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Combinatorics & Power Consumption

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Abstract— **in this paper the power consumption possible combinations for a number of loads and a single power source are discussed. The proposed mathematical model depends basically on combinatorics to estimate how many ways are possible to operate different loads powered by a single power source. This paper takes into consideration the different conditions and limitations to operate n number of loads limited by their minimum and maximum values of power consumption. Four different cases of operations are discussed and modeled. A flowchart and a step-by-step method to do the calculations for a general case is also illustrated at the end of this paper.**

Index Terms— **combinatorics, combinations, load, model, operation, power consumption.**

I. INTRODUCTION

A power consumption model is proposed in this paper. The model focuses basically on calculating the number of the ways possible to operate a system of a specific number of power consuming devices. Combinatorics has been used by researchers and engineers in many fields of science. Combinatorics-based methodology is proposed in [1] to calculate pulse repetition frequency (PRF) set quality. In [2] realizing electronic circuit components is based on a combinatorics based algorithm and the field programmable analog array (FPAA). Detecting objects via generation of high level combinatorics and applying relational operators is discussed in detail in [3], the proposed method is simple but yet powerful for object detection. To assist in solving combinatorics problems, researchers in [4] have built a virtual environment which helps in modeling, learning and solving such problems. Solving complex problems and counting of possible combinations of solutions can be done by analytic techniques of computer science and combinatorics [5]. Combinatorics are usually found in solving complex systems such as graph theory applications and NP-complete problems [6]. Sparse signal recovery is achieved in [7] by combining two approaches, geometry and combinatorics.

On the other hand, power consumption models have been proposed for different applications and systems. A realistic power consumption model for wireless sensor network devices proposed in [8] can assist in designing the hardware and the software of more energy efficient wireless devices and their power management systems. Another power consumption model of wireless networks is designed, implemented and validated in [9] using wireless devices from the market. Models to forecast power consumption of power systems are proposed in [10] to serve the major purpose in commercial relations of regional systems with the wholesale power market.

This paper discusses four possible cases of operating number of n loads powered by a single source.

II. POSSIBLE CASES OF POWER CONSUMPTION COMBINATIONS

a. Case1: loads have just two states: on or off

For this case we assume that each load can be either turned on (i.e. load number i is in On-State and consuming power P_i), or turned off (i.e. load number i is in Off-State and consuming no power $P_i = 0$), and any load can be turned on or off without any condition or limitation. In this case the number of power consumption combinations/possibilities is given in (1):

$$(2)^n = \sum_{k=0}^{n} \binom{n}{k} \qquad (1)$$

2: Indicates that the load has two states: on & off states.
2^n : Indicates the number of all the possible combinations of the operation of n loads. This number equals exactly the number of all the possible values of a digital word of n bits. Table 1 shows that the combinations of loads being on or off can be understood as the possible values of an n-bit digital number.

Table 1: Load combinations as an n-bit digital number

Possible state	On	On	On	. .	On
	Off	Off	Off	. .	Off
Loads	P1	P2	P3	. .	Pn
Digit Position	1	2	3	. .	n
Possible state	1	1	1	. .	1
	0	0	0	. .	0

$$\sum_{k=0}^{m} \binom{n}{k}$$ is the sum of the possible combinations

Where:

$$\sum_{k=0}^{n} \binom{n}{k} = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n}$$

and:

$$\binom{n}{k} = \frac{n!}{(n-k)! \, k!}$$

So, no load is turned on (i.e. no power consumption at all); or one load out of n loads is turned on, two loads out of n loads is turned on, . . . , or all the loads are turned on. Depending on load combinations, each load combination has its associate total power consumption Pt, assuming that there is no limitation on the power source capacity (i.e. the power source can provide power for the worst case scenario when all the loads are turned on and consuming power. Hence, there are 2^n possible values, assuming that the power supply can accommodate any possible power consumption needs. What if the power source cannot deliver more than a specific value Pt, and the total power consumption in some cases when ,for instance, all the loads are turned on exceeds the power source capacity? Case2 discusses how many load combinations there are in this case.

b. Case2: limited power supply capacity & unlimited load values

This case can be solved using the linear equation model. The instantaneous total power consumption is given in (2):

$$P1 + P2 + P3 + \ldots + Pn = Pt \qquad (2)$$

Assuming that P1, P2, .., Pn >= 0 and the power consumption of all the loads can be dimmed to an integer value, for example: the power consumption of load Pi can be 0, 1, 2, . . , Pt Watts. The number of possible load consumption combinations for this case is given in (3):

$$\binom{n + Pt - 1}{Pt} = \frac{(n + Pt - 1)!}{(n-1)! \, Pt!} \qquad (3)$$

c. Case3: limited power supply capacity & load values with a limit on their minimum values

This case can still be solved using the linear equation model given in (2). Assuming that:

$$P1 \geq P1\_min, P2 \geq P2\_min, \ldots, Pn \geq Pn\_min$$

and that the power consumption of any load can be dimmed to an integer minimum value, for example: the power consumption of load Pi can be 5, 6, 7, . . . , Pt Watts if Pi_min is 5.

Case3 can be converted to Case2 by writing the linear equation as in (4):

$$(P1 - P1\_min) + (P2 - P2\_min) + \ldots + (Pn - Pn\_min) =$$
$$Pt - (P1\_min + P2\_min + \ldots + Pn\_min) = Ptt \qquad (4)$$

Where:

$$Ptt = Pt - (P1\_min + P2\_min + \ldots + Pn\_min)$$

Since:

$$(P1 - P1_{min}) \geq 0, \quad (P2 - P2_{min}) \geq 0, \ldots$$
$$\ldots (Pn - Pn\_min) \geq 0$$

Then the solution of this equation (which is the number of possible load consumption combinations for this case) is:

$$\binom{n + Ptt - 1}{Ptt} = \frac{(n + Ptt - 1)!}{(n-1)! \, Ptt!}$$

d. Case 4: limited power supply capacity & load values with a limit on their minimum and maximum values

This case is more complicated than Case3 and can be solved using the linear equation model and the principle of inclusion & exclusion.

The instantaneous total power consumption is always given by the same equation in (2):

$$P1 + P2 + P3 + \ldots + Pn = Pt$$

Assuming that:

$P1\_max \geq P1 \geq P1\_min, \quad P2\_max \geq P2 \geq P2\_min, \ldots$

$$\ldots, Pn\_max \geq Pn \geq Pn\_min,$$

and that the power consumption of all the loads is an integer value, for example: the power consumption of load Pi can be 5, 6, 7, . . . , 12 Watts if Pi_min is 5 Watts and Pi_max is 12 Watts. Subtracting Pi_min from all the load power values will result in the equation given in (5):

$$(P1 - P1\_min) + (P2 - P2\_min) + \ldots + (Pn - Pn\_min) =$$

$$Pt - (P1\_min + P2\_min + \ldots + Pn\_min) \tag{5}$$

That yields to:

$$P1\_new + P2\_new + \ldots + Pn\_new = Ptt$$

The total number of solutions for this problem if there were no other conditions on the maximum values of Pi_new is:

$$\binom{n + Ptt - 1}{Ptt} = N = \frac{(n + Ptt - 1)!}{(n - 1)! \, Ptt!}$$

But:

$$Pi\_new = Pi - Pi\_min$$

with the following conditions:

$$(Pi\_max - Pi\_min) \geq Pi\_new \geq 0$$

For $i = 1, 2, 3, \ldots, n$

To solve this equation the opposite conditions are used instead, which are:

$$Pi\_new > (Pi\_max - Pi\_min)$$

Or

$$Pi\_new >= (Pi\_max - Pi\_min + 1)$$

$$Pi\_new >= Pi\_c$$

$$Pi\_c = (Pi\_max - Pi\_min + 1)$$

So the opposite conditions are:

$$C1: P1\_new >= P1\_c$$

$$C2: P2\_new >= P2\_c$$

.

$$Cn: Pn\_new >= Pn\_c$$

And in general, the opposite condition for load number i is:

$$Ci: Pi\_new >= Pi\_c$$

Supposing that Ci" is the original condition for load number i, which means Ci" is the opposite condition of Ci and given in (6):

$$Ci''_i: Pi\_new < Pi\_c \tag{6}$$

Then the solution of Case4, which is the general case, is given in (7):

$$|C1''. C2'' \ldots Cn''| = N$$

$$- \sum_{i=1}^{n} |Ci|$$

$$+ \sum_{1 \leq j1 < j2 \leq n} |Cj1.Cj2|$$

$$- \sum_{1 \leq j1 < j2 < j3 \leq n} |Cj1.Cj2.Cj3|$$

$$+ \ldots$$

$$+ (-1)n . |C1. C2 \ldots Cn| \tag{7}$$

Where:

$\sum_{i=1}^{n} |Ci|$ is the sum of numbers of solutions to the formulas satisfying conditions Ci from 1 to n and only once at a time

$\sum_{1 \leq j1 < j2 \leq n} |Cj1.Cj2|$ is the sum of numbers of solutions to the formulas satisfying two conditions Cj1 and Cj2, where 1<= j1 < j2 <= n

$\sum_{1 \leq j1 < j2 < j3 \leq n} |Cj1.Cj2.Cj3|$ is the sum of numbers of solutions to the formulas satisfying three conditions Cj1, Cj2 and Cj3, where 1<= j1 < j2 < j3 <= n

$|C1.C2 \ldots Cn|$ is the number of solutions to the problem satisfying all the conditions C1 through Cn.

To find |Ci|, condition Ci only has to be satisfied:

$$P1\_new + P2\_new + \ldots + (Pi\_new - Pi\_c) + \ldots$$

$$\ldots + Pn\_new = Ptt - P1\_c = Ptt'$$

Then:

$$|Cj| = \binom{n + Ptt' - 1}{Ptt'} = \frac{(n + Ptt' - 1)!}{(n-1)! \, Ptt'!}$$

To find |Cj1. Cj2| both Cj1 and Cj2 have to be satisfied together. Then:

$$P1\_new + P2\_new + \ldots$$

$$\ldots + (P|1\_new - P|1\_c) + \ldots + (P|2\_new - P|2\_c) + \ldots$$

$$\ldots + Pn\_new = Ptt - P|1\_c - P|2\_c = Ptt''$$

The yields to:

$$|Cj1. Cj2| = \binom{n + Ptt'' - 1}{Ptt''} = \frac{(n + Ptt'' - 1)!}{(n-1)! \, Ptt''!}$$

And the same method can be used to calculate $|Cj1. Cj2. Cj3|$ through $|C1. C2. ..Cn|$.

An algorithm for calculating all the possible combinations of Case4 is illustrated in fig. 1.
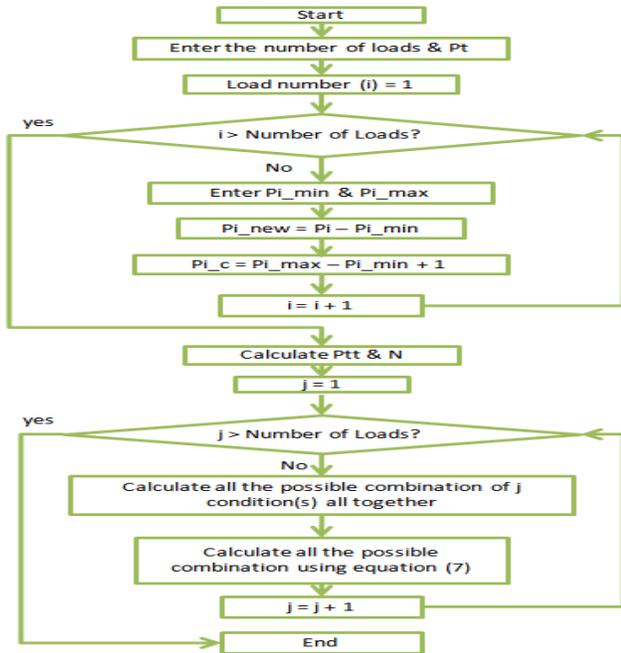

Figure 1: A flowchart for calculating all the possible combinations of the operating modes of number of loads

III. CONCLUSION

This paper has shown the possible four cases for operating a number of loads powered from a single power source. Case4 provides general equations and a method to calculate all the possible operating modes possible of the loads. Case4 can be used for any other type of loads such as mechanical loads, it can also be used to assist other applications and systems, such as power management systems, to give, for instance, a quick pointer/indicator whether another possible combination is possible, so that the power management system will try turning on/off and/or dimming loads to achieve the optimal power consumption according to the conditions and limitations of the system.

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