' experience with mathematical modeling, mathematical analysis, numerical methods, computation, engineering and communication. In the first semester, students have short modules (2–4 weeks) that include relatively simple experiments and numerical simulations. This prepares students for the second semester, when students work in teams to perform and analyze experiments of greater complexity using more advanced mathematical skills. At the end of the second semester, students present their research results both orally and in writing.

FALL SEMESTER

Texts

Haberman, *Mathematical Models: Mechanical Vibrations, Population Dynamics and Traffic Flow*
Farlow, *Partial Differential Equations for Scientists and Engineers*

Experimental Apparatus

Vernier LabPro—Data acquisition and analysis software, Accelerometer, Photogates, Temperature probe, Masses, Springs, Pendulum, Cycloid track, Power supply, voltmeter, conductive paper and pens

Syllabus

Unit I: Introduction—Math Modeling, Gravity and Newton's Law of Cooling

Week 1: Review of Differential Equations, Introduction to Mathematical Modeling and Applied Problems
Physical Experiment 1: Newton's Law of Cooling—is the power really 1?

Week 2: Equilibrium and Stability in one dimension (1st order), Newton's Law of Cooling Review vector calculus, Newton's laws, conservative systems

Week 3: Least squares fitting for realistic data
Project 1: Mathematical modeling and Newton's Law of Cooling experiment analysis

Unit II: Mechanics I—The Brachistochrone

Week 4: Calculus of Variations

Week 5: Derivation of the Nonlinear Differential Equation governing the Brachistochrone (Curve for which a ball travels from one point to another in the fastest time under the influence only of gravity), Solution to the Nonlinear Ordinary Differential Equation (Parametric Equations)
Physical Experiment 2: Timing a trajectory: the Brachistochrone vs. the line

Week 6: Tautochrone property of the Solution, Analysis for the line and of the cycloid for different height/length ratios
Project 2: Calculus of variations, Brachistochrone experiment and analysis of the cycloid

Week 7: Review and Midterm and Going over Midterm

Unit III: Mechanics II—Mass-Spring Systems

Week 8: Review Midterm, Second order ODEs and harmonic motion, Dimensional Analysis

Week 9: Derivation and solution of undamped and damped single mass-spring systems
Physical Experiment 3: Single vertical mass-spring setup

Week 10: Phase plane analysis, Double mass-spring system, Non-linear oscillations and the Pendulum
Project 3: Measuring the spring constant, frequency and evaluating linearity of a spring and other mass-spring analysis

Week 11: Linear Stability and Linearization (higher order), Energy Conservation and Energy Curves, Numerical Methods for ODEs
Physical Experiment 4: Double mass-spring and its frequencies
Project 4: Double mass-spring and its frequencies; how initial conditions influence the dynamics of the double mass-spring; nonlinear springs

Week 12: Phase curves for the damped pendulum, The Spring Pendulum Project + Physical Experiment 5: Timing the pendulum, analysis of the nonlinear pendulum and linearized pendulum equations

Unit IV: Electrostatics and Incompressible Fluids

Week 13: Derivation of Laplace equation for potential flow, Electrostatic potential, Properties of the Laplace equation, Elliptic PDEs

Week 14: Separation of Variables, Solutions in Rectangular and Cylindrically symmetric regions

Week 15: Finite difference methods, Review
Physical Experiment 6: Electrostatic Field Mapper experiment
Project 6: Analytic and Experimental Solution of Laplace's equation for electrostatics problems (equipotential and flux lines)

Grading Policy

The final grade in this course will be determined as follows:

Homework/Projects: 66% Midterm and Final Exams: 34%

SPRING SEMESTER

General Description

In the spring semester, students learn more advanced methods from classical mechanics and use them to study problems that have attracted more recent interest: dynamical bias in coin tosses, as shown by Diaconis et al., chaos in the double pendulum, and the dynamics of simple walking toys.

Text

H.C. Corben and Philip Stehle, *Classical Mechanics XYZ*

Expository Articles

Keller, "The Probability of Heads," Amer. Math. Mo., (93) 1986

Diaconis, Holmes, Montgomery, "Dynamical Bias in the Coin Toss," preprint, 2004

Halir & Flusser, "Numerically stable direct least squares fitting of ellipses," Proc 6th Intl. Conf. in Central Eur. On Computer Graphics, 1998

McGeer, and the Ruina lab, papers on walking toys

Experimental Apparatus

Matlab image processing toolbox, digital camera, high-speed video camera & software, gyroscopes, coins, plates, and pendula

Syllabus

Unit I: Rigid Body Mechanics

Week 1: Course overview, introduction to Matlab's image processing toolbox, Keller's "no dynamics" coin-toss model
Project 1: Use Matlab image processing software to track object in video

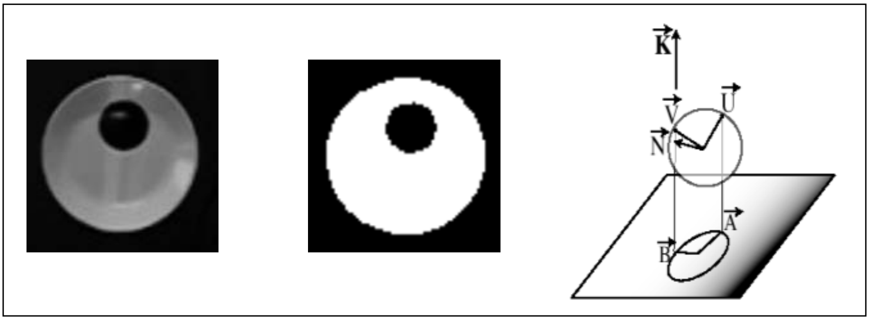
Week 2: Review vector calculus, Newton's laws, conservative systems

Week 3: The gravitational potential, review of variational methods, Hamilton's principal and derivation of equations of motion as Euler-Lagrange equations
Project 2: Extend project 1 to calculate gravitational acceleration from a video of a bouncing ball

Week 4: Conservation laws and symmetry, rigid rotations in two dimensions, moments of inertia, parallel axis theorem

Week 5: Rigid rotation in 3D, parallel axis theorem, body frame & fixed frame, rotational kinetic energy & the inertia tensor, angular momentum. More image processing, least squares fitting & specialized methods for fitting ellipses
Project 3a: Feynman's plate experiment part I: shoot and analyze video of thrown dinner plate, detect edges and fit to ellipses

Figure 1: Video capture from plate experiment and reconstruction of its position



Week 6: Euler's equations, the rotator, the symmetric free top, Feynman's plate experiment, geometry of three-dimensional reconstruction of plate from image
Project 3b: Feynman plate II: reconstruct plate positions, verify analytic predictions

Week 7: The asymmetric free top, stability of motion about axes, the Poinsot sphere
Project 3c: Experimental verification of stability and instability

Week 8: Moving between fixed and body frame, the body cone & space cone, the Diaconis et al. result

Begin big project A: Shoot and analyze several high-speed videos of coin tosses to verify the Diaconis result and get a probability distribution of biases

Week 9: parallel axis theorem for inertia tensors, the "heavy top" (gyroscope)
Project 4: The gyroscope

Unit II: Pendulums and Nonlinear Oscillators

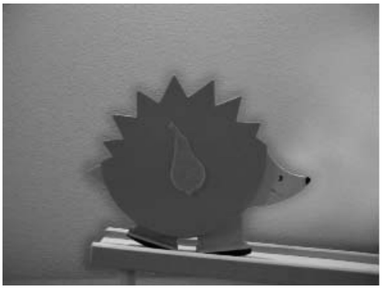
Week 10: Forced damped linear and nonlinear oscillators, Poincare maps, chaos
Project 5: Forced damped linear and nonlinear oscillators

Week 11: Stabilization of the inverted pendulum by rapid oscillation of support (with demonstration!)

Week 12: The double pendulum, Lyapunov exponents
Project 6: Numerical and experimental demonstration of chaos using Lyapunov exponents

Weeks 13 & 14: Modeling and experiments with a simple walking toy, reference to Ruina lab

Figure 2: A simple walking toy



Week 15: Practice project presentations

Grading

The final grade in this course will be determined as follows:
Homework exercises: 25% Projects and Presentations: 75%

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