1975

Work and Energy

WORK AND ENERGY

INTRODUCTION

Energy is much in the news lately. The term "energy" usually refers to the inherent ability of a material system, such as a person, a flashlight battery, or rocket fuel, to bring about changes in its environment or in itself. Some common sources of energy are the fuel used to heat hot water, the gasoline that propels a car, the dammed water that drives the turbine in a hydroelectric plant, and the spinning yo-yo that can climb up its own string. Inanimate energy sources are of central importance in raising the standard of living of mankind above the subsistence level.

The physicist distinguishes among several types of energy, including kinetic energy (associated with a flying arrow or other moving object), elastic energy (associated with stretched or compressed strings), chemical energy (associated with fuel-oxygen systems or a storage battery), thermal energy (associated with the sun and other objects that are hotter than their surroundings), and nuclear energy. Applications of the energy concept in the science of mechanics, which you are studying now, usually concentrate on kinetic energy, potential energy (to be introduced in the module Conservation of Energy), and work (the transfer of energy by the action of a force). Sometimes the phrase "mechanical energy" is used to refer to the forms of energy of importance in mechanics.

When you get on your bicycle, you have undoubtedly noticed that it takes a good deal of effort to get yourself moving rapidly. If you exert yourself very strenuously, you can reach a given speed after a short distance; or you can take it easy and pedal over a longer distance to reach the same speed. In some sense it always takes the same amount of "work" to reach a given speed - either a large exertion for a short distance or a small exertion for a long distance. You may also have noticed that if you are carrying a passenger on your bike, then it takes more "work" to reach the same speed.

It turns out that these intuitive relationships among the "work" done on a system, its mass, and changes in its speed can be sharpened into a precise statement, called the work-energy theorem. (One caution, though: the technical definition of work needed for this precise statement is different from its everyday usage and physiologic meanings; e.g., you do no work on a heavy box by merely holding it still.) As you will begin to see in the present module, this relationship between work and mechanical energy gives you a new and powerful tool for the solution of many problems, a tool that is often easier to use than a direct application of Newton's second law.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Calculate the dot (scalar) product of two vectors, given rectangular or polar descriptions (needed for Objectives 1, 3, 4 of this module)	Vector Multiplication Module
*Distinguish between displacement and position, speed and velocity (needed for Objectives 1, 3, 4 of this module)	Rectilinear Motion Module
*Given position or velocity as functions of time, find the velocity or position (needed for Objective 4 of this module)	Rectilinear Motion Module
*Construct free-body diagrams (needed for Objectives 1, 3, 4 of this module)	Newton's Laws Module
*Apply Newton's second law of motion (needed for Objectives 1, 3, 4 of this module)	Newton's Laws Module
*Calculate the definite integral of polynomial functions (needed for Objectives 1, 3 of this module)	Calculus Review
*Find the derivatives of polynomial and rational functions of one variable (needed for Objectives 1, 3, 4 of this module)	Calculus Review

LEARNING OBJECTIVES

After you have mastered the content of this module you will be able to:

1. Work - Define the work done by a force and the work done on a particle; calculate the work done by a constant or variable force oriented parallel or obliquely to the displacement of the particle.

2. Kinetic energy - Define and calculate the kinetic energy of a particle or system of several particles, given their masses and velocities.

3. Work-energy principle - Relate the work done on a particle to the change in its kinetic energy, and solve problems of particle motion in one dimension using this relationship.

4. Power - Define power and apply the relationships of power to work, force, velocity, and kinetic energy in connection with the motion of a particle in one dimension.

GENERAL COMMENTS

Your text deals exclusively with the energy concept in the context of Newtonian mechanics. Actually, energy is much broader than that, and your understanding will be helped if you are aware of other scientific uses of the energy concept.* The text's definitions of work and kinetic energy are chosen to be useful in mechanical theory, and therefore may sound very abstract to you.

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*For instance, see Introductory Physics: A Model Approach [by Robert Karplus (Benjamin, New York, 1969)], Chapters 4, 9, 11, and 14, especially if you have not had physics in high school.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

After reading the General Comments, please read each of the comments below along with the text section primarily dealing with the objective as keyed in the table. Your text readings are from Chapter 8. Follow the order of the objectives and pay special attention to the examples in the text and to the additional Problems with Solutions in the study guide. After concluding the reading and examples, work out the assigned problems later in the study guide. Hints for solving these, if needed, are given at the end. Finally, check your learning by taking the Practice Test.

In relation to Objective 1, note that the physicist's technical definition of work is introduced gradually by proceeding from a simple case (constant force) in Section 8.1 to the general case (variable force) in Eq. (8.3). The integral in Eq. (8.3) is called a line integral because the integration extends along the line (path) of the particle's motion. In this course you will only be asked to evaluate the work for particle motion along a straight line. (Skip Section 8.2 for now.) Note that the vector force in Eq. (8.3) may vary in magnitude and in direction; what matters is the component along the path $d\vec{s}$. (Note: we shall use $d\vec{l}$ for line integrals because $d\vec{s}$ is more commonly reserved for areas.)

BUECHE

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text*	Assigned Problems: Study Guide	Additional Problems (Chapter 8)
1	Secs. 8.1, 8.3	A, B, C	Illus. 8.3, 8.4	I, J	1 to 4
2	Sec. 8.4	D	Illus. 8.5	K	
3	Sec. 8.4	E, F	Illus. 8.5	L, M	5 to 8, 14
4	Sec. 8.2	G, H	Illus. 8.1, 8.2	N, O	Quest. 7 Probs. 9 to 14

*Illus. = Illustration(s). Quest. = Question(s).

For Objective 2 don't be put off by the reference to impulse at the beginning of Section 8.4. The same result follows directly from Newton's law as stated in Eq. (5.1) in Section 5.1. You may be surprised by the integral over the variable v on page 116. The purely mathematical manipulation is an application of the abstract definition of the definite integral to the sum that has to be evaluated. The kinetic energy $K = (\frac{1}{2})mv^2$ is defined in the text after Eq. (8.5). It is a scalar quantity that may be found for a system of several particles by merely adding the kinetic energies of the individual particles.

Objective 3 is treated only briefly on p. 117; you will gain your understanding of the work-energy theorem by working on problems or by looking at another text.

For Objective 4 go back to Section 8.2. Equation (8.2c) shows that the power P is constant if both force and velocity are constant, and this applies in Illustration 8.1. When a car or other vehicle has an engine that delivers constant power and the velocity changes, then so does the force, as in Problem H. If the force is constant and leads to an acceleration of the moving object, then the power must vary according to Eq. (8.2c) as in Problem G.

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

After reading the General Comments, please read each of the comments below along with the text section in Chapter 6 primarily dealing with the objective as keyed in the table. Pay special attention to the examples in the text and to the additional solved problems in the study guide. After concluding the reading and examples, work out the assigned problems later in this study guide. Hints for solving these, if needed, are given at the end. Finally, check your learning by taking the Practice Test.

In relation to Objective 1, note that the definition of work is gradually introduced in Sections 6-2 to 6-4, proceeding from the constant-force case in Eq. (6-2) to the variable-force case in Eq. (6.10). Equation (6-9) apparently has an infinitesimal left-hand side and finite right-hand side.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text (Examples)	Assigned Problems: Study Guide	Additional Problems*
1	Secs. 6-2 to 6-4	A, B, C	1, 2	I, J	Quest. 1 to 5, 9, Probs. 1 to 10
2	Sec. 6-5	D		K	Quest. 7, Probs. 13, 15
3	Secs. 6-5, 6-6	E, F	3, 4	L, M	Quest. 6, Probs. 11, 12, 14, 16 to 24
4	Sec. 6-7	G, H	5	N, O	Probs. 25 to 36

*Quest. = Question(s).

It is a mixture of two equations:

$$\Delta W = \vec{F}\cdot\Delta\vec{r} = F \cos\phi\, \Delta r (\text{finite } \Delta r)$$

and (1)

$$dW = \vec{F}\cdot d\vec{r} = F \cos\phi\, dr \ (\text{infinitesmal } dr).$$

The latter is applicable in the limit as Δr approaches zero. By integrating both sides you obtain Eq. (6-10). As explained in the Introduction, the work concept is especially useful when forces in a problem vary as functions of position, because the integration over position in Eq. (6-10) can easily take this variation into account. The integral in Eq. (6-10) is called a line integral because the integration extends along the line (path) of the particle's motion. In this course you will only be asked to evaluate the work for motion along a straight line.

A very important point to remember in applications of Eqs. (6-2) (constant force) or (6-10) (variable force) is that the force may be the resultant force acting on a particle, or it may be a particular force of interaction of the particle with another object (pull of a rope, friction, force of gravity). The two very good examples in Section 6-2 both happen to have zero resultant force (zero acceleration), and that will not always be true (Examples 3 and 4).

Objective 2 is satisfied by Eq. (6-12) and the last paragraph of Section 6-5. Note that kinetic energy $K = (\frac{1}{2})mv^2$ is a scalar quantity, not a vector, so that the relative directions of the velocities of many particles in a system are immaterial as far as the total kinetic energy is concerned.

Objective 3 is stated in Eq. (6-14) and illustrated by Examples 3 and 4. Applications of the work-energy theorem are very diverse and can involve the use of Newton's laws to help find the forces that must be known if the work is to be calculated. Here you must again remember to distinguish between the resultant force on a moving body (this accelerates the body and affects its kinetic energy) and the individual forces of interaction with various separate objects that may do positive or negative work on the moving body. Free-body diagrams will help you keep these in mind.

Objective 4 is treated in Section 6-7, and one example with a constant force is given. If the force is not constant, Eq. (6-15) applies. Together with Eq. (1) in this study guide, you can show that the first equation in Example 5 is true in general:

$$P = dW/dt = (\vec{F}\cdot d\vec{r})/dt = \vec{F}\cdot(d\vec{r}/dt) = \vec{F}\cdot\vec{v}. \qquad (2)$$

If a car or other vehicle has an engine that delivers constant power (constant P), and the velocity changes, then the resultant force $\vec{F}$ acting on it must vary; if the resultant force is constant and the velocity varies, then the power P must vary. Keep all the possibilities in mind, and look closely at the study guide Problems G and H keyed to this objective.

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

After reading the General Comments, please read each of the comments below along with the text section in Chapter 7 primarily dealing with that objective as keyed in the table. Pay special attention to the examples in the text and to the additional solved problems in the study guide. After concluding the reading and examples, work out the assigned problems given later in this study guide. Hints for solving these, if needed, are given at the end. Finally, check your learning by taking the Practice Test.

We suggest that you begin reading Section 7-2, which deals with Objective 1. Note that the physicist's technical definition of work is contrasted carefully with the everyday notion. We consider the expression "work done by force" to be better physics idiom than "the work of a force" used to introduce Eq. (7-3). The integral in Eq. (7-3) is called a line integral because the integration extends along the (possibly) curved line (path) of the particle's motion. In this course you will only be asked to evaluate the work for motion along a straight line.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems
1	Sec. 7-2	A, B, C	7-1, 7-2	I, J	7-1 to 7-6, 7-15, 7-21
2	Secs. 7-1, 7-3	D	Example (Sec. 7-3)	K	7-7 to 7-9
3	Sec. 7-1	E, F		L, M	7-16 to 7-19(a), (b), (d)
4	Secs. 7-9, 7-10	G, H	Example (Sec. 7-9)	N, O	7-38 to 7-45(a), (c), 7-46, 7-47

For Objective 2, return to Section 7-1, where the kinetic-energy formula is derived. You may have been surprised to see the integral over the variable v on p. 94. This purely mathematical manipulation applies the definition of the definite integral as the limit of a certain sum, provided that the limits of integration correspond on the two sides of the equation, as pointed out in the text just before Eq. (7-1). Continue with Section 7-3. For a system consisting of several particles, the total kinetic energy is the sum of the individual particles' kinetic energies.

Objective 3 requires you to put together Eqs. (7-2), (7-3), and the unnumbered equation in the box just before Eq. (7-2):

$$\int_{\vec{s}_1}^{\vec{s}_2} \vec{F} \cdot d\vec{s} = \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2. \quad (1)$$

For Objective 4, turn ahead to Sections 7-9 and 7-10. The important result is presented in Eq. (7-17). The power P is, of course, constant when both vector force and velocity are constant. When a car or other vehicle accelerates with an engine that delivers constant power, however, then the velocity changes and so does the force (Problem H). If the force is constant and leads to acceleration of the moving object, then the power must vary according to Eq. (7-17) (Problem G).

In connection with all objectives, note how free-body diagrams are used in Figures 7-3 and 7-4.

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

After reading the General Comments, please read each of the comments below along with the text section in Chapter 9 primarily dealing with the objective as keyed in the table. Pay special attention to the examples in the text and to the additional Problems with Solutions in the study guide. After concluding the reading and examples, work out the assigned problems later in this study guide. Hints for solving these, if needed, are given at the end. Finally check your learning by taking the Practice Test.

We suggest that you begin by reading Section 9-1, which deals with Objective 2 by means of an abstract mathematical manipulation. For a system consisting of several particles, the total kinetic energy is the sum of the individual particles' kinetic energies.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text (Examples)	Study Guide	
1	Secs. 9-2, 9-3, 9-5	A, B, C	9-1(a), 9-2	I, J	9-2, 9-5, 9-6, 9-9, 9-10, 9-13 to 9-16(a), 9-17, 9-20
2	Sec. 9-1	D		K	
3	Secs. 9-3, 9-4, 9-5	E, F	9-1(b), 9-3, 9-4	L, M	9-3, 9-4, 9-7, 9-8, 9-11, 9-26
4	Sec. 9-6	G, H		N, O	9-22 to 9-25

To deal with Objectives 1 and 3, continue your reading in Sections 9-2 and 9-3.* Note that one is looking for changes in speed and kinetic energy, hence one selects the component of force parallel to the direction of motion for the left-hand side of Eq. (9-4), which is called the work. Section 9-2 considers only the change in kinetic energy due to a force acting on a particle over an infinitesimal displacement $d\vec{r}$, a limitation not clearly visible from the right-hand sides of Eqs. (9-4) and (9-6).

*Seven lines from the bottom on p. 134, the first equation should read $v_{f1} = v_{i2}$, not $v_{fi} = v_{i2}$.

The limitation of infinitesimal displacements is removed in Section 9-3 where there is a careful derivation of the work:

$$W_{i\to f} = \int_i^f \vec{F}\cdot d\vec{r} \qquad (1)$$

done on a particle by the resultant force $\vec{F}$. Two very useful theorems follow, concerning the work done by each individual force of interaction ($\vec{F}_1$, $\vec{F}_2$, ...) and the work done by each rectangular component (F_x, F_y, F_z) of the force.

Regarding the statement after Eq. (9-7a), consider this remark: The relation between work and kinetic energy holds regardless of the path followed by the particle, but the numerical values of the work done and kinetic energy change will usually differ for different paths. Study the informative examples at the end of the section carefully. Sections 9-4 and 9-5 add more details concerning Objective 3. You may skip the first subsection "Work Energy and Impulse Momentum" in Section 9-4.

For Objective 4, turn to Section 9-6. The important result is presented in Eq. 9-13, which is illustrated by Problems G and H in this study guide. The power P, according to the equation, is constant when both vector force and velocity are constant. When a car or other vehicle is accelerated by an engine that delivers constant power, however, the velocity changes and so does the force. If the force is constant and leads to acceleration of the moving object, then the power must vary according to Eq. (9-13).

In connection with all the objectives, note how free-body diagrams are used in Figures 9-4, 9-5, 9-6, and 9-8.

PROBLEM SET WITH SOLUTIONS

A(1). The forces $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$ act on a particle of mass M. The forces vary with the particle's position.

(a) State the work done by force $\vec{F}_2$ as the particle moves along curve C from position $\vec{r}_1$ to position $\vec{r}_2$.

(b) State the total work done on the particle when it moves subject to the three forces as described above.

Solution

(a) $$W_a = \int_{\vec{r}_1 \text{ (along C)}}^{\vec{r}_2} \vec{F}_2 \cdot d\vec{r}$$

(b) $$W_b = \int_{\vec{r}_1 \text{ (along C)}}^{\vec{r}_2} (\vec{F}_1 + \vec{F}_2 + \vec{F}_3) \cdot d\vec{r}.$$

B(1). A particle moves along the y axis from y_1 to y_2 according to the force law given below. Find the work done on the particle and by the particle in each case. The constant y_0 equals 3 m.

(a) $F_y(y) = [(y/y_0)^2 - 16]$ N: $y_1 = -y_0$, $y_2 = 2y_0$;

(b) $F_y(y) = [(y/y_0)^3 - 3(y/y_0)^2 + 6(y/y_0) + 3]$ N: $y_1 = 0$, $y_2 = 2y_0$.

Solution

$$W = \int_{y_1}^{y_2} F(y)\, dy$$ is work done on particle.

$-W$ is work done by particle.

(a) $$W = \int_{-y_0}^{2y_0} [(\frac{y}{y_0})^2 - 16]\, dy = [\frac{1}{3}(\frac{y}{y_0})^3 - 16(\frac{y}{y_0})]\, y_0 \Big|_{-y_0}^{2y_0}.$$

$$W = [\tfrac{1}{3}(8) - 16(2)]y_0 - (-\tfrac{1}{3} + 16)y_0 = -45y_0 = -135 \text{ J}; \quad (-W) = +135 \text{ J}.$$

Since W is negative and (-W) is positive, work is done by the particle.

(b) $W = \int_0^{2y_0} [(\frac{y}{y_0})^3 - 3(\frac{y}{y_0})^2 + 6(\frac{y}{y_0}) + 3]\, dy = [\frac{1}{4}(2)^4 - (2)^3 + 3(2)^2 + 3(2)]y_0.$

$W = (4 - 8 + 12 + 6)y_0 = 14y_0 = 42$ J $\quad (-W) = -42$ J.

Since W is positive, work is done <u>on</u> the particle.

(<u>Note</u>: The integration may be carried out with the help of the substitution $y/y_0 = x$, $dy = y_0\, dx$.)

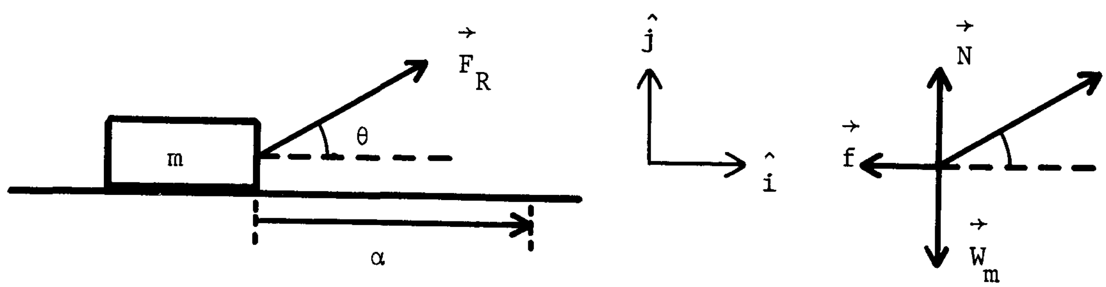
C(1). A block of mass m = 3.00 kg is drawn at constant speed a distance d = 4.0 m along a horizontal floor by a rope exerting a constant force of magnitude F_R = 8.0 N making an angle θ = 15.0° with the horizontal. Sketch a free-body diagram for the block, then compute:

(a) the work done on the block by the resultant force;

(b) the work done by the rope on the block;

(c) the work done by friction on the block.

<u>Solution</u>



Data: m = 3.00 kg $\quad |\vec{F}_R| = 8.0$ N (rope)

θ = 15.0° $\quad \vec{F}_R = F_{Rx}\hat{i} + F_{Ry}\hat{j}$

d = 4.0 m $\quad \vec{f} = -f\hat{i}$ (friction)

$\Delta\vec{r} = \Delta x\hat{i}$

(a) Since the block was pulled at constant speed, $\vec{F} = \vec{F}_R + \vec{f} + \vec{W}_m + \vec{N} = 0$.

Total work $W = \int \vec{F}\cdot d\vec{r} = 0$.

(b) The work done by the rope:

$W_R = \int \vec{F}_R\cdot d\vec{r} = F_R \cos\theta\, d$

$= (8.00)(0.966)(4.0)$

$= 30.9$ J.

(c) The work done by friction $W_f = \int \vec{f}\cdot d\vec{r} = -fd = -(7.74)(4.0) = 30.9$ J.

(From the x component of $\vec{F} = 0$, we find $F_{Rx} - f = 0$, $f = F_{Rx} = 7.73$ N.)

Comparing the answers of (a), (b), and (c), we find $W = W_R + W_f$ checks.

D(2). A 1000-kg sportscar moving at 30.0 m/s and 5000-kg truck moving at 12.0 m/s approach an intersection at right angles. What is the total kinetic energy of the two-vehicle system?

Solution

The directions of motion are irrelevant, since kinetic energy is a scalar and depends only on v^2. Call the sportscar #1, the truck #2:

$$K = K_1 + K_2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}(1000)(30.0)^2 + \frac{1}{2}(5000)(12.0)^2$$

$$= (500)(900) + (2500)(144) = 4.5\times10^5 + 3.6\times10^5 = 8.1\times10^5\text{ J}.$$

E(3). A 2000-kg car travels at 20.0 m/s on a level road. The brakes are applied long enough to do 1.20×10^5 J of work.

(a) What is the final speed of the car?

(b) What further distance is required to stop the car completely if the brakes are applied again and the constant decelerating force on the car is 4.0×10^3 N? (Use two methods and compare the results.)

(c) What ultimately happens to the car's initial kinetic energy?

Solution

(a) The kinetic energy of the car (mass M = 2000 kg) is converted to thermal energy of the brake drums and linings by friction. The work done $W = 1.20\times10^5$ J reduces the car's kinetic energy from the initial K_i to the final $K_f = K_i - W$. The speed drops from the initial v_i = 20.0 m/s to the final v_f = ?, where

$$K_i = ½Mv_i^2,\quad K_f = ½Mv_f^2,$$

$$v_f = \sqrt{2K_f/M} = \sqrt{2(K_i - W)/M} = \sqrt{v_i^2 - 2W/M}$$

$$= \sqrt{400 - \frac{2(1.2\times10^5)}{2\times10^3}} = \sqrt{400-120} = \sqrt{280} = 16.7\text{ m/s}.$$

(b) With applied force F = 4.0 x 10^3 N, the car moves a distance d, so work in the amount W' = Fd = K_f is done to stop the car.

$$d = K_f/F = ½Mv_f^2/F = \frac{1}{2}\times\frac{2\times10^3\times280}{4.0\times10^3} = 70\text{ m}.$$

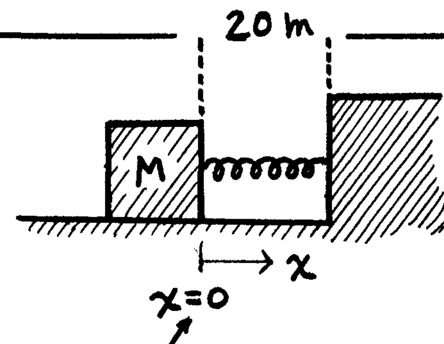
Find deceleration a, time to stop t, and distance moved d at the average speed $v_{av} = (1/2)v_f$. Use Newton's second law:

$$a = F/M = -\frac{4000}{2000} = -2\ \text{m/s}^2;\quad t = -v_f/a = \frac{16.7}{2.0} = 8.35\ \text{s};$$

$$d = v_{av}t = \tfrac{1}{2}(16.7)(8.35) = 70\ \text{m}.$$

(c) Thermal energy in brakes - see part (a).

F(3). The block of mass M in the diagram rests on a frictionless surface. The spring exerts a force $F_x = -kx$ on the block, where k = 0.75 N/m and positive x is measured to the right. The spring is <u>not</u> fastened to the block.



(spring <u>not</u> compressed)

(a) How much work must be done on this system by an external agent to move the block slowly from x = 0 to x = 0.80 m?

(b) If the block is now released, how much work will the spring do on the block?

(c) If the block has a mass M = 0.330 kg, what is its final speed?

<u>Solution</u>

(a) The force $F'_x = -F_x = kx$ must be exerted on the block to move it slowly, compressing the spring. Hence the work W is

$$W = \int_0^{0.80} F'_x\,dx,$$

$$W = \int_0^{0.80} kx\,dx = \tfrac{1}{2}kx^2\Big|_0^{0.80} = \tfrac{1}{2}(0.75)\,[(0.80)^2 - (0)^2],$$

$$W = 0.24\ \text{J}.$$

Since W is positive, work is done by the external agent.

(b) Assuming the spring has zero mass, the force it exerts on the block is F_x, equal in magnitude to the force F' exerted by the agent in (a). Hence the work it will do on the block is W = 0.24 J, which will slide away when it passes x = 0. The block acquires kinetic energy in this process.

(c) Kinetic energy:

$$K = \frac{1}{2}Mv^2 = W = 0.24 \text{ J}.$$

$$\text{Final Speed:} \quad v = \sqrt{\frac{2K}{M}} = \sqrt{\frac{(2)(0.24)}{(0.33)}} = \sqrt{1.44} = 1.2 \text{ m/s}.$$

G(4). An 1800-kg elevator starts from rest and is pulled upward at a constant acceleration of 2.50 m/s^2.

(a) Draw a free-body diagram for the elevator.

(b) Find the upward speed of the elevator as function of time after it started.

(c) Find the power required to raise the elevator as function of time.

(d) Find the kinetic energy of the elevator as function of time.

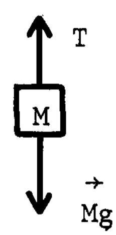
(e) Evaluate the power and kinetic energy numerically for the time when the elevator has risen 20.0 m.

(f) Describe the relation between the power in (c) and the kinetic energy in (d).

Solution

(a) Newton's second law:

$$\vec{F} = \vec{T} - M\vec{g} = M\vec{a}.$$



(b) $v(t) = at$ (since a is constant)

$= 2.5t$ m/s.

(c) $P(t) = Tv(t) = M(g+a)at$

$= (1800)(9.8+2.50)2.5t = 5.5 \times 10^4 t$ W.

(d) $K = (1/2)Mv^2 = (1/2)Ma^2t^2 = 5.6 \times 10^3 t^2$ J.

(e) Height $h = (1/2)at^2$, $h = 20.0$ m, $a = 2.50$ m/s,

$$t = \sqrt{2h/a} = \sqrt{40.0/2.5} = \sqrt{16.0} = 4.0 \text{ s}.$$

$$P = M(g+a)at = (1800)(9.8 + 2.50)(2.50)4 = 2.2 \times 10^5 \text{ W}.$$

$$K = (1/2)Ma^2t^2 = (1/2)(1800)(2.5)^2(4)^2 = 0.90 \times 10^5 \text{ J}.$$

(f) Because of the gravitational force, not all the power is available to increase K, but some is used to raise the elevator.

H(4). The following questions deal with a sports car of mass 1500 kg (including the driver and a companion). Its maximum power is 7.2×10^4 W:

(a) How long would it take the car to accelerate from 15.0 m/s to 30.0 m/s on a straight, level road, if the engine delivered maximum power continuously?

(b) What is the speed $v(t)$ of the car as function of time after the accelerator is "floored" (i.e., maximum power P, initial speed $v_0 = 15$ m/s)?

(c) What is the accelerating force $\vec{F}(t)$ exerted on the car in (b)?

Solution

(a) Call speeds $v_0 = 15.0$ m/s and $v_1 = 30.0$ m/s. Gain in kinetic energy $K_1 - K_0$ comes from power delivered up to time t. Maximum power is $P = 7.2 \times 10^4$ W.

$$K_1 - K_0 = Pt; \quad t = (K_1 - K_0)/P = \frac{1}{2} M(v_1^2 - v_0^2)/P;$$

$$t = (1/2)(1500)(30^2 - 15^2)/7.2 \times 10^4 = 7.0 \text{ s}.$$

(b) $v(t)$ can be found from $K(t)$ (kinetic energy):

$$\tfrac{1}{2}Mv^2 = K(t) = K_0 + Pt; \quad v(t) = \sqrt{2(K_0 + Pt)/M} = \sqrt{v_0^2 + 2Pt/M},$$

$$v(t) = \sqrt{225 + (2)(7.2 \times 10^4)t/(1.5 \times 10^3)} = \sqrt{225 + 96t}$$

$$= (15\sqrt{1 + 0.43t}) \text{ m/s}.$$

(c) $$P = Fv \quad \text{or} \quad F = \frac{P}{v} = \frac{7.2 \times 10^4}{15\sqrt{1+0.43t}}$$

$$= \left(\frac{4.8 \times 10^3}{\sqrt{1 + 0.43\,t}}\right) \text{ N}$$

directed along the road.

Problems

I(1). A particle subject to several forces moves at constant speed along the y axis. One force is $\vec{F} = (ax + b)\hat{i} + (cy^2 + d)\hat{j}$, where $a = 10.0$ N/m, $b = 5.0$ N, $c = 9.0$ N/m^2, and $d = 26.0$ N. Find the amount of work done by the force $\vec{F}$ on the particle as it moves from $y = 2.00$ m to $y = 4.0$ m.

J(1). A 3.5-kg block attached to a rope is allowed to slide down a plane inclined at 30° to the horizontal. The coefficient of friction is 0.40 and the tension applied to the rope is 8.0 N, in the direction parallel to the incline.

(a) How much work is done by the rope's tension on the block while it slides a distance of 2.00 m along the plane?

(b) How much work is done on the block by the force of gravity while it slides a distance of 2.00 m along the plane?

(c) Compare the results of (a) and (b), and comment on the difference.

K(2). A girl on a motor scooter chasing a horse has one-fourth the kinetic energy of the horse, which has 10 times her mass. She speeds up by 1.00 m/s and then has one-third the horse's kinetic energy. What were the original speeds of (a) the girl (on the motor scooter) and (b) the horse?

L(3). A 7.0-kg block slides down a rough plane inclined at 60° to the horizontal. After sliding a distance of 15.0 m from its starting point, its speed is 12.0 m/s. The coefficient of friction is 1.00.

(a) Draw a free-body diagram for the block.

(b) Evaluate the work done by each of the forces acting on the block.

(c) Calculate the initial velocity of the block.

M(3). A block of mass 1.80 kg slides on a horizontal frictionless table. It has the initial speed 5.0 m/s and is aimed end-on at the free end of a spring capable of exerting the force -kx when it is deformed by the distance x (positive for stretching, negative for compression) from its unextended length L. The constant k = 180 N/m. The other end of the spring is firmly nailed to the table. The spring has mass m_s.

(a) How much work must the block do on the spring in order to come to rest?

(b) How far will the spring be compressed when the work in (a) is done on it?

(c) How does the mass of the spring affect the answers to (a) and (b)?

N(4). A 5000-kg truck has an engine that can deliver a maximum of power of 5.0×10^5 W. The driver steps on the gas when the truck is moving at 15.0 m/s and keeps the accelerator floored (i.e., maximum power output - assume it is constant) for 5.0 s.

(a) What will be the truck's speed at the end of the 5.0 s?

(b) What will be the speed of the truck as function of time after the accelerator is floored?

(c) (Optional difficult problem) How far does the truck travel during the 5.0 s while the accelerator is floored?

O(4). A satellite rocket of 5.0×10^4 kg mass acquires a speed of 2.0×10^3 m/s in one minute after launching. The rocket motor functions at constant thrust (constant force), and the rocket's mass change in one minute is negligible.

(a) What is the average power developed by the rocket during the first minute of flight? (Gravity should be ignored.)

(b) What instantaneous power is developed by the rocket as function of time after launch?

(c) Compare the instantaneous power after one minute of flight with the average power in (a) and comment on the relationship.

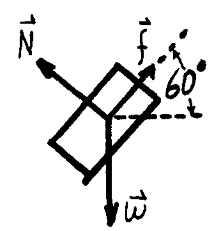
Solutions

I(1). The other forces and the motion of the particle can be ignored; only F_y contributes to the work, which is calculated from an integral along the y axis. Answer: W = 220 J.

J(1). The friction and motion of the block are not needed to find the work required. Applying the definition of work to the tension and the force of gravity will give the answers directly, since the block's displacement is stated. Answers: (a) W = -16.0 J; (b) W = +34 J.

K(2). Introduce as variables the girl's and horse's masses and speeds. Use the information given to set up three equations. The mass of the motor scooter does not enter and one mass cancels. Answers: (a) v = 6.5 m/s; (b) v = 4.1 m/s.

L(3). There are three forces: gravity, friction, and the normal force due to the inclined plane. Since the displacement is given, the net force, acceleration, and speed do not need to be calculated. Answers: (b) 890 J, -520 J, 0; (c) v_i = 6.0 m/s.



M(3). To come to rest, the block has to lose its kinetic energy. Since the spring is at rest at the end, when the block is at rest, any work done to give the spring kinetic energy will be returned. Hence the mass of the spring does not affect the result. Answers: (a) 22.5 J; (b) 0.50 m.

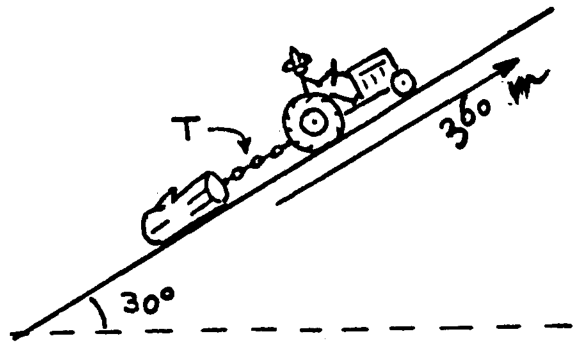
N(4). At constant power, the kinetic energy increases at a uniform rate. The speed can be found from the uniform rate of kinetic energy increase. Answers: (a) 35 m/s; (b) $(225 + 200\,t)^{1/2}$; (c) 130 m.

O(4). The average power can be found from the total kinetic-energy change during the time interval. The instantaneous power must be found from the force and instantaneous velocity. Answers:

(a) 1.7×10^9 W; (b) 5.6×10^7 t.

PRACTICE TEST

1. A tractor pulls a log of mass 200 kg up a 30° slope at constant speed of 1.20 m/s. The log is moved a distance of 360 m.



   (a) Draw a free-body diagram for the log.

   (b) Introduce symbols to state an expression for the work done by the force of gravity on the log during the operation.

   (c) Evalute the work stated in (b) numerically and give an interpretation of the sign of the result.

   (d) What is the power supplied by the tractor, assuming that the log moves without friction.

2. A brick with mass 4.0 kg is pushed along a horizontal wooden plank with the initial speed of 1.50 m/s. The coefficient of friction with the plank is 0.250.

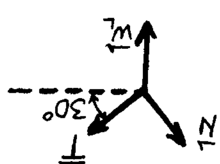
   (a) What is the brick's initial kinetic energy?

   (b) How far along the plank will the brick slide before coming to rest?

   (c) What happens to the initial energy of the brick?

Practice Test Answers

1. (a)



   (b) $W = \int_0^{360} \vec{W}_L \cdot \hat{i}\, dx.$

   (c) -3.5×10^5 J.

   (d) 1180 W.

2. (a) 4.5 J.

   (b) 0.46 m.

   (c) Heat up the brick and plank.

Date____________________________

WORK AND ENERGY

pass recycle

Mastery Test Form A

1 2 3 4 5

Name________________________________ Tutor______________________________

Dennis, an impish 30.0-kg boy, runs down a grassy slope (30° from the horizontal) at 3.00 m/s. Suddenly he falls down and continues to slide on the seat of his pants (coefficient of friction 0.8).

1. Make a sketch and free-body diagram for Dennis while he is sliding.
2. How far does he slide before coming to a stop? Use the work-energy theorem.
3. How much work is done by the force of gravity on the boy? Set up the work integral carefully and be prepared to explain the sign of this work.
4. What is the power developed by the force of friction when his pants first make contact with the ground?
5. Describe qualitatively how this power varies during his slide.

Date______________________

WORK AND ENERGY

pass recycle

Mastery Test Form B

1 2 3 4 5

Name______________________________ Tutor______________________________

A block of mass 4.0 kg is drawn across a rough horizontal floor by a force of 12.0 N applied at an angle of 37° with the horizontal. The coefficient of friction is 0.200. The block starts from rest and is moved a total of 7.0 m before the force ceases to act suddenly.

1. Draw a sketch of the phenomenon and a free-body diagram for the block.
2. Find the work done on the block by the applied force and by friction after setting up the work integral.
3. What is the kinetic energy at the moment the force ceases acting?
4. What is the power expended by the applied force at the moment before it ceases acting?
5. State the work-energy theorem (as applied to a particle) in the form of an equation and briefly identify the meaning of each symbol used.

Date____________________

WORK AND ENERGY

pass recycle

Mastery Test Form C

1 2 3 4 5 6

Name__________________________________ Tutor__________________________________

A 5.0-kg piece of ice can slide in a 2.00 m long frictionless chute, at an angle of 60° with the horizontal.

1. Draw a sketch of the situation, and a free-body diagram of the ice while a person is pushing it up the chute at constant speed.
2. How much work is done on the ice by the force of gravity when it is pushed from the bottom to the top of the chute? Set up and evaluate the work integral.
3. With what kinetic energy will the ice arrive at the bottom of the chute if it starts at the top with a speed of 2.0 m/s and slides down freely? Apply the work-energy theorem.

A 20 000-hp (1.5×10^7 W) railroad locomotive accelerates a 10 000-ton (1.00×10^7 kg) train from a speed of 10.0 m/s to 30.0 m/s at full power along a straight track.

4. How long a time interval is required for this process, neglecting friction?
5. Find the kinetic energy and the speed of the train as functions of time during the interval.
6. Find the force accelerating the train as function of time during the interval.

Instructor__________________ Date____________________________

WORK AND ENERGY

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Mastery Test Form D

1 2 3 4 5

Name__________________________ Tutor______________________________

Starting from rest, a block of 20 kg mass is pushed 20 m up a 37° inclined plane by a constant force of 200 N acting parallel to the surface. The coefficient of kinetic friction between the contact surfaces is 0.3.

1. Make a free body diagram of the block. Indicate the forces which are involved in energy changes.
2. Find the work done by the acting force.
3. Find the increase in potential energy.
4. Find the work done against friction.
5. Find the increase in kinetic energy.
6. Compare the sum of the answers in 3,4, and 5 with the answer in 2. What are your comments?
7. What is the velocity at the instant 20 m is reached? What is the instanteous power rating at this point? What is the average power rating during the time of travel?

Courtesy of the University of Missouri-Rolla

Instructor________________________ Date____________________

WORK AND ENERGY

Mastery Test Form E

pass recycle
1 2 3 4 5

Name___________________________ Tutor__________________________

A sled with its load has a mass of 60 kg and is started in motion by a boy who exerts a constant horizontal force of 98.0 newtons over a horizontal distance of 5 meters. The sled then starts down a slope which is 50 meters long and has a vertical height of 10 meters. The sled then reaches a horizontal terrain. The coefficient of kinetic friction is 0.02 between sled runners and the snow over the entire distance.

1. Make a free body diagram of the sled for the first 5 meters of travel. Indicate which forces are involved in energy changes.

2. How much work does the boy do in starting the sled? What becomes of this work?

3. What is the change in potential energy of the sled between the top of and the end of the slope?

4. What is the change of kinetic energy between the top of the slope and the end of the slope?

5. How far does the sled go on the horizontal terrain?

6. How much total work is done against friction?

7. What was the velocity of the sled at the instant it started on the horizontal terrain? What is the power rating for the sled at this instant?

Courtesy of the University of Missouri-Rolla.

Instructor____________________ Date____________________

WORK AND ENERGY

Mastery Test Form F

pass recycle

1 2 3 4 5

Name__________________________ Tutor____________________

When a quarterback throws a quick pass he exerts a constant horizontal force of 144 newtons on the ball, which has a mass of 0.9 kgm. The force acts for 0.1 sec and his hand moves forward 0.8 meters.

1. Make a free body diagram of the football while being thrown. Indicate the force or forces which do not influence the energy transformations.

2. How much work does he do on the ball in throwing the pass?

3. What is the kinetic energy of the ball as it leaves his hand?

4. What is the average power developed by the quarterback while throwing the ball?

5. A 1000 kilogram automobile has a speed of 30 m/sec on a horizontal road when the engine is developing 35 kilowatts. For the same power and frictional forces, what will be its speed on a hill which has a rise of 1 meter in 20 meters of road?

MASTERY TEST GRADING KEY - Form A

What To Look For | **Solutions**

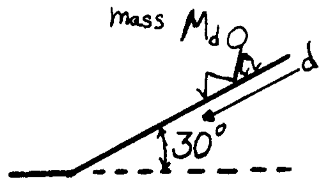
What To Look For

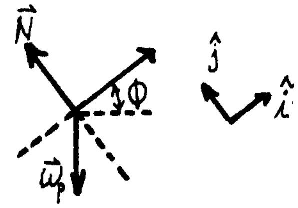
1. Three forces $\vec{N}$, $\vec{f}$, $\vec{W}_D$: (Normal, friction, weight).

   Angle of slope.

   Equations (1) to (4) not required -- prepares for later parts.

Solutions





1. Resultant force:

$$\vec{F} = \vec{N} + \vec{f} + \vec{W}_D = F_x\hat{i} + F_y\hat{j}.$$

$$F_x = M_D a_x \quad F_y = 0 \qquad (1)$$

$$F_x = f - W_D \sin\phi F_y = N - W_D \cos\phi, \qquad (2)$$

$$\vec{f} = \mu\vec{N}, \quad W_D = M_D g, \qquad (3)$$

$$\text{Data: } M_D = 30.0 \text{ kg}, \quad \phi = 30°, \qquad (4)$$

$$v_i = -3.00 \text{ m/s}, \quad g = 9.8 \text{ m/s}^2.$$

What To Look For

2. Setting up W and K for theorem; correct solution of resultant F; correct use of resultant, not merely friction, in work-energy theorem.

Solutions

2. Initial kinetic energy K_i is dissipated through friction, but gravitational force tends to maintain it. K is lost owing to (-) work done by net force:

$$W = K_f - K_i = -K_i \quad \text{since } K_f = 0,$$

$$W = \int \vec{F}\cdot d\vec{r} = -F_x d; \quad K_i = \tfrac{1}{2}M_D v_i^2,$$

$$F_x = \mu N - M_D g \sin\phi = M_D g(\mu\cos\phi - \sin\phi).$$

(<u>Note</u>: $F_y = 0$, $N = W_D \cos\phi = M_D g \cos\phi$.)

$$-F_x d = -K_i, \quad d = \frac{K_i}{F_x} = \frac{\tfrac{1}{2}M_D v_i^2}{M_D g(\mu\cos\phi - \sin\phi)},$$

$$d = \frac{v_i^2}{2g(\mu \cos \phi - \sin \phi)}$$

$$= \frac{(3.00)^2}{(2 \times 9.8)(0.8 \times 0.866 - 0.50)}$$

Two significant figures OK.

$d = 2.4$ m.

3. Definition of work

3. $W_{gravity} = \int \vec{F}_D \cdot d\vec{r}, \quad d\vec{r} = dx\hat{i} + 0\hat{j},$

$\vec{F}_D = -M_D g(\hat{i} \sin \phi + \hat{j} \cos \phi).$

This can be obtained as indicated, or from $dW = F\, dr \cos \theta$, where this angle θ is 90° more than ϕ in the problem.

Limits of $\int$: $x = 0$ to $x = -d$,

$\vec{F}_D \cdot d\vec{r} = -M_D g \sin \phi\, dx,$

$$W_{gravity} = \int_0^{-d} (-M_D g \sin \phi\, dx)$$

$= M_D g \sin \phi d$

$= (30.0)(9.8)(0.50)(2.4)$

$= 350$ J (Positive).

Sign is +, two significant figures.

The force of gravity does work on Dennis, thus decreasing the rate of kinetic energy loss.

4. Definition $P = (dw/dt)f$ is less convenient here, but OK. (-) sign since f, v opposite. N comes from Problem 2. Two significant figures.

4. $P = \vec{f} \cdot \vec{v}$ ($\vec{f}, \vec{v}$ oppositely directed)

$= -\mu N v_i$

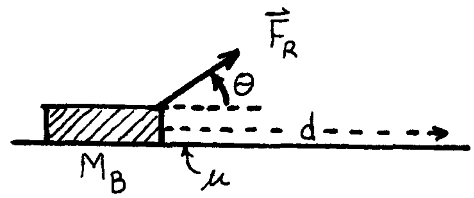
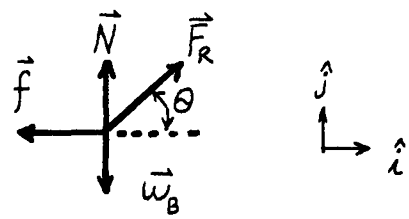
$= -\mu(W_D \cos \phi)\, v_i$

$= -(0.8)(30.0)(9.8)(0.866)(3.00)$

$= 610$ W

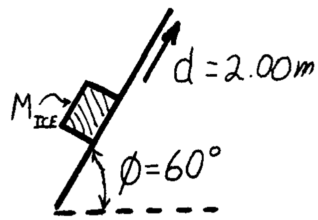
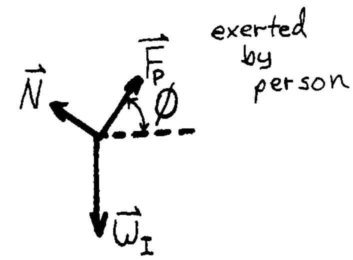
5. The power decreases as speed decreases; f stays constant until he stops.

MASTERY TEST GRADING KEY - Form B

What To Look For	Solutions
1. Four forces: weight $\vec{W}_B$; normal $\vec{N}$, rope $\vec{F}_R$; friction $\vec{f}$. Note angle of rope.	1.   Data: $M_B = 4.0$ kg, $\mu = 0.200$, $f = \mu N$, $F_R = 12.0$ N, $d = 7.0$ m, $\theta = 37°$, $g = 9.8\ m/s^2$, $W_B = M_B g$
2. Definitions of W; note sign (+) and angle. Note sign (-) only. Use correct f.	2. $W_R = \int \vec{F}_R \cdot d\vec{r} = \int_0^{7.0} F_R \cos\theta\, dx$ $= F_R \cos\theta d = (12.0)(0.8)(7.0) = 67$ J. $W_F = \int \vec{f} \cdot d\vec{r} = \int_0^{7.0} (-\mu N)\, dx$ $= -\mu[W_B - F_R \sin\theta]\, d$ $= -(0.20)[(4.0)(9.8) - (12)(0.6)]\ 7.0 = -45$ J.
3. Use this, not laws of motion (latter OK but tell student). More significant figures useful for subtraction, but not required.	3. Use the work-energy theorem: $W = K_f - K_i = W_R + W_f$, $K_i = 0$, $K_f = W = 22$ J. $W_R + W_f = 67.2 - 44.8 = 22.4$ J (from Problem 2).

4. Recognize formula for instantaneous power, NOT average $P_{av} = W/(\text{time for movement})$.	4. $P = \vec{F}_R \cdot \vec{v}_f = (F_R \cos\theta)v_f$ $= (12.0)(0.08)(3.35) = 32$ W. $W = K_f = \frac{1}{2}M_B v_f^2$
Use kinetic energy to find v.	$v_f = \sqrt{2W/M_B} = \sqrt{(2)(22)/4.0} = 3.3$ m/s.
5. Resultant force should be identified. Integral optional.	5. $W = K_f - K_i$ W: work done by resultant force on particle; K_f, K_i: final, initial kinetic energy
Initial and final K should be identified in relation to work. Ask for oral explanation if the written answer is incomplete.	$= \int \vec{F}\cdot d\vec{r}$.

MASTERY TEST GRADING KEY - Form C

What To Look For	Solutions
1. Note three forces: gravity, person, normal. Angle of slope. $W_I = M_I g$.	1.  
2. General integral. Note angle ϕ, is usually θ in definition minus 90° or $\theta = \phi + 90°$.	2. $W_g = \int \vec{W}_I \cdot d\vec{r} = -W_i d \sin\phi$ $= -M_I g d \sin\phi = -(5.0)(9.8)(2.00)(0.866)$, $W_g = -85$ J. (Ask Student to interpret sign).
3. Source of W.	3. $W = K_f - K_i$. Work done during slide is done by force of gravity. Since motion is down, $W = -W_g$ from Problem 2. $K_i = \frac{1}{2} M_I v_i^2$, $K_f = W + K_i = 85 + (\frac{1}{2})(5.0)(2.0)^2$ $= 85 + 10 = 95$ J.
4. Apply work-energy theorem and definition of power (for constant P).	4. Symbols: M_T = mass of train, i = initial, F = resultant force, P = power, t = time, f = final Speed is related to kinetic energy. Use $W = K_f - K_i = P\,\Delta t$ since P is constant. $K_i = \frac{1}{2} M_t v_i^2$, $K_f = \frac{1}{2} M_T v_f^2$, $\Delta t = (K_f - K_i)/P = \frac{1}{2} M_T (v_f^2 - v_i^2)/P$ $\Delta t = (\frac{1}{2})(10^7)\ \frac{30^2 - 10^2}{1.5 \times 10^7} = \frac{800}{3} = 270$ s.

5. Introduce time variable. Use initial speed. Solve for v(t).

5. $K(t) = K_i + Pt = \frac{1}{2}M_T v_i^2 + Pt$

$= (\frac{1}{2})(10^7)(10)^2 = 1.5 \times 10^7 t$

$= 10^7(50 + 1.5t)$ J (t in seconds).

$K(t) = \frac{1}{2}M_T v(t)^2$,

$v(t) = \sqrt{2K(t)/M_T} = \sqrt{100 + 3.0t}$ m/s .

6. Two methods possible. Use v(t) from Problem 5 for both. Get same result. F NOT constant; a NOT constant $\neq (v_f - v_i)/t$.

6. $P = Fv, \quad F = P/v,$ (1)

$F = M_T a = M_T dv/dt.$ (2)

OR

$$F = \frac{P}{v} = \frac{1.5 \times 10^7}{(100+3.00t)^{\frac{1}{2}}} \text{ N (t in seconds)} \quad (1)$$

$$F = M_T a$$

$$= M_T \frac{d}{dt} \sqrt{100+3.00t} = \frac{1}{2} \frac{3.00\ M_T}{(100+3.00t)^{\frac{1}{2}}}$$

$$= \frac{1.5 \times 10^7}{(100+3.00t)^{\frac{1}{2}}} \text{ N.} \quad (2)$$