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A Model for Predicting the Multiscale Crack Growth Due to an Impact in Heterogeneous Viscoelastic Solids

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Abstract
A two-scale model for predicting the multiple crack growth in viscoelastic solids due to an impact is presented. The cracks are considered only at the local scale through the use of a micromechanical viscoelastic cohesive zone model. The multiscale model has been implemented in a finite-element code. In order to minimize the computation time, the local finite-element meshes are solved in parallel by multiple processors. An example problem is given in order to demonstrate the capabilities of the model.

Keywords: multiscale model, heterogeneous viscoelastic media, dynamic/impact loading, fracture

Introduction

A variety of inelastic heterogeneous solids subjected to an impact undergo a significant load-induced energy dissipation due to the development of multiple cracks and/or voids. Indeed, the evolution of new internal boundaries can occur on multiple length and time scales, and often takes place without the loss of structural integrity, due to the load redistribution and material inhomogeneities. Although the predictions of such events may seem to be very costly and complex, there are many structural applications where an effective model would save a considerable expense. Such examples include the geologic salt, sea ice, polymer composites, tank armor, and plastic ballistic explosives. In all of these materials, experimentally based design methodologies are extremely costly, therefore suggesting the need for improved models.

In inelastic media, the problem of crack growth is particularly complicated by the fact that there are at least two competing energy mechanisms that occur essentially simultaneously whenever one or more cracks run: the bulk material dissipation and the fracture energy. Thus, it can be quite difficult to separate the various sources of energy dissipation by experimental methods. Furthermore, due to material heterogeneities that can occur on multiple length scales, cracks can coexist over a broad range of length scales simultaneously, and the energy dissipated on these differing length scales can nevertheless be of the same order of magnitude. Finally, since the material viscoelasticity by its very nature implies that there are governing time scales in the material behavior, widely differing fracture events can be observed on differing time scales.

Multiscale algorithms for problems of this type have until recently been untenable. This is due to the fact that the problem must be solved recursively on any and all length scales and at all locations in which the material microstructure is evolving in time. However, due to the developments mentioned above, as well as the vast

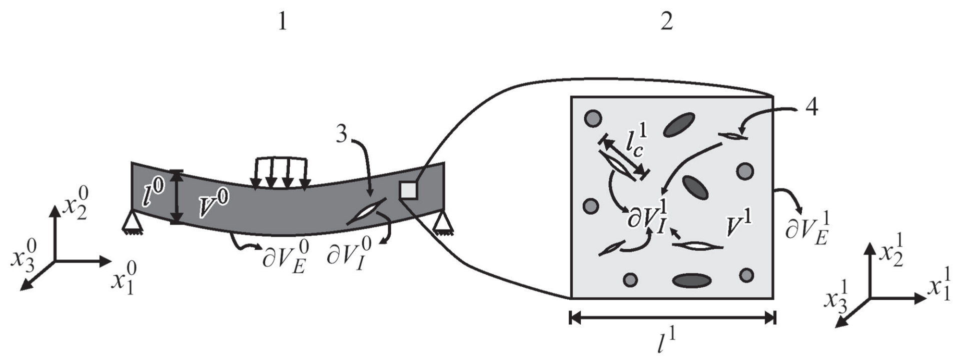

Figure 1. Description of a two-scale problem: 1) global scale, 2) local scale, 3) macrocrack, and 4) microcrack.

improvement in computing power of the last decade, it is now possible to solve two-dimensional problems on two, three, and even four length scales by utilizing a desktop computer, single- or multi-processor. Similarly, solutions in three dimensions are possible on multiprocessor computers. These solutions are obtained by using standard time marching multiscale algorithms, linked by appropriate homogenization theorems [1-6], to solve problems that permit the prediction of the evolution of hundreds, even thousands of cracks simultaneously. In this paper, an example solution for a viscoelastic media with local cracks is given in order to demonstrate the capabilities of the model.

Multiscale Modeling

Multiscale modeling relies generally on the assumption that the global constitutive behavior of heterogeneous materials can be determined simultaneously throughout an analysis based on the behavior of the constituents and their interactions at the local scale. This approach is especially advantageous in problems with evolving microstructure due to the formation and growth of cracks, since the evolution of the microstructure is necessarily both space- and time-dependent. A more detailed description of multiscale models can be found in [7-12].

Consider a structural part which is statistically homogeneous at the global scale but microscopically heterogeneous, as shown in Figure 1, where the microstructure may contain inclusions as well as growing cracks.

The superscripts refer to a scale index, where 0 refers to the global scale and 1 refers to the local scale; V^{μ}, ∂V_E^{μ}, and ∂V_I^{μ} are the volume and the external and internal boundary surfaces of a scale μ, respectively; l^{μ} is the length scale associated with the scale μ; l_c^{μ} is the length scale associated with the cracks at the scale μ. The global-scale initial boundary-value problem (IBVP) can be posed as follows:

- conservation of the linear momentum

$$\sigma^0_{ji,j} + \rho^0 b^0_i = \rho^0 \frac{d^2 u^0_i}{dt^2} \quad \text{in } V^0 ,$$

where σ^0_{ji} is the Cauchy stress tensor, ρ^0 is the mass density of the statistically homogeneous object, b^0_i is the body force vector per unit mass, u^0_i is the displacement vector, and V^0 is the volume of the object at the global length scale;

- conservation of the angular momentum

$$\sigma^0_{ij} = \sigma^0_{ji} \quad \text{in } V^0 \;;$$

- small strain–displacement relation

$$\varepsilon^0_{ij} = ½\,(u^0_{i,j} + u^0_{j,i}) \quad \text{in } V^0,$$

where ε^0_{ij} is the infinitesimal strain tensor defined on the global length scale;

- constitutive equations

$$\sigma^0_{ij}(t) = \Omega^{\tau=t}_{\tau=-\infty}\{\varepsilon^0_{ij}(\tau)\} \quad \text{in } V^0,$$

where $\Omega^{\tau=t}_{\tau=-\infty}$ is a functional mapping that accounts for history-dependent effects, such as the viscoelasticity, and is determined by locally averaging the response at the local scale.

Now, considering that continuum mechanics is still valid at the microscale, assuming that the global length scale is much larger than the local one, $l^0 \gg l^1$, that the length scale associated with cracks at the local scale, l_c^1, is much smaller than the local length scale, $l^1 \gg l_c^1$, that these cracks are homogeneously distributed at the local scale, and finally, that the length of the wave propagating on the global scale, l_ω^0, is much larger than the local scale length, $l_\omega^0 \gg l^1$, the local IBVP can be approximated as a quasi-static problem given by:

- conservation of the linear momentum

$$\sigma^1_{ji,j} + \rho^1 b^1_i = 0 \quad \text{in } V^1,$$

- conservation of the angular momentum

$$\sigma^1_{ij} = \sigma^1_{ji} \quad \text{in } V^1 \;;$$

- small strain–displacement relation

$$\varepsilon^1_{ij} = ½\,(u^1_{i,j} + u^1_{j,i}) \quad \text{in } V^1,$$

- constitutive equations

$$\sigma^1_{ij}(t) = \Omega^{\tau=t}_{\tau=-\infty}\{\varepsilon^1_{ij}(\tau)\} \quad \text{in } V^1, \tag{1}$$

where, in the case of the local scale, it is assumed that $\Omega^{\tau=t}_{\tau=-\infty}$ is known a priori for all constituents;

- fracture criterion

$$G^1_i = G^1_{ci} \quad \text{in } V^1.$$

where G^1_i is the fracture energy release rate at the local scale, G^1_{ci} is the critical energy release rate of the material, and the index *i* refers to the mode of fracture.

Homogenization of Local Fields

The homogenization principles can now be used to establish the relationships connecting both length scales. If all the assumptions herein made hold, it can be shown that

$$\varepsilon^0_{ij} = \frac{1}{V^1}\int_{\partial V^1_E} ½\,(u^1_i n^1_j + u^1_j n^1_i)\, dS\,,$$

where ∂V^1_E is the external boundary of the local length scale, and n^1_i is the outward unit normal vector to this external boundary surface;

$$\varepsilon^0_{ij} = \bar{\varepsilon}^1_{ij} = -\alpha^0_{ij},$$

where $\bar{\varepsilon}^1_{ij}$ is the volume-averaged strain at the local length scale, and α^0_{ij} is the internal boundary average of displacements at the local length scale, which are given by

$$\bar{\varepsilon}^1_{ij} = \frac{1}{V^1}\int_{V^1} \varepsilon^1_{ij}\, dV, \quad \alpha^0_{ij} = \frac{1}{V^1}\int_{\partial V^1_i} \tfrac{1}{2}(u^1_i n^1_j + u^1_j n^1_i)\, dS,$$

where ∂V^1_i is the internal boundary of the local length scale, and n^1_i is the outward unit normal vector to this internal boundary surface.

Now consider the following mathematical expansion for the global length-scale stresses in terms of the local stresses:

$$\sigma^0_{ij} = \bar{\sigma}^1_{ij} = \frac{1}{V^1}\int_{V^1} \sigma^1_{ij}\, dV = \frac{1}{V^1}\int_{\partial V^1_E} \sigma^1_{ki}\, n^1_k x^1_j\, dS, \tag{2}$$

where $\bar{\sigma}^1_{ij}$ is the volume-averaged stress at the local scale, and the right-hand side of the previous equation is the external boundary-averaged stress.

Similarly,

$$\rho^0 = \lim_{l^1/l^0 \to 0} (\rho^0) = \bar{\rho}^1 = \frac{1}{V^1}\int_{V^1} \rho^1\, dV.$$

Finally, the constitutive relationship at the global length scale can be found by direct substitution of the local-scale constitutive equation, Equation (1), into the volume-average of stresses, Equation (2). But there is no need to determine the global-scale constitutive relation a priori, since it is determined concurrently as the multiscale analysis is performed.

Crack Modeling at the Local Scale

The crack propagation is herein modeled by using the micromechanical viscoelastic cohesive zone model developed by Allen and Searcy [13, 14], which is formulated in such a way that the material parameters required to characterize the model can be obtained directly from microscale experiments. Briefly, the cohesive zone is postulated to be represented by a fibrillated zone that is small compared to the total cohesive zone area. The length scale of this IBVP is one length scale be low that of the smallest local scale required in the multiscale problem. The solution to this IBVP has been obtained and homogenized [13], thus leading to the following traction–displacement relation in the cohesive zone:

$$T^1_i = [\,1 - \alpha(t)]\frac{1}{\lambda^1} \cdot \frac{\delta^1_i}{\delta^{*1}_i}\left[T^f_i + \int_0^t E^1_c\,(t-\tau)\,\frac{\partial\lambda^1}{\partial\tau}\, d\tau \right] \text{ on } \partial V^1_{Ic},$$

where $E^1_c(t)$ is the uniaxial viscoelastic relaxation modulus of the undamaged cohesive zone material, $\partial V_{Ic}{}^1$ is the part of the internal boundary on which cohesive zones are active, δ^1_i is the crack opening displacement vector in the coordinate system aligned with crack faces, $\delta^{*}{}^1_i$ is a material length parameter, λ^1 is the Euclidean norm of the crack opening displacement vector, T^f_i is the value of traction at which the cohesive zone initiates, and $\alpha(t)$ is the damage parameter, defined by

$$\alpha(t) \equiv \frac{A_0 - \sum_{k=1}^{nf} A_k(t)}{A_0},$$

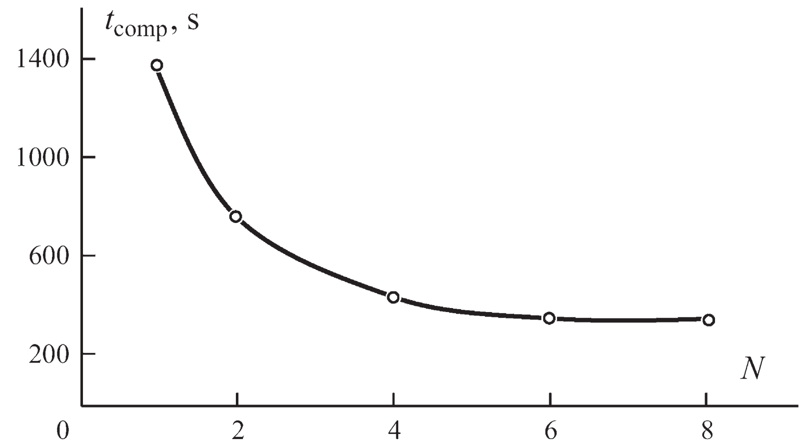

Figure 2. Computation time t_{comp} in relation to the number N of processors.

where A_0 is the undamaged planform cross-sectional area of a representative area of the cohesive zone, $A_k(t)$ represents the cross-sectional area of a k-th fibril, and nf is the number of fibrils contained in the representative area.

Although the evolution of $\alpha(t)$ can be determined experimentally, in the current paper, it is assumed to follow the phenomenological law [13, 15] given by

$$\dot{\alpha} = A[\lambda(t)]^m, \quad \text{if} \quad \dot{\lambda} > 0 \;\text{ and }\; \alpha < 1,$$

$$\dot{\alpha} = 0, \quad \text{if} \quad \dot{\lambda} \leq 0 \;\text{ or }\; \alpha = 1,$$

where A and m are the microscale phenomenological material constants.

The computational model presented here uses an algorithm to automatically insert cohesive zone elements into the finite-element mesh at the instant of time at which the criterion for cohesive zone initiation is satisfied, say $T_i^1(t) \geq T_i^f$. This minimizes the maximum band width of the stiffness matrix, as opposed to the meshes with cohesive zone elements embedded a priori. The Lagrange multipliers are also used to enforce contact conditions at the cohesive zones, avoiding interpenetration. Further details of this approach may be found in [16].

Multiscale Algorithm

The multiscale model described here has been implemented based on the finite-element method (FEM). In this case, the global and local length scales are solved by means of explicit and implicit quasi-static FEM solution schemes, respectively. It should be noted that parallel programming can significantly reduce the computation time necessary for the solution of multiscale problems. The concept of parallel programming is extremely suitable for multiscale algorithms in which the local scales can be solved simultaneously by different processors and independent of each other. The current computational algorithm developed by the authors is now parallelized using the Open MPI libraries [17]. Figure 2 shows the reduction in the computation time t_{comp} due to the use of multiple processors for a test case consisting of a global mesh of 374 degrees of freedom and 120 local meshes of 186 degrees of freedom, where an automatic insertion of cohesive zones is allowed in the local scale. A problem consisting of 500 time steps was solved in a Dell workstation with eight processors at 3.00 GHz and 32 GB RAM memory running on Red Hat Linux 5.0. Note that, as the number of processors is increased, the time spent on message passing increases, and the number of local problems per processor decreases, so that, for a given problem, there will be an optimum number of processors for which there will be no gain if more processors are used, as shown in Figure 2

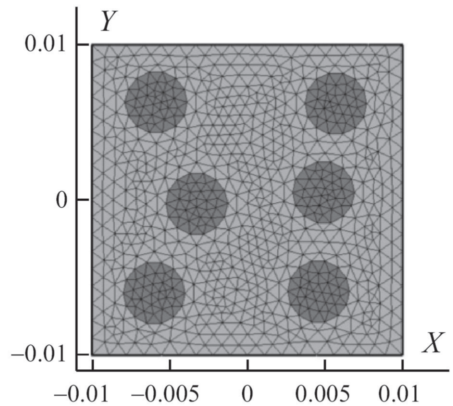

Figure 3. Geometry and the finite-element mesh of a local-scale unit cell.

Table 1. Material Properties

Bulk materials	E, Pa	ν	ρ, kg/m^3	i	E_i , Pa	ρ_i , s
Fiber	550	0.30	30,000	∞	350	–
Matrix		0.30	30,000	1	50	1×10^{-2}
				2	50	1×10^{-1}
				3	50	1×10^{0}
				4	50	1×10^{1}
				5	50	1×10^{2}
				6	50	1×10^{3}

Table 2. Properties of Materials in the Cohesive Zone

Interface	δ_n mm	δ_t mm	T_n^f Pa	T_t^f Pa	A	M
Matrix-matrix	0.001	0.001	35	1000	2.5	1.5
Fiber-matrix	0.001	0.001	20	200	5.0	1.5

Note: The viscoelastic properties are the same as of the bulk matrix.

Example Problem

The example problem considered consists of an impact of an elastic cylinder on a heterogeneous viscoelastic plate. The geometry of the local scale used for this example problem is shown in Figure 3, where the units are given in mm. The fiber is assumed elastic, but the matrix linear viscoelastic. The material properties used are given in Table 1. It is also assumed that the viscoelastic relaxation moduli can be expressed in the form of a Prony series.

Impact of a Cylinder on a Plate

This problem consists of the impact of a homogeneous elastic cylinder (1.0 cm in diameter, Young's modulus of 500 MPa, Poisson's ratio of 0.35, and mass density of 80,000 kg/m^3) on a rectangular (4.0 × 1.0 cm)

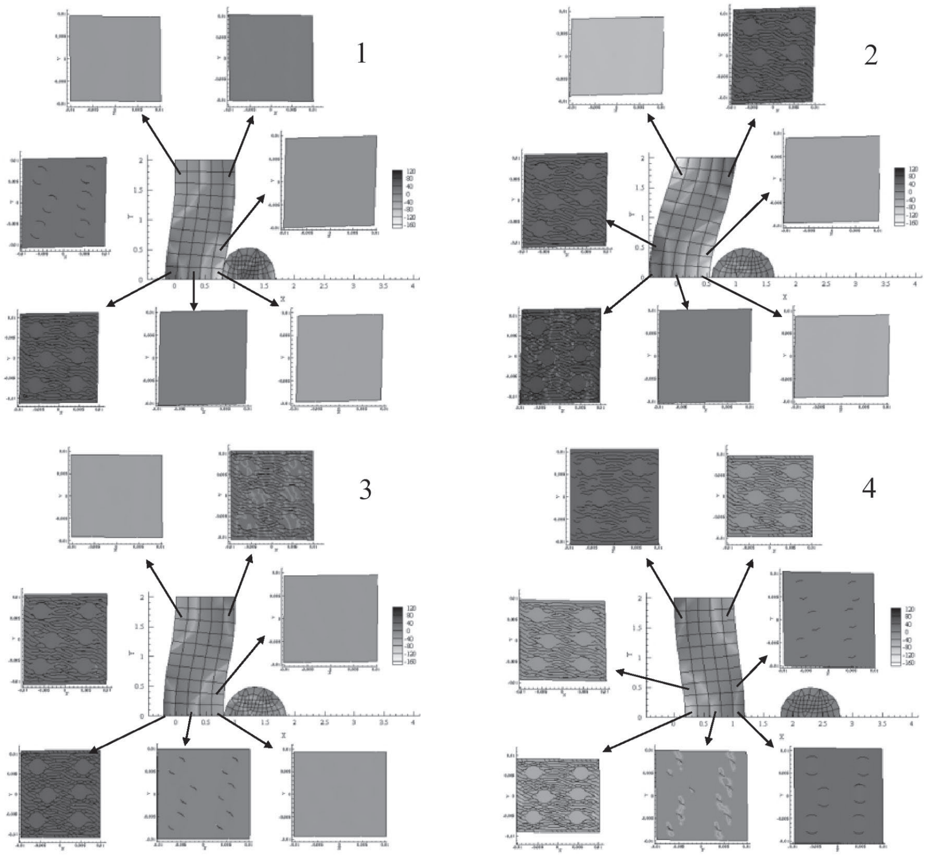

Figure 4. Snapshots of global and selected local meshes for the viscoelastic impact problem with cracks at the local scale: t = 1.20 (1), 1.57 (2), 2.00 (3), and 2.65 s (4).

heterogeneous plate. Plane stress conditions were assumed, and the symmetry of the problem was taken into account to minimize the computation effort. The initial speed of the elastic cylinder was 2 cm/s. A local mesh (Figure 3) was attached to every integration point of this global mesh, totaling 200 local meshes. The Lagrange multipliers were also used to enforce noninterpenetrating frictionless contact conditions between the plate and the cylinder.

Figure 4 presents snap shots of the global and selected local meshes at different times. It can be observed that the multiscale model presented is able to predict the space-dependent (with respect to the global position) damage evolution at the local scale. As a consequence, a damage-induced anisotropy was produced at the local scale due to orientation of the cracks.

It is important to notice that the bulk viscoelastic behavior of the matrix, as well as of the viscoelastic cohesive zones, dissipate a considerable amount of energy, so that, in the regions of the plate that experience late tensile waves (the front of the plate), few internal boundaries are formed, as can be observed in Figure 4. This is a desirable feature of the computational models intended for the design of viscoelastic impact devices.

It is also important to note that some of the internal boundaries shown in Figure 4 are not cracks, but the cohesive zones formed, which will become cracks when the damage parameter $\alpha(t)$ reaches unity.

Conclusions

This paper presented a multiscale model for predicting the performance of heterogeneous viscoelastic materials under an impact loading. Automatically inserted cohesive zone elements are used in order to model the evolution of microcracks in the material. The material viscoelasticity is another source of energy dissipation accounted for in the model. A multiscale finite-element code has been developed in order to allow the modeling of complex problems that have no available analytic solution.

Some of the capabilities of the model have been demonstrated by an example problem on a cylinder/plate impact, for which multiple energy dissipation mechanisms have been considered, i.e., the bulk and cohesive zone viscoelastic dissipation, as well as the fracture energy. Besides the fact that multiscale models can account for multiple sources of energy dissipation on the local scale, it has other important features in the design of composite materials, such as the fact that material characterization is needed only at the local scale and that important design variables, such as the volume fractions of constituents and the particle orientation, have been incorporated into the model.

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