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Written Communication in a Sixth-Grade Mathematics Classroom

Mary Schneider
Holdrege, NE

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Mary Schneider

Holdrege, NE

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Department of Mathematics

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Written Communication in Mathematics

Abstract

In this action research study of my sixth grade mathematics classroom, I investigated what happened to students' mathematical achievement when they had increased practice on written explanations to problems. I wanted to see if writing out solutions to problems helped them overall in daily mathematics. By using specific mathematic vocabulary more frequently and deliberately during my instruction, I wanted to investigate whether students would correctly use specific math vocabulary in their written explanations. I also increased my expectations of the students' written explanations throughout the research project. I wanted to determine whether students would try to meet or even exceed my expectations. I discovered that students used vocabulary more frequently in their written explanations by providing definitions of vocabulary versus using the vocabulary in context. I found little to no evidence suggesting that my students' mathematical achievement changed through more practice on written communication; however, I did find as my expectations for the quality of students' written explanations increased, most of my students improved their written explanations of problems and my teaching became more deliberate and specific. As a result of this research, I plan to continue having students communicate their mathematical ideas through written communication while continuing to focus on specific mathematical vocabulary and its purpose in written communication.

Introduction

I selected the research topic of written communication in mathematics, specifically in the area of problem solving. This is an area of concern because students often struggle when it comes to explaining their line of thinking. I believe it is important to be able to express ideas in a clear manner so others can understand thought processes. As a math teacher, it is easier to ask students to just solve computational problems than it is to teach kids how to think and write. What I believe about teaching and learning math does not always occur in my classroom. It is important to have my students connect mathematical ideas through oral and written communication; however, time constraints and the pressure of "covering" material often prevent me from doing this.

I wanted to improve written communication in relation to problem solving in my classroom. The evidence I had that this was a problem was when I had required my students, on occasion, to write out an explanation, they struggled. My students often did not know how to begin writing out an explanation for their solution. They did not know what information to include or how to explain their reasoning. Sometimes students "knew" they needed to divide, for example, because other operations did not make any sense. However, that alone did not indicate they understood the problem or solution. Other students simply offered another form of a computation problem as the explanation. For example, if students solved an addition problem, their "explanation" might have included subtraction to prove it was correct. They also did not use specific vocabulary to support their reasoning.

Using specific mathematical vocabulary is something I wanted students to demonstrate in their written communication. Does knowing and using specific mathematical vocabulary increase students' understanding of problems? We "studied" mathematical vocabulary, and students were able to give definitions and draw examples of vocabulary, but I wanted them to be able to apply it appropriately in written context. I thought I would have a clearer idea of my students' true knowledge of math concepts through the use of written communication in regards to problem solving. My ideal classroom would be one where students could concisely explain their reasoning for solving a problem and use their ever-developing mathematics vocabulary while explaining, either in oral or written form. I wanted to have this form of communication generate discussions on the different ways of solving problems where students did not think their way was the only way but also understood multiple solutions. Thus, written communication became an obvious choice for me as I considered a topic for an action research project in conjunction with my master's degree program.

I teach at a rural middle school in central Nebraska. Our school has grades five through eight. Homeroom groups of fifth and sixth graders travel together to the different subjects while the seventh and eighth graders each have individualized schedules. The population of the middle school was approximately 330 students during the year of my study. The K-12 district population was approximately 1,100 students with 93% Caucasian and 5% Hispanic. Our free and reduced lunch student population was about 30% and our English Language Learner (ELL) population was about 2.5%, with Spanish being the predominant language spoken of that group. My research took place during the spring semester of the 2008-2009 school year.

Problem Statement

According to the National Council of Teachers of Mathematics (NCTM), it is important that students are competent in the content and process standards (2000). "School mathematics curricula should focus on mathematics content and processes that are worth the time and attention of students" (p. 15). It is important that students have a working knowledge of the content standards. Two content standards that I planned to focus on during this project were number and operations and geometry. My previous focus as a teacher has already been on the content standards, and I think this should be just a part of students' mathematical experience. Besides the two aforementioned content standards, I also planned to use my study to address the principles of equity and learning. I believe by holding all students accountable for written explanations of problem solving, I will be raising my expectations. I recognize I will need to assist students who struggle with written communication as well as processing ideas. I sometimes fall into the trap of believing that if students get the right answer, this is an indication that they understand the concept; this is not necessarily true. Students using their prior knowledge and incorporating that into their written explanations will give me a better idea of where they are coming from in a problem and what their actual understanding is. Students may not use my preferred approach, but it may be a viable one that I had not thought of previously. I can use written explanations as an instructional strategy to identify students' strengths and weaknesses.

My intent is to have students improve in the area of communication through the use of problem solving. Communication is one of the process standards (NCTM, 2000). My goal is to have students be able to communicate their ideas clearly to myself and other students. I frequently talk about how there is more than one way to solve a problem, but students have a hard time seeing others' explanations because the solutions are not written or stated coherently. Students often assume that others will "just know" a part of their solution without actually writing it down for others to read. This leads to misunderstandings between the writer and the reader. Written communication is important to me in my own teaching because it will help me become a better assessor of student knowledge. I will get to know how my students are thinking as opposed to just seeing their final answer. It will also give me insight as to whether students are making any connection of mathematics vocabulary to "real" life. Studying this problem will benefit others as written communication is not just used in mathematics. It is a skill that is needed in every aspect of life from comprehension questions to science experiments.

I hope there to be carryover between what happens with writing in my classroom and with other places in my students' lives. I also believe it will benefit my students' in future mathematics classes as the curriculum becomes more challenging. Students will have practice in expressing themselves in something besides just numbers. I believe the data gathered by this action research problem will benefit teachers of any subject matter. My desire is that through my research project, students will come to better understand the meaning behind math problems by using written communication and vocabulary. They will also be able to communicate their ideas more clearly with others.

Literature Review

Pugalee (2004) stated that, "[T]hough the literature asserts the power of writing as a learning tool advancing the writing across the disciplines movement, it has not gained wide acceptance in mathematics classrooms" (p. 28). There are many forms of communication available in the mathematics' classroom: oral, visual, and written. Students are more experienced and comfortable with the visual and oral explanations of how to do a math problem than with writing a formal explanation. According to research, the type of instruction many mathematics educators received as students was one of computation and memorization with no value in explanations and reasoning (Baxter, Woodward, & Olson, 2005). "Thus, many students develop a vision of mathematics as a collection of arbitrary rules that make little sense" (p. 119). The purpose of my research was to investigate the relationship between increased written communication in mathematics and student understanding. Common themes I found in the literature were: written communication in mathematics and problem solving, math vocabulary in written communication, written communication and improved mathematical understanding, written versus oral communication, and types of journal writing including transactional and expressive.

Written Communication in Mathematics and Problem Solving

Written communication is not the focus in many mathematics classrooms; yet it still takes place. The topic of a study done by Albert (2000) was the relation between oral and written thought processes of seven middle grade students. Albert looked at the problem-solving procedures and strategies used by students to explain their understanding of mathematical concepts. Students worked independently and organized their thinking about mathematical concepts through writing and graphic representations. Higher-level thinking skills, such as analysis and synthesis of information, were more apparent when students were required to write explanations. Albert compared written communication to a private conversation with oneself; explanations are put down on paper, and it helps make students' answers more visual and concrete. Another important insight from Albert's research was the strength of a "self-talk" technique allowing students to think to themselves about how to solve a problem and then putting it down in writing. It was a way for students not only to solve problems with specific steps and procedures, but also to think about what they are doing, how they are doing it, and why they are doing it.

Draper and Siebert (2004), two educational researchers, used the method of cooperative inquiry to discover the connection between literacy and mathematics in a standards-based classroom. The study occurred in an undergraduate, inquiry-based mathematics classroom. The topic of their study was the similarities and differences for mathematics and literacy educators in regards to instructional goals in mathematics. Draper analyzed lessons in regards to literacy, vocabulary, and the effect it had on student learning, while Siebert focused more on the mathematical content of the lesson. Even though they viewed the lessons differently, each agreed that literacy is a part of mathematics, just as mathematics involves literacy. These two ideas cannot be separated as oral and written communication is intertwined. Draper and Siebert agreed that students in mathematics classrooms must learn both the mathematics and how to use textbooks to be able to reason and communicate mathematics.

According to a study done by Baxter, Woodward, and Olson (2005), some students who did not orally participate in mathematical discussions did respond when they were asked to write about mathematical ideas in their journals. The subjects of this study were four seventh grade students from a middle school in the Pacific Northwest. The four students selected were special education students enrolled in a general math class. Researchers frequently observed the classroom and the four students rarely participated in oral discussions. However, in journal explanations written by these four students, Baxter et al. were surprised at the depth of the explanations. One particular student connected the lesson to a situation familiar to himself to explain his understanding. He included how he used rounding to estimate the amount of money needed for a candy purchase. He clearly described the situation as well as his mathematical understanding. The research of Baxter et al. suggests that teachers should not underestimate the abilities of their students to do writing in a mathematics classroom.

Even though written communication is just one way of sharing mathematical ideas, the three research studies all concluded that it may be the opportunity needed for some of the students who do not verbalize well or choose not to participate in class. The subjects of the three studies were very different: one study focused on seven middle grade students; another study used undergraduate mathematics students; and the third studied four special education students in the seventh grade. All three studies came to the conclusion that writing is a way for students to analyze and synthesize information even though they may not know they are doing it.

Math Vocabulary in Written Communication

When writing in mathematics, there is a specialized vocabulary that needs to be incorporated. Burton and Morgan (2000) looked at the language used in 53 published mathematics research papers. Seventy research mathematicians were interviewed for the broader study. The focus was on the natural language of mathematics, which included special vocabulary and symbols. The mathematicians studied made it clear that it was important to use mathematical language deliberately in order to get across the particular meaning.

In order for students to understand this careful construction and use of mathematics vocabulary in their writing, vocabulary must be emphasized on a daily basis. Draper and Siebert (2004) found it was important for instruction to focus on the students' acquisition of vocabulary words and meanings for those words. Students needed to have a connection between the symbolic representations and their mathematical meanings. One teacher in this study emphasized vocabulary through visuals, games, and activities to help students see the relevance of learning them. According to the NCTM (2000), it is important for students to be able to express their mathematical ideas clearly and specifically. Students need to be able to communicate to peers, teachers, and others by using the precise language of mathematics.

Using precise vocabulary in order to communicate mathematical ideas was a common thread in the research studies. Not only did the researchers conclude that students needed to know the mathematics vocabulary, but also the students needed to attach meaning to it. Emphasizing vocabulary on a daily basis was something Burton and Morgan (2000) discovered was pertinent to learning vocabulary, while Draper and Siebert (2004) found that students needed to see the relevance of vocabulary knowledge and usage.

Written Communication and Improved Mathematical Understanding

It is important to establish a real purpose for the writing, according to Shield and Galbraith (1998). The subjects for their study were students from three eighth grade classrooms from two different schools. Shield and Galbraith's goal in the study was to come up with a particular coding scheme to analyze student writing, something they felt had been ignored in previous research. Student letters were used as data for their study. Students were to write letters to friends who were absent from class that day explaining a mathematical concept that was studied. Shield and Galbraith believed that whenever possible, it was important to establish a real purpose for the writing. While some of the students' writing became clearer in showing understanding over the three-month study, many students' writing displayed little change in understanding. Shield and Galbraith attributed part of this lack of change to teachers not purposely developing the writing of students.

As noted earlier, Albert (2000) found results contrary to those of Shield and Galbraith (1998). Recall that Albert found that the writing deepened the students' understanding of the mathematics. Students also commented that writing their ideas in math helped them keep track of their thinking and solutions and helped them understand the math more clearly. However, when asked how writing helped them understand better, many students had a hard time verbalizing reasons. Albert concluded that writing allowed students to go back and read their chain of thoughts, which helped them understand the mathematical concepts better.

Pugalee (2004) examined 20 students, all ninth-graders, enrolled in a high school algebra class. Pugalee and a university mathematics educator chose 20 open-ended problems from the curriculum that likely would provide students opportunity to produce descriptive written or oral explanations. The results of the study showed that the students who wrote their problem-solving processes had correct solutions at a statistically higher rate than students who verbalized their processes and solutions. Pugalee believed that words are a means for students to communicate with themselves and others. Written communication gave students the chance to really think about what they are doing, why they are doing it, and how they are doing it. These suggestions by Pugalee are similar to what Albert (2000) proposed when he concluded that students could use their writing to reconstruct or reorganize the information given to help them understand the problem more clearly. Students could ask themselves, ‘What do I know about this problem?’ ‘What do I need to know to solve this problem?’ and ‘What strategy do I need to solve this problem?’

While the research done by Shield and Galbraith (1998) did not come to a distinct conclusion about writing improving the understanding of mathematics, the studies done by Albert (2000) and Pugalee (2004) did. Shield and Galbraith agreed that writing is a classroom activity that allows for a deeper understanding of the mathematical concepts students are learning; it can be used for students to reflect and expand their ideas in a way to extend their understanding. However, their research showed little evidence to support that written communication in mathematics increased students’ understanding of the material. Albert, as well as Pugalee, both concluded that writing allowed students to track their thoughts and processes and therefore understand the mathematics more thoroughly.

Written versus Oral Communication

Pugalee’s (2004) research compared writing explanations to the think-aloud method. He explained think-alouds as the verbalizations of what students are thinking in their heads. Pugalee (2004) stated that “thinking aloud on paper” (p. 29) is what writing has been deemed at times. It was used as a way to gather more in-depth data about processes that students use to solve problems. The subjects in the think-aloud group in Pugalee’s study were given a problem to solve; they were given paper and pencil and told that they could use the materials as needed to solve the problem but to also verbalize everything they were thinking out loud. If the student did not speak for 15 seconds, they were prompted to tell what they were thinking. As stated earlier, Pugalee discovered that students who wrote out their explanations versus verbalized them produced correct solutions at statistically higher rates.

Writing gives students time to really think through their solutions without the fear of saying something in front of their peers that is incorrect. With writing, one can go back and edit the mathematical solutions. Baxter et al. (2005) agreed that it is a chance for students to justify and support their explanations that can lead to a more extensive verbal conversation later. They noticed that it gave some of their subjects the opportunity to move from a passive listener to a more active role when they were given the opportunity to have something down in writing first. Written explanations were not only in the form of words but also included drawings, symbols, and graphs.

In the study by Baxter et al. (2005), the teacher was able to get a deeper understanding of the students' needs in her class by reading the written explanations of students. The subjects of this study were reluctant to verbally participate in class. When given certain journal prompts such as the use of calculators in class, students who had never spoken in class expressed strong feelings about the topic in their journals. Writing may be one way for students to make connections in mathematics and help them stay positive about the subject matter. Students who were given the opportunity to work more on problem-solving with written solutions showed an improved or positive attitude toward mathematics (Albert 2000).

Each of the studies above concluded that written communication in mathematics was a way that allowed some students to be more successful than oral communication. Writing enabled students to gather their thoughts and edit their ideas before presenting them formally to a teacher or verbally to peers. In some of the research studies, students even became more active participants in the classroom when they were given the opportunity to write down their ideas and solutions.

Types of Journal Writing

There can be many forms and purposes for writing in mathematics. Journals can be used to take notes, write down feelings, ask questions, write explanations, or to describe a method of solving a problem. Communication is one of the NCTM's K-12 process standards (NCTM, 2000). It is stated that students should be able to "communicate their mathematical thinking coherently and clearly to peers, teachers, and others" (p. 402). Journaling is one means of doing this. One type of journal use is for transactional or public writing. Fried and Amit (2003) would describe this type of writing as "summaries, questions, explanations, definitions, and word problems" (p.105). In the study done by Fried and Amit, classroom notebooks were reviewed in two different high school math classes. Focus groups were chosen from each class to be interviewed. In both classrooms, the notebooks were used for transactional writing. Students copied lessons off the board precisely as the teacher had written them. In one classroom, the notebook would be inspected occasionally, while in the other classroom the notebooks always were inspected. The students never wrote down any reasoning as to why the problems worked; they just had the solutions. Journals contained the how and very little of the why. This not only limited students' ability to think, it also limited the possibility of creativity in solving problems.

Expressive writing or private writing is another type of journal writing. In the communication standard for grades six to eight in the NCTM Principles and Standards (2000), it is stated that, "To help students reflect on their learning, teachers can ask them to write commentaries on what they learned in a lesson or a series of lessons and on what remains unclear to them" (p. 271). This is an example of expressive writing. It is believed that expressive writing can improve transactional writing (Fried & Amit, 2003). Writing can start out as expressive and lead into transactional as the student thinks about the problem, rewrites it, and edits it.

As with any form of journal writing, there always comes the discussion whether journals should be graded, and if so, how? For journals to be completely private and most useful to students, Fried and Amit (2003) believe journals should only serve as self-assessment so students will be honest in their thoughts. Baxter et al. (2005) discovered that students who used journals in their study often were making connections to the teacher. Not only were they using them for mathematical purposes, but they were also writing down their feelings about math or other topics. There are two important things to remember when using journals to study students' mathematical thinking according to Baxter et al. First, a student's written response is only a partial picture of the student's thinking. It is one version of the student's ideas. Secondly, a certain writing prompt geared to elicit a particular response may not be what the student writes about at all.

While the studies above all used journal writing as a means of communication, the purpose of the journals was different. In the study conducted by Fried and Amit (2003), journals were used to write down solutions, with no reflections or feelings, also known as transactional writing. While in the study done by Baxter et al. (2005), the journals were used for solutions as well as feelings about the mathematics. When students were allowed to write down their feelings (also known as expressive writing), it allowed the teacher to better connect with the students in the classroom. Fried and Amit do agree with Baxter et al. that expressive writing can be used to improve transactional writing.

Summary of Related Literature

Communication, in one form or another, is vital in order to express our concerns and ideas to one another. For my action research project, I chose to study the relationship between increased written communication in mathematics and students' understanding. I have become an advocate for writing out explanations to mathematical problems ever since I became involved with the Math in the Middle Institute Partnership. I became more confident in my ability to understand and solve problems, as well share my ideas with others. Explaining solutions to mathematical problems is a method to gather thoughts and ideas and express them for others to understand. After reading many research articles, I found no research on writing in sixth-grade mathematics, the course I teach. My role will be not only researcher, but also teacher, which was different from all of the studies reviewed. The literature did show me the benefits of discussing appropriate mathematical language and formatting for students' written explanations as found by Draper & Siebert (2004). Following my literature review, I wanted my project to combine transactional writing and expressive writing. Thus, these two types of writing would become the focus of my research.

Purpose Statement

The purpose of my action research project was to try to improve student achievement through written communication and vocabulary instruction in a problem-solving context. I examined three research themes: the quality of written communication in homework and problem solving, the quality and quantity of precise mathematical vocabulary terms used in students' written communication, and students' mathematical achievement when higher expectations are placed on the quality of written communication. I was seeking to answer the following research questions:

- What will happen to students' vocabulary usage in written explanations after they receive specific vocabulary instruction?
- What will happen to students' mathematical achievement through increased practice on written explanations?

- What happens to my mathematics teaching when I increase expectations for the quality of student written explanations, including the correct usage of precise mathematical vocabulary?

Method

My research began in February 2009 and concluded in April 2009. I began my research by choosing my first period students, my homeroom group of sixth grade students, as my research subjects. The students were a mixture of low-, middle-, and high-ability learners. There were 17 students in the class, with 10 boys. One student received special education services in math. Two students were classified as ELL students; however, neither one of them received language services in math. One of the ELL students was on a consultation basis with the ELL teacher, while the other student met daily with that teacher.

I used a survey as one method to collect data. A pre-survey was given on February 10, 2009, followed by the post-survey on April 16, 2009 (Appendix A). This survey consisted of three questions in which the students chose whether they strongly agreed, agreed, were undecided, disagreed, or strongly disagreed, with statements about written explanations and vocabulary. The other two questions were open-ended with one question related to written explanations versus computation problems and the other about how teachers could better help students in math.

From February 2, 2009, through April 6, 2009, I collected students’ written explanations to problems from the textbook lessons and problem-solving problems (Appendix B). I had originally planned to gather two or even three examples of written explanations weekly, but I found out quickly how overwhelming it was to score and keep track of that much data. I revised my goal to collect a written explanation to one problem from the lesson and one problem from the problem solving set each week. There were several weeks, however, that I was unable to collect these two samples from my students. We had a shortened week during February 9-13, due to parent-teacher conferences. Therefore, I only collected one example of student work that week. During the week of February 23-27, 2009, I again only collected one piece of data. I wrote in my teacher journal, “This was a recovery week for me. I was so overwhelmed by everything that I had to step back as a researcher. My teacher role took over as did my mom role” (Personal journal, week of February 23-27). We had a three-day week during March 2-6 due to spring break. I again did not collect data. Achievement tests were given the week of March 16-20, thus no data was collected. We had one more shortened week during April 6-8; thus, I was able to only collect one piece of data. I quickly found out while doing my research that there were so many interruptions in schedules that changed my plans.

I used a rubric to score students’ written explanations (Appendix C). I also used a cover sheet for each problem to keep track of how students were scoring on each area of the rubric (Appendix D). I kept track of their scores on the 13 written explanations collected using a blank spreadsheet with the students’ names. The problems from the textbook and other problem-solving problems I used are included in Appendix B.

I had passed out the IRB consent forms before I began my research. Students were asked to take them home and return them to the sixth grade English teacher if their parents would allow them to participate in my research project. The English teacher then gave me a list of seven students to interview. I interviewed five students; I had attempted to interview six students, but one refused to be interviewed. I individually interviewed the five students, using a tape recorder, on Tuesday, February 17, 2009, and again on Tuesday, April 14, 2009. I asked each of the five students a subset of the list of questions in the first interview (Appendix E). I did not ask my students the questions in the same order for each student. I discovered the lack of consistency in order made it much more difficult to analyze the responses as I listened to them on the tape recorder. For the second interview, I did ask them all the same questions in the same order. I also added a few additional questions to the list that were pertinent to my research. The questions I added were specific to the work we had been doing on problem solving during the research project. I asked my students what we had done with problem solving that had helped them or what they liked, what part of problem solving was hard for them, what types of problems were easier or harder to write up and why, and whether it was helpful when I had shown them an example of a strong and weak write-up and we scored it as a class using the rubric. I listened to the first interview immediately afterward and recorded each of the students' responses; I followed the same procedure for the second interview.

I wrote in my teacher journal on a weekly basis throughout my research project. I included in my journal entries student quotes, my quotes, thoughts of mine regarding instruction, and overall impressions of how the day or week went. I included how I adapted the next day's lesson based on the previous day, how I felt about the needs of the students, and episodes I classified as successes as well as failures.

Findings

I arranged my classroom in three sections with each section having four pairs of desks. Students sat with a partner almost daily. A typical day in my classroom during this research action project began with a "question of the day." I wrote a review question on a small whiteboard in my room. The questions varied from a computational type of problem to vocabulary on which we had been working. Students answered these questions in their assignment books. If a problem was computational or vocabulary based, students would raise their hands as they completed the problem. I would check one student's answer and then assign them to be the *checker* for their partner or entire section. I had previously trained my students to help other students by referring peers to the student journals, where notes and examples were written for concepts studied in class prior to conducting this research project. At times, the “question of the day” would require students to describe how they felt about their written explanations or how well they worked with their partner from the previous day. I would personally read each student’s response to these types of questions.

Every other day, I gave my students a timed facts review of three minutes followed by mental math problems. This was how a lesson began in the Saxon course 1 textbook (Hake, 2007). I would either have the students work on the mental math by reading the problems out of their textbooks, or I would read the problems orally. Facts and mental math were usually checked the next day. Students would share their strategies for problems while they checked the mental math. They would then record their scores on a tracking sheet as well as write descriptions of the types of mental math problems they missed. There was a special section on the tracking sheet where students wrote a specific explanation of the types of problems missed.

I then continued the lesson by allowing students to ask questions about problems from the previous day’s before we checked the lesson. Some students asked for a whole problem to be worked out while others asked about specific parts of a problem (e.g., the label). Sometimes I worked the problem on the board; other times I had students share how they worked it. After all questions were answered, we then checked the assignment. If students were also working on problem solving, I would then give the opportunity for students to ask questions over what was unclear. I also would have other students share “clues” on what they found to be helpful in solving the problem. During this time, I would reiterate specific vocabulary words for my students to include in their written explanations or redirect them to the specific question being asked. This was also the time that I would do some reteaching if I had encountered a lot of questions the day before over a specific concept.

For the day's that I would introduce a new lesson, I usually put up an example problem on the board and asked, "What do you think about this kind of problem? What would you do to solve it?" I then would give my students a few minutes to talk it over with their partner, and they would write ideas down on their small whiteboards. I would have students share ideas before I would specifically give instruction on the new concept. Students then would practice problems that I had created or found in the practice set of the textbook lesson. Students would either work individually or with their partners on these practice problems. I always encouraged my students to ask their neighbor for help if I was helping someone else. I concluded the lesson by having my students complete the lesson from the book, which was 30 questions in length.

I allowed students to work with a partner on most problem-solving days if they wanted to. The majority of students always chose to work with a partner. They could sit anywhere in the room to work on the problem. Some chose to work on the small whiteboards first and then transfer their ideas to their paper while others worked directly on their papers the entire time. I moved around the room to answer questions. Students were given the opportunity the following day to share their solutions and ideas. Depending on time, I would draw name sticks from the can and only those students or pairs of students could share their solutions. I was amazed to see how disappointed some students became when there was not enough time for *them* to share *their* ideas.

Students' Use of Vocabulary

The first research question I investigated was, *what will happen to students' vocabulary usage in written explanations after they receive specific vocabulary instruction?* I found through this research that even though students knew the vocabulary, they struggled with including specific vocabulary in the context of their writing. Before this project when students would present solutions or ideas, I found myself allowing them to just use any words to help them explain, instead of using specific vocabulary. Throughout this project, however, I became very particular about the use of appropriate mathematical vocabulary. I forced myself and my students to use appropriate mathematical vocabulary whether it was during whole class instruction or one-on-one instruction. While sharing problem-solving ideas as a whole class, I made a conscious effort to point out how one student had worked the vocabulary into his explanation by using it within his work and write up. One student commented, “I never think of it when I am trying to solve the problem. I am so focused on finding the answer” (Personal Journal, Week of April 6). Many students vocalized agreement with this statement.

Even though students felt it was important to know specific math vocabulary in understanding problems, they were still reluctant to use specific vocabulary in their formal problem solving write-ups, even after receiving specific vocabulary instruction. I wanted my students to approach one particular problem as if they were teaching it to someone else. The following is the problem: *Freddy drove a stake into the ground, looped a 12-foot-long rope over it and walked around the stake to mark off a circle. What was the ratio of the radius to the diameter of the circle?* Students had difficulty incorporating the vocabulary in their written explanations without just defining the words.

> As a class, we talked about possible vocabulary words to incorporate into the explanations. Students even listed these words on their paper to help them do this. Listing the words turned out to be a waste of time for most students as very few incorporated the vocabulary, even though we brainstormed words to help them get started. (Personal Journal, Week of March 9)

I asked students in the second interview, *of the four parts that you were scored on using the rubric, was there any one part that was harder for you?* One student replied, “The only hard part was the vocabulary and I think it was that for all of the kids because maybe we already know the words.” Another commented, “It was hard to work words into the explanation if you know what they mean already.” This told me that even though students knew the definitions of vocabulary words, they did not necessarily know how to apply the words in written explanations. An area I needed to work on more in class was writing up explanations to model how to include specific mathematical vocabulary.

All students interviewed agreed that it was important to know mathematical vocabulary in order to understand the problem and get the answer right. In addition to this, when students were asked on the post-survey if understanding specific math vocabulary was important in understanding problems, most students strongly agreed. No students disagreed. The following graph shows the pre- and post-survey results.

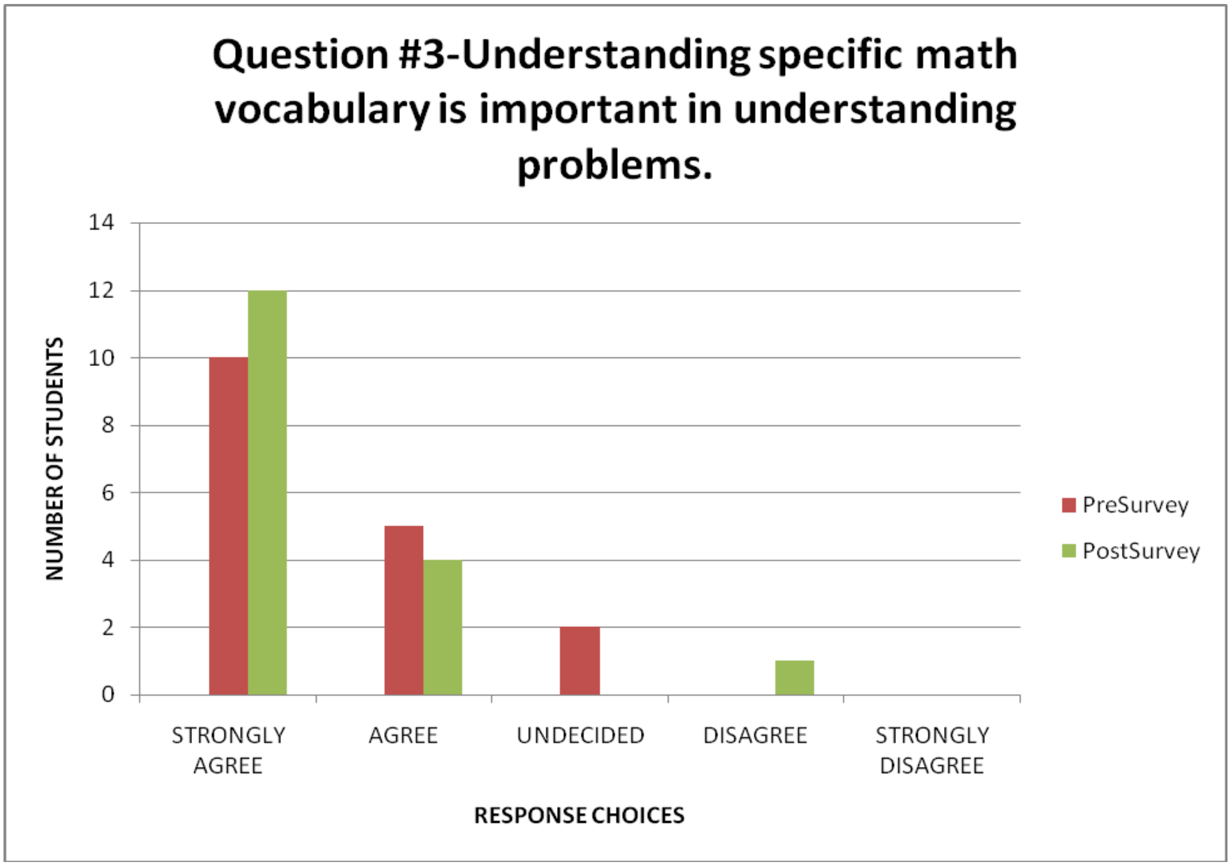


Table 1: Understanding specific math vocabulary is important in understanding problems. The results of this graph suggested that students thought it was important to understand the vocabulary, but it did not necessarily mean they thought it was important to use the vocabulary in written explanations to help them understand. Students were not using vocabulary in their written explanations very well. Just because students had received specific vocabulary instruction did not mean that they would use the vocabulary in their written explanations.

Written Explanations and Mathematical Achievement

Through an increase in written explanations, I was curious to see if my students' understanding of math and problem solving would improve. That led me to my next research question of, *what will happen to students' mathematical achievement through increased practice on written explanations?* I was curious to see if an overall increase in student scores would emerge as I looked across the averages, over time, for the 13 problems I asked students to write up during my research period. The following graph illustrates the data collected for the 13 problems.[1]

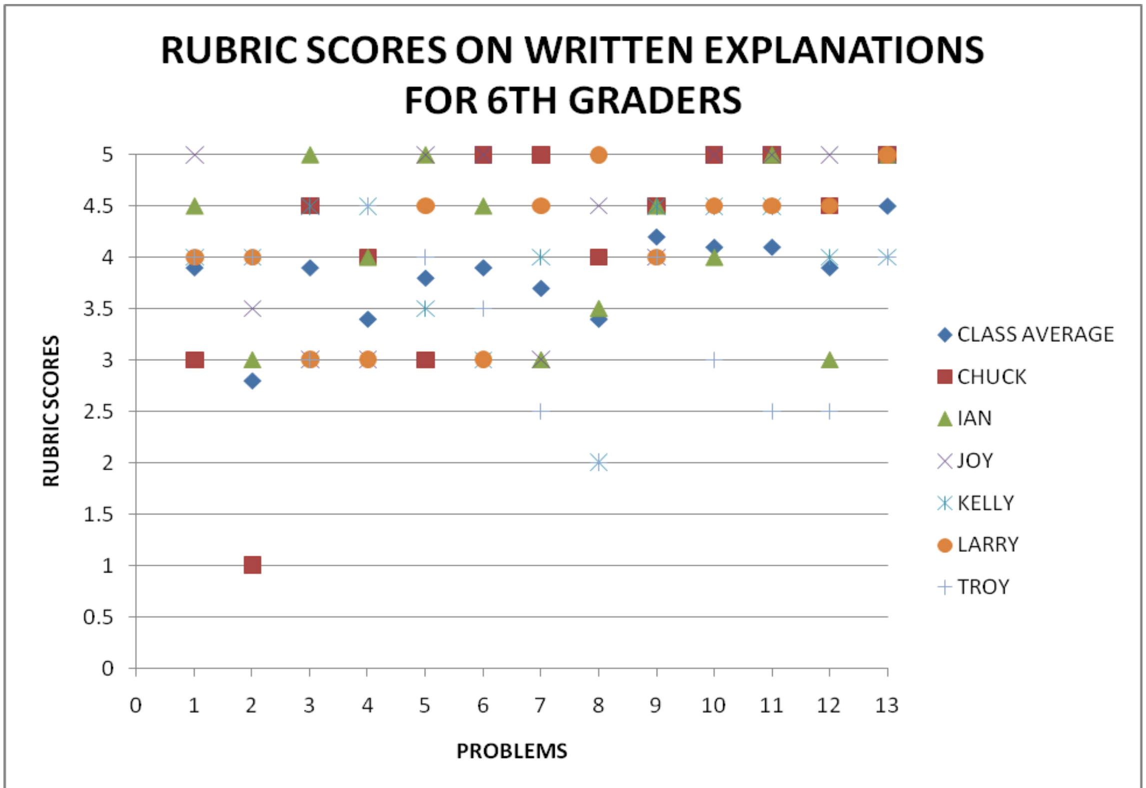


Table 2: Rubric Scores on Written Explanations for 6th Graders

The data showed that the class average for the first problem was considerably higher than the second problem. I found myself giving more hints on the first problem than later on in the research so I made the assertion that was why the scores were higher. Students were frustrated with the first problem not only on having to write a complete explanation, but the problem itself was challenging. The scores on the second problem surprised me as it was similar to problems we had done before in class. The class average seemed to fluctuate more from problems one through eight; the class average on problems nine through 13 were all around four. Not one individual student’s average consistently increased over the 13 problems. Chuck, after the fifth problem, never scored below a

[1] All student names are pseudonyms.

four, and Larry, after the sixth problem never scored below a four. As I looked at the fluctuation of averages, I decided to plot the class averages for the two different types of problems assigned to be written up; there was a difference. Of the 13 problems, six of them were from the student lesson in the Saxon course 1 textbook (Hake, 2007). The other seven were problem-solving problems from the lessons where there had not been previous instruction. The following graph shows the results.

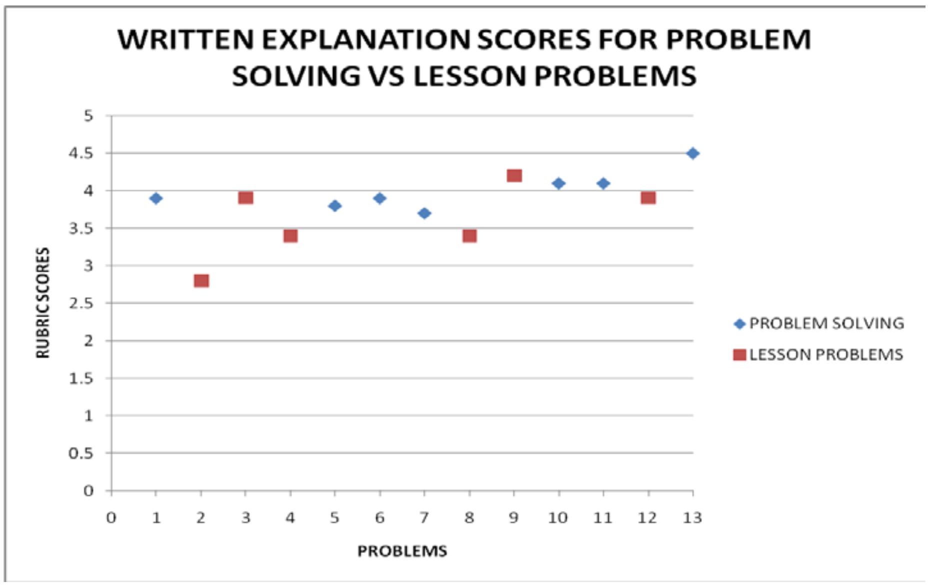


Table 3: Written Explanations for Problem Solving vs Lesson Problems

Overall students had better written explanations on the problem-solving problems than on the problems from the lessons. I would have expected it to be different as students previously had practice and instruction on the lesson problems. This led me to believe that there was little transference of knowledge from the text to a problem-solving assignment. One specific piece of evidence that supported this was one particular problem assigned to students, to find the area and side of a square given the perimeter. This was the eighth problem I assigned during my research project. We had worked on this type of problem in class and had drawn pictures to help “see” the problem. When it came time to write up an explanation, the scores were some of the lowest yet. The average score was 3.4 out of 5. “We even went over the problem later as a class to help students understand it better. The same type of problem came up two lessons later, and 14 out of 60 students missed it" (Personal journal, Week of March 9-13). In the post interview I asked students what types of problems were easier or harder for them to write up. Joy responded:

> Definitely the lesson ones were easier because we had done something like them before and if we didn't remember then we could look in the lesson. But writing up ones we knew were a lot harder because I couldn't find the words. The harder ones were easier to write up the explanations because I talked my way through the steps. (Student interview, April 14, 2009)

Joy's comments were representative of many students. If they already knew how to do the problem, then writing it out step by step was difficult for my students, because it was "automatic." Joy's perception was the problem itself was easier, but the write up was more difficult. I found it interesting that even though average class scores did not show steady improvement, the majority of students said they strongly agreed or agreed that writing out explanations helped them understand math problems better. This is shown in the survey below.

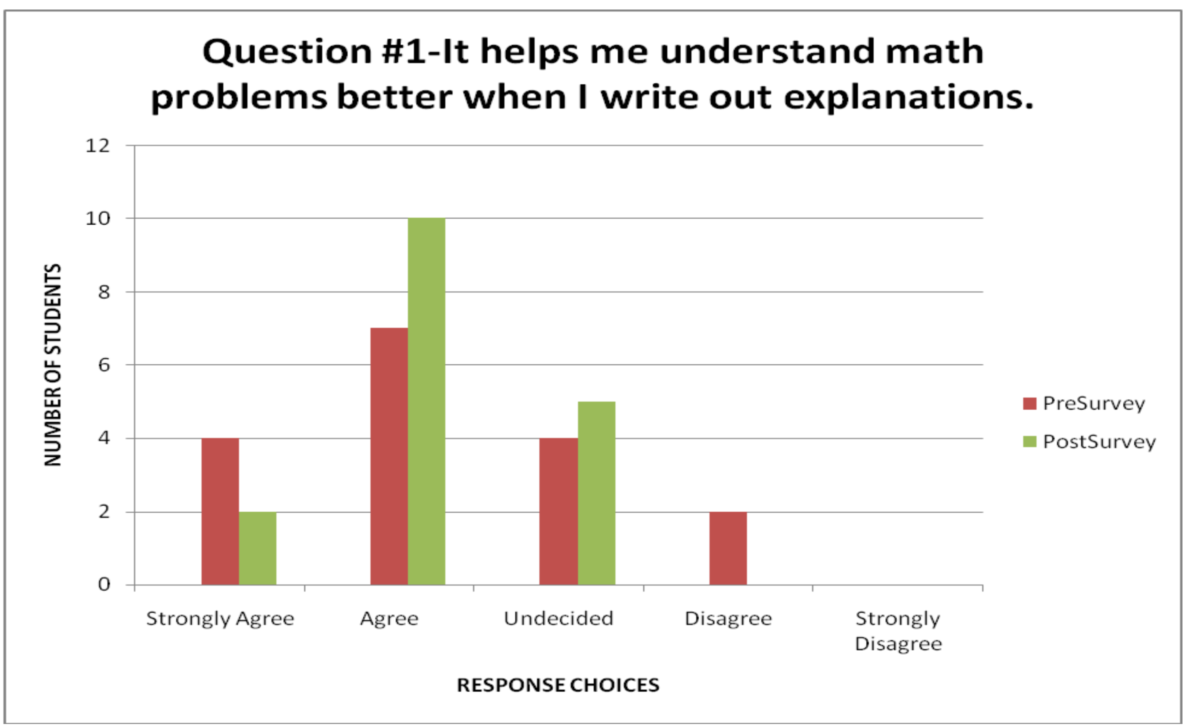
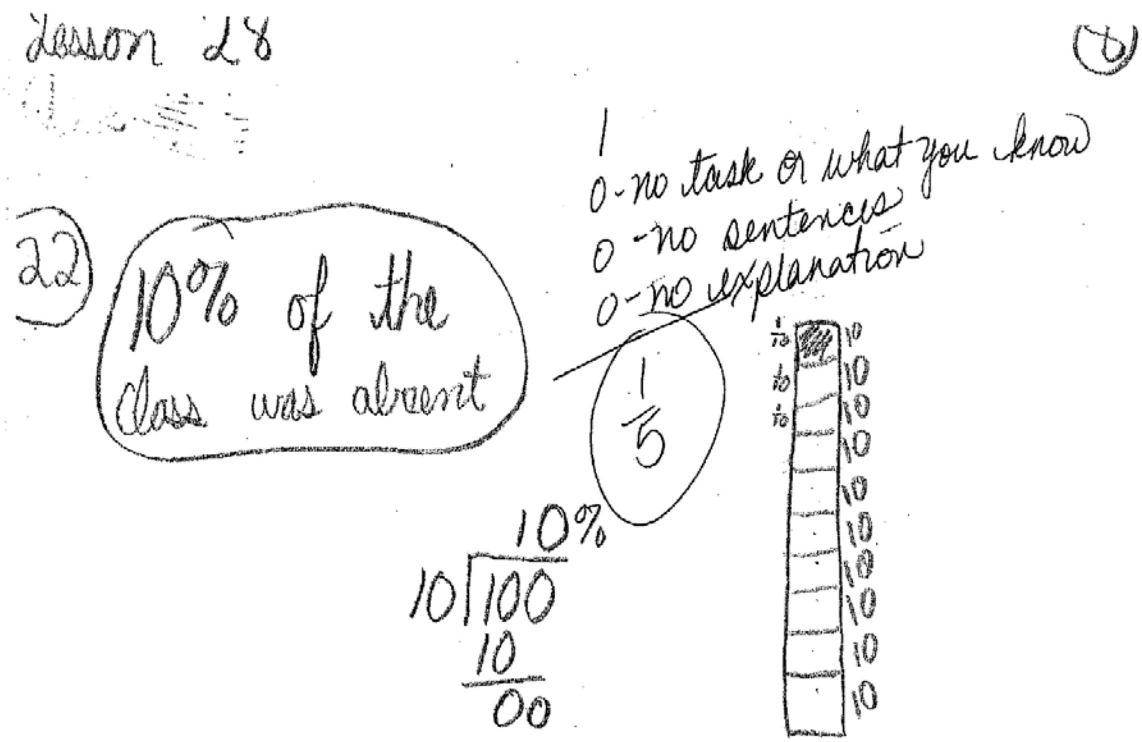


Table 4: It helps me understand math problems better when I write out explanations.

Eleven out of 17 students on the pre-survey strongly agreed or agreed that writing out explanations to math problems helped them understand, while four were undecided and two disagreed. No one strongly disagreed. According to the post-survey, 12 out of 17 students strongly agreed or agreed that written explanations in math helped them understand better. Five students were undecided, and no one disagreed or strongly disagreed in the post-survey. This told me that the majority of students felt that writing out explanations to math problems helped them understand better. Even though the average scores may not show overall growth for the whole class, there were definitely some students who improved their written explanations for problems. Larry scored three out of five points on problems three, four, and five, but scored four and above on problems seven through 13. Chuck scored low on the first two written explanations (i.e., a three out of five and a one out of five), but he never scored lower than a four out of five on problems six through 13. See scatter plot on rubric scores for student written explanations on Table 2. Chuck's work on problem number two is below.



During the student post-interview on April 14, 2009, I asked Chuck if his attitude had changed about having to explain a math problem this year. He responded, “At first, it was really boring and hard, but now I like it. It is fun. I feel like I have improved.” He also stated that he became more willing to draw pictures and use diagrams to help solve the story problems.

On the post-survey question, *I like to write out explanations to mathematical problems,* six students strongly agreed or agreed that they liked to write out explanations to mathematical problems, while four disagreed and no one strongly disagreed.

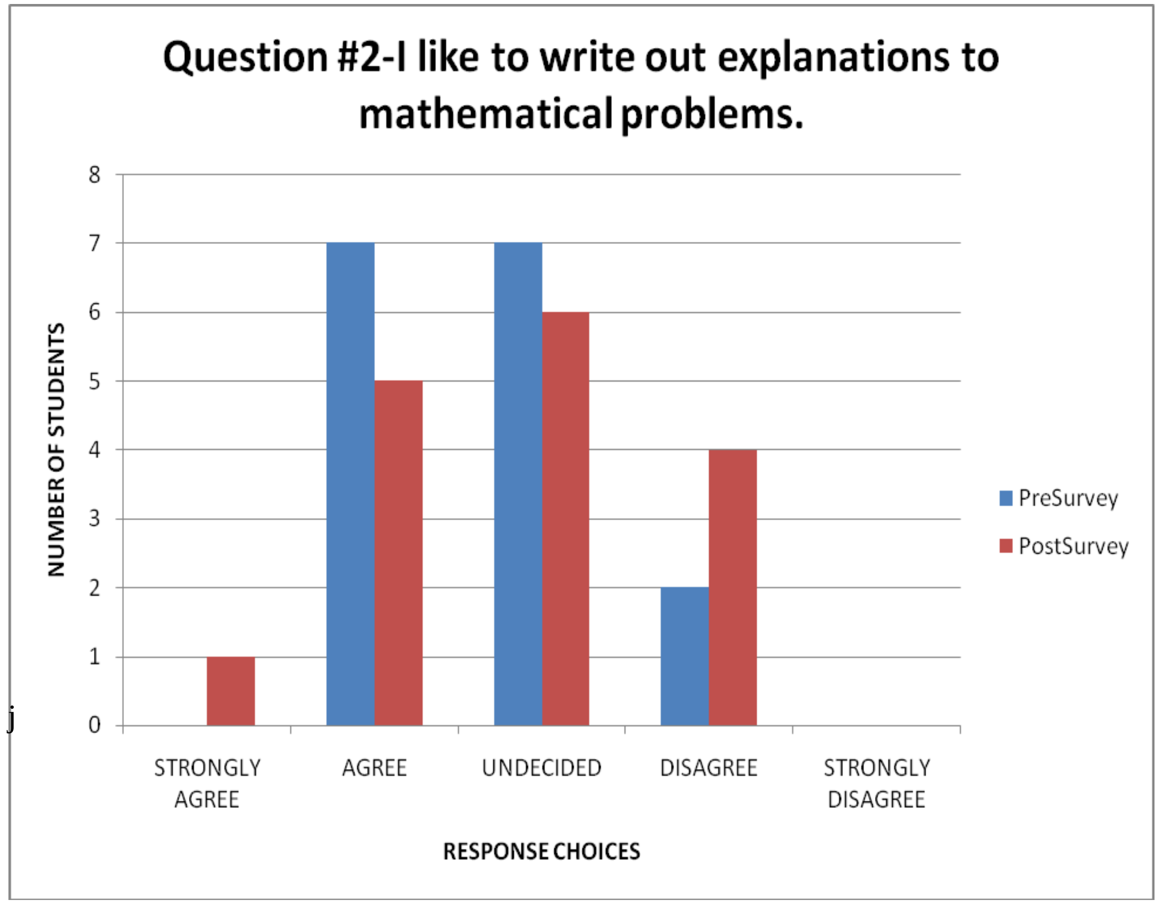


Table 5: I like to write out explanations to mathematical problems.

This data suggested that after working on written explanations during my research project, the attitude about writing explanations changed for some students. In the pre-survey, seven students agreed, and said they liked to write out explanations to mathematical problems, and only two disagreed. The post-survey showed that six strongly agreed or agreed, and four disagreed. This concerned me that after spending time writing up explanations more students disliked writing up explanations. I had noted in my teacher journal, “For some students, I feel the writing up of the problems is souring them on the math” (Personal Journal, Week of March 9-13). In my teacher journal, I had more evidence that some students were frustrated with the written explanations. “One of the special education paraprofessionals told me one of the students said to her, ‘This is math.

Why to we have to write in sentences?' I am afraid that I am turning off some students to math because of the writing part of it" (Personal Journal, Week of March 30-April 3).

I was also surprised over the course of the research project how the attitude about written explanations improved for some students. In the second interview I asked, "Has your attitude changed about having to explain a math problem this year?" Joy responded, "Yeah, at the beginning I was really bad at them but now I am getting better. I am probably more positive because I didn't want to do them but now they are alright." When Troy answered the same question, he said, "Yeah, (i.e. attitude change), more positive because I didn't used to want to write out a lot but now it's a lot easier" (Student interview, April 14, 2009).

Increased Expectations and Mathematics Teaching

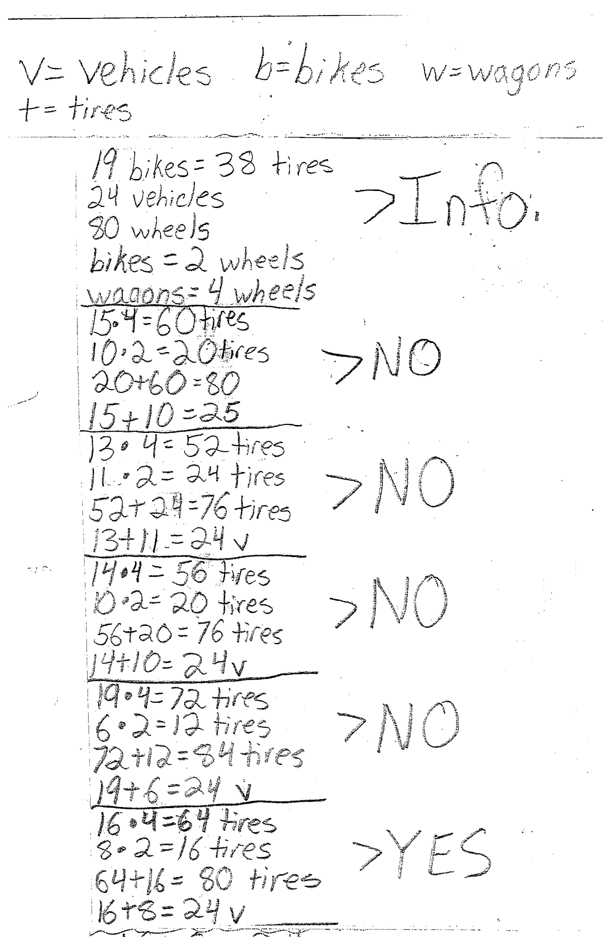
I found that my teaching changed during my research project. My third research question was, *what happens to my mathematics teaching when I increase expectations for the quality of student written explanations, including the correct usage of precise mathematical vocabulary?* I discovered that when I increased the expectations for the quality of student written explanations, students increased the expectations of themselves and developed confidence. When asked in the post-interview how students felt when they were required to give a written explanation Chuck said, "Confident because I just keep reading over it and if I don't get it, I will ask for help." Troy stated, "I feel confident because I have done it more." As Troy worked on a problem one day, I noticed he had set up a table. I asked why he had done this and he said, "It has helped me on other problems so I thought I would try it again." He also asked, "Will we be able to share our solutions? I want to share mine" (Personal Journal, Week of March 23, 2009). He had been a student who had been somewhat turned off to written explanations because he just wanted to give a solution.

Students gained confidence because more of them wanted to share their solutions. Many students wanted to explain their ideas to their classmates even when they had not received the full 5 points on a written explanation. In fact, on one particular day, I had so many volunteers that I had to limit the number sharing.

> On Tuesday, it was time to share some of our ideas for the last two problem solvings. I was amazed at how many wanted to share their ideas or solutions. I had to draw from the can (names on sticks) for three students to share each problem (Personal Journal, Week of April 6, 2009).

During my research project, my teaching became much more deliberate. After a week of writing up explanations, I decided it was time to look at a “strong” solution and a “weak” solution. I decided to share two examples of student write-ups, without sharing the students’ names. I planned to have the class help me use the rubric to score them. I thought this would tell me if students even understood what was on the rubric in the first place. I was amazed at how well the students scored these two problems using the rubric (Appendix C). I also talked about comments I would make under each section and some abbreviations I would use. When I passed back the problem from the previous week, students took the time to check their explanation against the rubric. I then asked students to write an explanation for problem #30 from Lesson #30. I was pleased with what I saw. Many of the students were looking not only at their rubric as they were writing but also at the problem from last week and how it had been scored. I heard one student say, “I need to make sure I write what I know and what I am to find out, cuz I didn’t get any points on that last time” (Personal Journal, Week of February 9). This helped support my claim that students’ level of writing explanations would increase as I increased my expectations.

As I looked back at Chuck's written explanation on problem number two as compared to problem number seven, I was impressed. He scored only a 1 on problem number two but a 5 on problem number seven. Out of the four sixth-grade math classes, I considered his write-up as one of the best for problem number seven. He had improved from no explanation whatsoever on problem number two to a very detailed account of his thinking on problem number seven. He even included a key for his work. Problem number seven was the following: *The playground is filled with bicycles and wagons. If there are 24 vehicles and 80 wheels altogether, how many bicycles are on the playground? How many wagons?* His work for problem number seven is below.



V= vehicles b=bikes w=wagons
t= tires

19 bikes= 38 tires
24 vehicles
80 wheels
bikes = 2 wheels
wagons= 4 wheels
>Info.

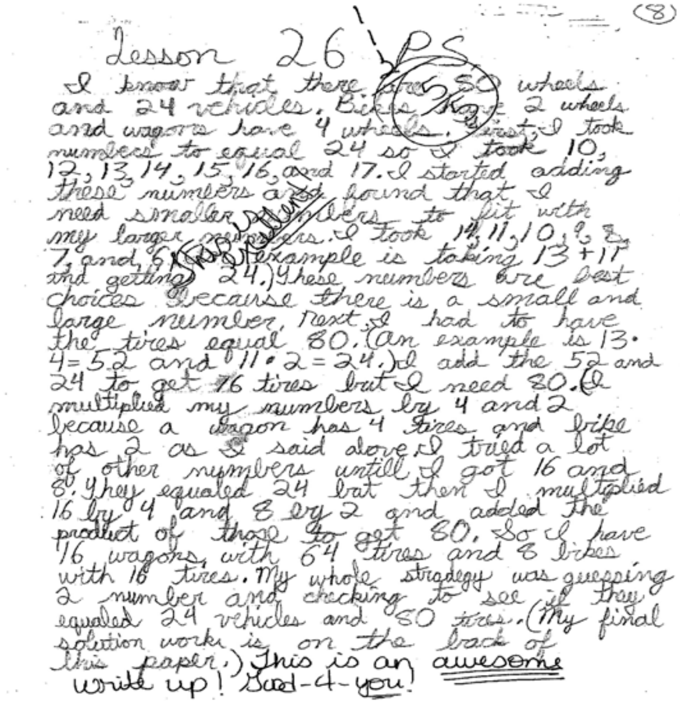
15•4=60 tires
10•2=20 tires
20+60=80
15+10=25
>NO

13•4=52 tires
11•2=24 tires
52+24=76 tires
13+11=24 v
>NO

14•4=56 tires
10•2=20 tires
56+20=76 tires
14+10=24 v
>NO

19•4=72 tires
6•2=12 tires
72+12=84 tires
19+6=24 v
>NO

16•4=64 tires
8•2=16 tires
64+16=80 tires
16+8=24 v
>YES



Lesson 26 PS (8)

I know that there are 80 wheels and 24 vehicles. Bikes have 2 wheels and wagons have 4 wheels. First I took numbers to equal 24 so I took 10, 12, 13, 14, 15, 16, and 17. I started adding these numbers and found that I need smaller numbers to fit with my larger numbers. I took 14, 11, 10, 9, 8, 7, and 6. (An example is taking 13 + 11 and getting 24.) These numbers are best choices because there is a small and large number. Next I had to have the tires equal 80. (An example is 13 • 4 = 52 and 11 • 2 = 24.) I add the 52 and 24 to get 76 tires but I need 80. I multiplied my numbers by 4 and 2 because a wagon has 4 tires and bike has 2 as I said above. I tried a lot of other numbers untill I got 16 and 8. They equaled 24 but then I multiplied 16 by 4 and 8 by 2 and added the product of those to get 80. So I have 16 wagons with 64 tires and 8 bikes with 16 tires. My whole stradegy was guessing a number and checking to see if they equaled 24 vehicles and 80 tires. (My final solution work is on the back of this paper.) This is an awesome write up! Good-4-you!

I was impressed with how Chuck explained his thinking as he worked through the problem. He used a table to keep track of his ideas, even the ones that did not work. He was determined to complete the problem on his own and come up with the solution. Chuck made the following statement around the midway point of my study:

> Okay, would you read through this? I think I am missing something, and I want to get all the points. I made sure I have the task and what I know, and my solution is in a complete sentence. But how about this part? Does it make sense? (Personal journal, Week of February 23, 2009)

By raising my expectations for complete and more precise written explanations, I found students took more pride in their work. One of the highlights with this research action project was with the stair step problem (Appendix B). There were two different pairs, composed of resource students, working on this problem. Both groups did a fabulous job of writing up their explanations. Many of the other students had a hard time stating simply the pattern they found in the table in a way that we could follow and understand. Both of these groups said, however, "It was easy; you just do…" (Personal Journal, Week of February 16-20). Some of my "top" math students were not able to write up the pattern in such a way that I could follow it. Again this supported the idea that if I raised my expectations for all students, many students should show improvements in their written explanations.

Conclusions

After implementing new teaching strategies in the area of written communications in mathematics, I found that it took a lot of time and patience. I discovered that even with specific vocabulary instruction, my students struggled with how to include vocabulary in a meaningful way in their written explanations. Students found it even more difficult to write meaningful explanations when they were more familiar with how to solve a problem in the first place. They could define the vocabulary, yet they could not use it. Draper and Siebert (2004) found that students needed to see the relevance of vocabulary knowledge and usage. I tried to be very deliberate in discussing the appropriate use of vocabulary throughout my research. I found that this did not necessarily carry over to my students' writing, yet I did hear an increase in vocabulary being used in the classroom. Emphasizing vocabulary on a daily basis was something Burton and Morgan (2000) discovered to be pertinent to learning vocabulary, while Draper and Siebert (2004) found that students needed to see the relevance of vocabulary knowledge and usage.

The more students worked on written explanations the easier it became for the majority of them. Having time to read the problem silently and then come up with their own idea on how to solve it before working with a partner was something of which I needed to do more. My students did successfully gather their own thoughts and ideas before sharing with someone else during the times I did allow this individual time. Albert (2000) found that the "self-talk" technique was very beneficial. It was a way for students not only to solve problems with specific steps and procedures but also allowed them to think about what they were doing, how they were doing it, and why they were doing it.

Just as the research done by Shield and Galbraith (1998) did not come to a distinct conclusion about writing improving the understanding of mathematics, neither did mine. I believe it was worth my class time, and students benefited from doing it. Shield and Galbraith agreed that writing is a classroom activity that allows for a deeper understanding of the mathematical concepts students are learning; it can be used for students to reflect and expand their ideas in a way to extend their understanding. However, their research showed little evidence to support that written communication in mathematics increases students' understanding of the material.

What amazed me the most about my research was the fact that when I raised my expectations for the quality of students' written explanations, many students became more concerned and involved in their learning. Students who at the beginning of the research only complained when they were assigned written explanations were asking when the next problem would be. By the end of the semester, there was not a single groan from a sixth grade student from four different classes when I said, "Today we are going to work on problem solving and written explanations." To me that was success in itself.

Implications

Because of this research action project, I plan to devote time to problem solving and written explanations next year. I believe it is a method of communication in mathematics that some students are very successful at even if they are not deemed a "good" math student. Some students who normally would not speak up in class are given the opportunity to shine because of their unusual way of solving problems. As vocabulary usage and understanding has become one of our math department's goals, I will continue to incorporate appropriate vocabulary whenever possible and require the students to do the same.

As I continue with the problem solving, I will adjust my scoring rubric to allow for more partial points for each scoring area and increase the overall point value from a total of 5 to 20 points per problem. Even though I never heard students question me on the low point value assigned to each problem, making the problems worth more points will likely allow students to earn more credit for their effort toward writing a solid explanation. Next year, I want to seek out more challenging problems for students to solve and also try to match the problems up to students' interests. This seems to be a motivator in itself as I discovered with the *die* problem (Appendix B). Whatever the mathematics' problem, I know that requiring students to write in mathematics is an important method for students to communicate their understanding to others.

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Appendix A

Data Collection Pre/Post Survey

1. It helps me understand math problems better when I write out explanations.
   SA A U D DS

2. I like write out explanations to mathematical problems.

   SA A U D DS

3. Understanding specific math vocabulary important in understanding problems.

   SA A U D DS

4. Given the option, would you rather give one written explanation for a problem or do more computation problems? Why?

5. What could teachers do to help students understand math better?

Appendix B

Problems Used

1.)Problem Solving -Lesson 6 –Feb. 2, 2009

Carissa's school library received a gift of 500 new reference books. The books were arranged on a bookcase as shown in the diagram below. How many books are on each shelf?

Shelf 1
Shelf 2
Shelf 3
Shelf 4
Shelf 5

Shelf 1 plus Shelf 2 = 270 books

Shelf 2 plus Shelf 3 = 230 books

Shelf 3 plus Shelf 4 = 180 books

Shelf 4 plus Shelf 5 = 130 books

2.)#22 from Lesson 28-Feb. 3, 2009

If 1/10 of the class was absent, what percent of the class was absent?

3.)#30 from Lesson 29-Feb. 10, 2009

What percent of a circle is 2/5 of a circle? Explain why your answer is correct.

4.)#30 from Lesson 30-Feb. 16, 2009

Four pennies are placed side by side as shown below. The diameter of one penny is ¾ inch. What is the length of the row of pennies?

5.)Problem Solving –Lesson 11-Feb. 19, 2009

Sitha began building stair-step structures with blocks. She used one block for a one-step structure, three blocks for a two-step structure, and six blocks for a three-step structure. She wrote the information in a table. Continue the table through a ten-step structure. After completing the table, do you notice a pattern?

Number of Steps	Total Number of Blocks	Blocks Added to Previous Structure
1	1	N/A
2	3	2
3	6	3

6.)Problem Solving-Lesson 31-2-24-09

Franki has 7 coins in her hand totaling 50 cents. What are the coins?

7.)Problem Solving –Lesson 36:3-10-09

The playground is filled with bicycles and wagons. If there are 24 vehicles and 80 wheels altogether, how many bicycles are on the playground? How many wagons?

8.)#8 from Lesson 34 : 3-13-09

If the perimeter of a square in 24 inches,

a. How long is each side of the square?
b. What is the area of the square?

9.)#30 from Lesson 36-- 3-24-09

Freddy drove a stake into the ground, looped a 12-foot-long rope over it and walked around the stake to mark off a circle. What was the ratio of the radius to the diameter of the circle?

10.)Problem Solving Lesson 14: 3-27-09

It takes the local hardware store 8 seconds to cut through a piece of round galvanized steel pipe. How long will it take to cut a piece of pipe in half? Into quarters? Into six pieces? (Each cut must be perpendicular to the length of the pipe.)

11.)Problem Solving Lesson 3: 3-31-09

Tad picked up a number cube. His thumb and forefinger covered opposite faces. He counted the dots on the other four faces. How many dots did he count?

12.)#28 from Lesson 39 : 4-03-09

Instead of solving the division problem 390 divided by 15, Roosevelt divided both numbers by 3 to form the division 130 divided by 5. Then he multiplied both of those numbers by 2 to get 260 divided by 10. Find all three quotients.

13.)Problem Solving-Lesson 48: 4-06-09

Joseph is making sack lunches. He has two kinds of sandwiches, three kinds of fruit, and two kinds of juice. If each sack lunch contains one kind of sandwich, one kind of fruit, and one kind of juice, how many different sack-lunch combinations can Joseph make?

Appendix C

Problem Solving Rubric

Rubric for scoring written explanations in homework and problem solving

1 point for solution

0 point – for no solution

.5 point – incomplete solution

1 point – complete and correct solution with label

1 point for stating what they know about the problem and the task

0 point – no statement about the problem and the task

.5 point – partial statement, necessary vocabulary words undefined, task not included

1 point – complete statement about the problem in their own words, necessary vocabulary explained, and task defined

1 point for overall sentence structure

0 point – no sentences

.5 point - missing capital letters and punctuation, incomplete sentences, run-on sentences

1 point – Clear, complete sentences

2 points for explaining their work / showing their work

0 point – no work

1 point – some relevant information, computation only, incomplete explanation, missing labels

2 points – thorough explanation, work shown with words, drawings, numbers etc., and complete labeling of work

TOTAL – 5 points

Appendix D

Problem Scoring Rubric Tallies

Problem Solving________________

Date_____________

	0	.5	1	2
SOLUTION				
TASK & WHAT YOU KNOW				
SENTENCE STRUCTURE				
EXPLAINING & SHOWING WORK				

Appendix E

Student Interview questions-*Second interview question additions in italics*

1. What does it look like when you justify answers on a homework assignment?
2. What are the benefits of justifying your answers on your homework assignments if any?
3. Given the option, would you rather give one written explanation for a problem or do more computation problems? Why?
4. When working a word problem do you think you know the meanings of most of the vocabulary words in each problem? Please give some examples.
5. Why is it important to know the meanings of vocabulary words you see in math?
6. Has your attitude changed about having to explain a math problem this year? Why or why not?
7. What makes math easy or difficult for you?
8. Have you ever had a really bad experience with math? If so, what happened?
9. What could/do teachers do to help students in math?
10. Did you enjoy working word problems before this year?
11. What do you like most about math? Least about math?
12. This semester I have changed some of my teaching practices. What advice would you give me about continuing these changes next year?
13. How do you feel when you are required to give a written explanation for a problem?
14. Is there any benefit to writing out an explanation for a problem? If so what are the benefits?
15. *What did we do with problem solving that you liked or that helped you?*
16. *Did it help you at all when I showed an example of a strong and weak write up and as a class we scored them using the rubric?*
17. *Of the 4 parts that you were scored on using the rubric, was there any one part that was harder for you?*
18. *What types of problems were easier of harder for you to write up?*