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Energy Efficiency of Fixed-Rate Wireless Transmissions under Queueing Constraints and Channel Uncertainty

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Abstract—¹ Energy efficiency of fixed-rate transmissions is studied in the presence of queueing constraints and channel uncertainty. It is assumed that neither the transmitter nor the receiver has channel side information prior to transmission. The channel coefficients are estimated at the receiver via minimum mean-square-error (MMSE) estimation with the aid of training symbols. It is further assumed that the system operates under statistical queueing constraints in the form of limitations on buffer violation probabilities. The optimal fraction of power allocated to training is identified. Spectral efficiency–bit energy tradeoff is analyzed in the low-power and wideband regimes by employing the effective capacity formulation. In particular, it is shown that the bit energy increases without bound in the low-power regime as the average power vanishes. On the other hand, it is proven that if sparse multipath fading with bounded number of independent resolvable paths is experienced, the bit energy diminishes to its minimum value in the wideband regime as the available bandwidth increases. For this case, expressions for the minimum bit energy and wideband slope are derived. Overall, energy costs of channel uncertainty and queueing constraints are identified.

I. INTRODUCTION

The two key characteristics of wireless communications that most greatly impact system design and performance are 1) the randomly-varying channel conditions and 2) limited energy resources. In wireless systems, the power of the received signal fluctuates randomly over time due to mobility, changing environment, and multipath fading. These random changes in the received signal strength lead to variations in the instantaneous data rates that can be supported by the channel [1]. In addition, mobile wireless systems can only be equipped with limited energy resources, and hence energy efficient operation is a crucial requirement in most cases.

To measure and compare the energy efficiencies of different systems and transmission schemes, one can choose as a metric the energy required to reliably send one bit of information. Information-theoretic studies show that energy-per-bit requirement is generally minimized, and hence the energy efficiency is maximized, if the system operates at low signal-to-noise ratio (SNR) levels and hence in the low-power or wideband regimes. Recently, Verdú in [2] has determined the minimum bit energy required for reliable communication over a general class of channels, and studied of the spectral efficiency–bit

energy tradeoff in the wideband regime while also providing novel tools that are useful for analysis at low SNRs.

In many wireless communication systems, in addition to energy-efficient operation, satisfying certain quality of service (QoS) requirements is of paramount importance in providing acceptable performance and quality. For instance, in voice over IP (VoIP), interactive-video (e.g., videoconferencing), and streaming-video applications in wireless systems, latency is a key QoS metric and should not exceed certain levels [3]. On the other hand, wireless channels, as described above, are characterized by random changes in the channel, and such volatile conditions present significant challenges in providing QoS guarantees. In most cases, statistical, rather than deterministic, QoS assurances can be given.

In summary, it is vital for an important class of wireless systems to operate efficiently while also satisfying QoS requirements (e.g., latency, buffer violation probability). Information theory provides the ultimate performance limits and identifies the most efficient use of resources. However, information-theoretic studies and Shannon capacity formulation generally do not address delay and quality of service (QoS) constraints [4]. Recently, Wu and Negi in [5] defined the effective capacity as the maximum constant arrival rate that a given time-varying service process can support while providing statistical QoS guarantees. Effective capacity formulation uses the large deviations theory and incorporates the statistical queueing constraints by capturing the rate of decay of the buffer occupancy probability for large queue lengths. The analysis and application of effective capacity in various settings have attracted much interest recently (see e.g., [5]–[9] and references therein).

In this paper, we study the energy efficiency in the presence of queueing constraints and channel uncertainty. We assume that the channel is not known by the transmitter and receiver prior to transmission, and is estimated imperfectly by the receiver through training. In our model, we incorporate statistical queueing constraints by employing the effective capacity formulation which provides the maximum throughput under limitations on buffer violation probabilities for large buffer sizes. Since the transmitter is assumed to not know the channel, fixed-rate transmission is considered.

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II. SYSTEM MODEL

We consider a point-to-point wireless link. It is assumed that the source generates data sequences which are divided into frames of duration T . These data frames are initially stored in the buffer before they are transmitted over the wireless channel. The discrete-time channel input-output relation in the i^{th} symbol duration is given by

$$y[i] = h[i]x[i] + n[i] \quad i = 1, 2, \dots \quad (1)$$

where $x[i]$ and $y[i]$ denote the complex-valued channel input and output, respectively. We assume that the bandwidth available in the system is B and the channel input is subject to the following average energy constraint: $\mathbb{E}\{|x[i]|^2\} \leq \bar{P}/B$ for all i . Since the bandwidth is B , symbol rate is assumed to be B complex symbols per second, indicating that the average power of the system is constrained by \bar{P} . Above in (1), $n[i]$ is a zero-mean, circularly symmetric, complex Gaussian random variable with variance $\mathbb{E}\{|n[i]|^2\} = N_0$. The additive Gaussian noise samples $\{n[i]\}$ are assumed to form an independent and identically distributed (i.i.d.) sequence. Finally, $h[i]$, which denotes the channel fading coefficient, is assumed to be a zero-mean Gaussian random variable with variance $E\{|h|^2\} = \gamma$. We further assume that the fading coefficients stay constant during the frame duration of T seconds and have independent realizations for each frame. Hence, we basically consider a block-fading channel model.

The system operates in two phases: training phase and data transmission phase. In the training phase, known pilot symbols are transmitted to enable the receiver to estimate the channel conditions, albeit imperfectly. We assume that minimum mean-square-error (MMSE) estimation is employed at the receiver to estimate the channel coefficient $h[i]$. Since the MMSE estimate depends only on the training energy and not on the training duration [12], it can be easily seen that transmission of a single pilot at every T seconds is optimal. Note that in every frame duration of T seconds, we have TB symbols and the overall available energy is $\bar{P}T$. We now assume that each frame consists of a pilot symbol and $TB - 1$ data symbols. The energies of the pilot and data symbols are

$$\mathcal{E}_t = \rho \bar{P}T, \quad \text{and} \quad \mathcal{E}_s = \frac{(1 - \rho)\bar{P}T}{TB - 1}, \quad (2)$$

respectively, where ρ is the fraction of total energy allocated to training. Note that the data symbol energy \mathcal{E}_s is obtained by uniformly allocating the remaining energy among the data symbols.

In the training phase, the receiver obtains the MMSE estimate \hat{h} which is a circularly symmetric, complex, Gaussian random variable with mean zero and variance $\sigma_{\hat{h}}^2 = \mathbb{E}\{|\hat{h}|^2\} = \frac{\gamma^2 \mathcal{E}_t}{\gamma \mathcal{E}_t + N_0}$, i.e., $\hat{h} \sim \mathcal{CN}\left(0, \frac{\gamma^2 \mathcal{E}_t}{\gamma \mathcal{E}_t + N_0}\right)$ [13]. Now, the channel fading coefficient h can be expressed as $h = \hat{h} + \tilde{h}$ where \tilde{h} is the estimate error and $\tilde{h} \sim \mathcal{CN}\left(0, \frac{\gamma N_0}{\gamma \mathcal{E}_t + N_0}\right)$. Consequently, in the data transmission phase, the channel input-output relation becomes

$$y[i] = \hat{h}[i]x[i] + \tilde{h}[i]x[i] + n[i] \quad i = 1, 2, \dots \quad (3)$$

Since finding the capacity of the channel in (3) is a difficult task², a capacity lower bound is generally obtained by considering the estimate error \tilde{h} as another source of Gaussian noise and treating $\tilde{h}[i]x[i] + n[i]$ as Gaussian distributed noise uncorrelated from the input. Now, the new noise variance is $\mathbb{E}\{|\tilde{h}[i]x[i] + n[i]|^2\} = \sigma_{\tilde{h}}^2 \mathcal{E}_s + N_0$ where $\sigma_{\tilde{h}}^2 = \mathbb{E}\{|\tilde{h}|^2\} = \frac{\gamma N_0}{\gamma \mathcal{E}_t + N_0}$ is the variance of the estimate error. Under these assumptions, a lower bound on the instantaneous capacity is given by [12], [13]

$$C_L = \frac{TB - 1}{T} \log_2 \left(1 + \frac{\mathcal{E}_s}{\sigma_{\tilde{h}}^2 \mathcal{E}_s + N_0} |\hat{h}|^2 \right) \quad (4)$$

$$= \frac{TB - 1}{T} \log_2 (1 + \text{SNR}_{\text{eff}} |w|^2) \quad \text{bits/s} \quad (5)$$

where effective SNR is

$$\text{SNR}_{\text{eff}} = \frac{\mathcal{E}_s \sigma_{\hat{h}}^2}{\sigma_{\tilde{h}}^2 \mathcal{E}_s + N_0}, \quad (6)$$

Note that the expression in (5) is obtained by defining $\hat{h} = \sigma_{\tilde{h}} w$ where w is a standard complex Gaussian random variable with zero mean and unit variance, i.e., $w \sim \mathcal{CN}(0, 1)$.

Since Gaussian is the worst uncorrelated noise [12], the above-mentioned assumptions lead to a pessimistic model and the rate expression in (5) is a lower bound to the capacity of the true channel (3). On the other hand, C_L is a good measure of the rates achieved in communication systems that operate as if the channel estimate were perfect (i.e., in systems where Gaussian codebooks designed for known channels are used, and scaled nearest neighbor decoding is employed at the receiver) [11].

Henceforth, we base our analysis on C_L to understand the impact of the imperfect channel estimate. Since the transmitter is unaware of the channel conditions, it is assumed that information is transmitted at a fixed rate of r bits/s. When $r < C_L$, the channel is considered to be in the ON state and reliable communication is achieved at this rate. If, on the other hand, $r \geq C_L$, we assume that outage occurs. In this case, channel is in the OFF state and reliable communication at the rate of r bits/s cannot be attained. Hence, the effective data rate is zero and information has to be resent. Fig. 1 depicts the two-state transmission model together with the transition probabilities. Under the assumption of independent realizations of blocks, it can be easily seen that the transition probabilities are given by

$$p_{11} = p_{21} = P\{r \geq C_L\} = P\{|w|^2 \leq \alpha\} \quad (7)$$

$$p_{22} = p_{12} = P\{r < C_L\} = P\{|w|^2 > \alpha\} \quad (8)$$

where

$$\alpha = \frac{2^{\frac{rT}{TB-1}} - 1}{\text{SNR}_{\text{eff}}}, \quad (9)$$

and $|w|^2$ is an exponential random variable with mean 1, and hence, $P\{|w|^2 > \alpha\} = e^{-\alpha}$.

²In [13], the capacity of training-based transmissions under input peak power constraints is shown to be achieved by an SNR-dependent, discrete distribution with a finite number of mass points. In such cases, no closed-form expression for the capacity exists, and capacity values need to be obtained through numerical computations.

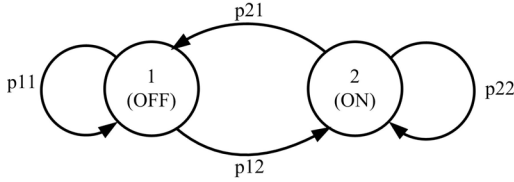


Fig. 1. ON-OFF state transition model.

III. EFFECTIVE CAPACITY AND SPECTRAL EFFICIENCY-BIT ENERGY TRADEOFF

In [5], Wu and Negi defined the effective capacity as the maximum constant arrival rate that a given service process can support in order to guarantee a statistical QoS requirement specified by the QoS exponent θ ³. If we define Q as the stationary queue length, then θ is the decay rate of the tail distribution of the queue length Q :

$$\lim_{q \rightarrow \infty} \frac{\log P(Q \geq q)}{q} = -\theta. \quad (10)$$

Therefore, for large q_{\max} , we have the following approximation for the buffer violation probability: $P(Q \geq q_{\max}) \approx e^{-\theta q_{\max}}$. Hence, while larger θ corresponds to more strict QoS constraints, smaller θ implies looser QoS guarantees. Similarly, if D denotes the steady-state delay experienced in the buffer, then $P(D \geq d_{\max}) \approx e^{-\theta \delta d_{\max}}$ for large d_{\max} , where δ is determined by the arrival and service processes [7].

The effective capacity is given by

$$-\frac{\Lambda(-\theta)}{\theta} = -\lim_{t \rightarrow \infty} \frac{1}{\theta t} \log_e \mathbb{E}\{e^{-\theta S[t]}\} \quad (11)$$

where $S[t] = \sum_{i=1}^t R[i]$ is the time-accumulated service process and $\{R[i], i = 1, 2, \dots\}$ denote the discrete-time stationary and ergodic stochastic service process. Note that in the model we consider, $R[i] = rT$ or 0 depending on the channel state being ON or OFF. In [10], it is shown that for such an ON-OFF model, we have

$$\frac{\Lambda(\theta)}{\theta} = \frac{1}{\theta} \log_e \left(\frac{1}{2} \left(p_{11} + p_{22} e^{\theta T r} + \sqrt{(p_{11} + p_{22} e^{\theta T r})^2 + 4(p_{11} + p_{22} - 1)e^{\theta T r}} \right) \right) \quad (12)$$

Note that $p_{11} + p_{22} = 1$ in our model. Then, for a given QoS delay constraint θ , the effective capacity normalized by the frame duration T and bandwidth B , or equivalently spectral efficiency in bits/s/Hz, becomes

$$R_E(\text{SNR}, \theta) = \max_{\substack{r \geq 0 \\ 0 \leq \rho \leq 1}} -\frac{1}{TB} \frac{\Lambda(-\theta)}{\theta} \quad \text{bits/s/Hz} \quad (13)$$

$$= \max_{\substack{r \geq 0 \\ 0 \leq \rho \leq 1}} -\frac{1}{\theta TB} \log_e (p_{11} + p_{22} e^{-\theta T r}) \quad (14)$$

$$= \max_{\substack{r \geq 0 \\ 0 \leq \rho \leq 1}} -\frac{1}{\theta TB} \log_e (1 - P(|w|^2 > \alpha)(1 - e^{-\theta T r})) \quad (15)$$

³For time-varying arrival rates, effective capacity specifies the effective bandwidth of the arrival process that can be supported by the channel.

$$= -\frac{1}{\theta TB} \log_e (1 - P(|w|^2 > \alpha_{\text{opt}})(1 - e^{-\theta T r_{\text{opt}}})) \quad (16)$$

Note that R_E is obtained by optimizing both the fixed transmission rate r and the fraction of power allocated to training, ρ . In the optimization result (16), r_{opt} and α_{opt} are the optimal values of r and α , respectively. The optimized r_{opt} can be found by solving

$$\frac{2^{\frac{rT}{TB-1}} T \log_e 2}{(TB-1)\text{SNR}_{\text{eff}}} (1 - e^{-\theta T r}) - \theta T e^{-\theta T r} = 0 \quad (17)$$

which is obtained from the first derivative of (15) with respect to r .

It can easily be seen that

$$R_E(\text{SNR}, 0) = \lim_{\theta \rightarrow 0} R_E(\text{SNR}, \theta) \quad (18)$$

$$= \max_{r \geq 0} \frac{r}{B} P \left\{ |w|^2 > \frac{2^{\frac{rT}{TB-1}} - 1}{\text{SNR}_{\text{eff}}} \right\}. \quad (19)$$

Hence, as the QoS requirements relax, the maximum constant arrival rate approaches the average transmission rate. On the other hand, for $\theta > 0$, $R_E < \frac{1}{B} \max_{r \geq 0} r P(|w|^2 > \alpha)$ in order to avoid violations of buffer constraints.

In this paper, we focus on the energy efficiency of wireless transmissions under the aforementioned statistical queueing constraints. Since energy efficient operation generally requires operation at low-SNR levels, our analysis throughout the paper is carried out in the low-SNR regime. In this regime, the trade-off between the normalized effective capacity (i.e., spectral efficiency) R_E and bit energy $\frac{E_b}{N_0} = \frac{\text{SNR}}{R_E(\text{SNR})}$ is a key tradeoff in understanding the energy efficiency, and is characterized by the bit energy at zero spectral efficiency and wideband slope provided, respectively, by

$$\frac{E_b}{N_0} \Big|_{R=0} = \frac{1}{\dot{R}_E(0)} \quad \text{and} \quad \mathcal{S}_0 = -\frac{2(\ddot{R}_E(0))^2}{\dot{R}_E(0)} \log_e 2 \quad (20)$$

where $\dot{R}_E(0)$ and $\ddot{R}_E(0)$ are the first and second derivatives with respect to SNR, respectively, of the function $R_E(\text{SNR})$ at zero SNR [2].

IV. OPTIMAL POWER ALLOCATION FOR TRAINING

In this section, we investigate the optimization problem in (15). In particular, we identify the optimal fraction of power that needs to be allocated to training while satisfying statistical buffer constraints.

Theorem 1: At a given SNR level, the optimal fraction of power ρ_{opt} that solves (15) does not depend on the QoS exponent θ and the transmission rate r , and is given by

$$\rho_{\text{opt}} = \sqrt{\eta(\eta + 1)} - \eta \quad (21)$$

where $\eta = \frac{\gamma TB \text{SNR} + TB - 1}{\gamma TB(TB - 2)\text{SNR}}$ and $\text{SNR} = \frac{\bar{P}}{N_0 B}$.

Proof: From (15) and the definition of α in (9), we can easily see that for fixed r , the only term in (15) that depends on ρ is α . Moreover, α has this dependency through SNR_{eff} . Therefore, ρ_{opt} that maximizes the objective function in (15) can be found by minimizing α , or equivalently maximizing SNR_{eff} .

Substituting the definitions in (2) and the expressions for σ_h^2 and σ_h^2 into (6), we have

$$\text{SNR}_{\text{eff}} = \frac{\mathcal{E}_s \sigma_h^2}{\sigma_h^2 \mathcal{E}_s + N_0} \quad (22)$$

$$= \frac{\rho(1-\rho)\gamma^2 T^2 B^2 \text{SNR}^2}{\rho\gamma TB(TB-2)\text{SNR} + \gamma TB\text{SNR} + TB-1} \quad (23)$$

where $\text{SNR} = \frac{\bar{P}}{N_0 B}$. Evaluating the derivative of SNR_{eff} with respect to ρ and making it equal to zero leads to the expression in (21). Clearly, ρ_{opt} is independent of θ and r .

Above, we have implicitly assumed that the maximization is performed with respect to first ρ and then r . However, the result will not be altered if the order of the maximization is changed. Note that the objective function in (15)

$$g(\text{SNR}_{\text{eff}}, r) = -\frac{1}{\theta TB} \log_e \left(1 - P \left(|w|^2 > \frac{2^{\frac{rT}{TB-1}} - 1}{\text{SNR}_{\text{eff}}} \right) \times (1 - e^{-\theta Tr}) \right) \quad (24)$$

is a monotonically increasing function of SNR_{eff} for all r . It can be easily verified that maximization does not affect the monotonicity of g , and hence $\max_{r \geq 0} g(\text{SNR}_{\text{eff}}, r)$ is still a monotonically increasing function of SNR_{eff} . Therefore, in the outer maximization with respect to ρ , the choice of ρ that maximizes SNR_{eff} will also maximize $\max_{r \geq 0} g(\text{SNR}_{\text{eff}}, r)$, and the optimal value of ρ is again given by (21). \square

V. ENERGY EFFICIENCY IN THE LOW-POWER REGIME

In this section, we investigate the spectral efficiency–bit energy tradeoff as the average power \bar{P} diminishes. We assume that the bandwidth allocated to the channel is fixed. With the optimal value of ρ given in Theorem 1, we can now express the normalized effective capacity as

$$\mathcal{R}_E(\text{SNR}, \theta) = \max_{r \geq 0} -\frac{1}{\theta TB} \log_e \left(1 - P \left(|w|^2 > \frac{2^{\frac{rT}{TB-1}} - 1}{\text{SNR}_{\text{eff, opt}}} \right) \times (1 - e^{-\theta Tr}) \right) \quad (25)$$

where $\text{SNR}_{\text{eff, opt}} = \frac{\phi(\text{SNR})\text{SNR}^2}{\psi(\text{SNR})\text{SNR} + TB - 1}$, and $\phi(\text{SNR}) = \rho_{\text{opt}}(1 - \rho_{\text{opt}})\gamma^2 T^2 B^2$, and $\psi(\text{SNR}) = (1 + (TB - 2)\rho_{\text{opt}})\gamma TB$. Note that $\text{SNR} = \bar{P}/(N_0 B)$ vanishes with decreasing \bar{P} . We obtain the following result on the bit energy requirement in the low-power regime as \bar{P} diminishes.

Theorem 2: In the low-power regime, the bit energy increases without bound as the average power \bar{P} and hence SNR vanishes, i.e.,

$$\left. \frac{E_b}{N_0} \right|_{R=0} = \lim_{\text{SNR} \rightarrow 0} \frac{E_b}{N_0} = \lim_{\text{SNR} \rightarrow 0} \frac{\text{SNR}}{\mathcal{R}_E(\text{SNR})} = \frac{1}{\mathcal{R}_E(0)} = \infty. \quad (26)$$

Theorem 2 follows by noting that $\text{SNR}_{\text{eff, opt}}$ scales as SNR^2 as SNR diminishes to zero, and by showing that this implies that \mathcal{R}_E scales as SNR^2 as well. Therefore, the first derivative of \mathcal{R}_E with respect to SNR is zero at $\text{SNR} = 0$, i.e., $\dot{\mathcal{R}}_E(0) = 0$, leading to the result that $\lim_{\text{SNR} \rightarrow 0} \frac{E_b}{N_0} = \infty$.

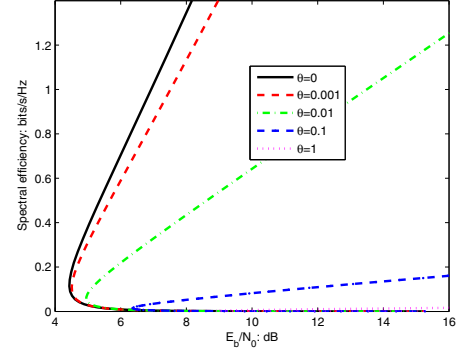


Fig. 2. Spectral efficiency vs. E_b/N_0 in the Rayleigh channel with $E\{|h|^2\} = \gamma = 1$. $B = 10^5$.

This result shows us that operation at very low power levels is extremely energy inefficient and should be avoided regardless of the value of θ . Note that the power allocated for training, $\mathcal{E}_t = \rho \bar{P} T$, decreases with decreasing \bar{P} . Hence, our ability to estimate the channel is hindered in the low-power regime while, as mentioned before, the system operates as if the channel estimate were perfect. This discrepancy leads to the inefficiency seen as \bar{P} approaches zero.

Fig. 2 plots the spectral efficiency vs. bit energy for $\theta = \{1, 0.1, 0.01, 0.001, 0\}$ when $B = 10^5$ Hz. As predicted by the result of Theorem 2, the bit energy increases without bound in all cases as the spectral efficiency $\mathcal{R}_E \rightarrow 0$. Consequently, the minimum bit energy is achieved at a nonzero spectral efficiency below which one should avoid operating as it only increases the energy requirements. Another observation is that the minimum bit energy increases as θ increases and hence as the statistical queueing constraints become more stringent. At higher spectral efficiencies, we again note the increased energy requirements with increasing θ .

VI. ENERGY EFFICIENCY IN THE WIDEBAND REGIME

In this section, we consider the wideband regime. We assume that the average power is kept constant. Note that as the bandwidth B increases, SNR approaches zero and we again operate in the low-SNR regime. Note also that flat fading assumption will not hold in the wideband regime as the bandwidth B increases without bound. On the other hand, if we decompose the wideband channel into N parallel subchannels, and suppose that each subchannel has a bandwidth that is equal to the coherence bandwidth B_c , then we can assume that independent flat-fading is experienced in each subchannel. If the fading coefficients in different subchannels are i.i.d. and the data and training energies are uniformly allocated over the subchannels, then the effective capacity of a wideband channel has an expression similar to that in (15) (see the journal version of this paper [14]). In [14], it is shown that if rich multipath is experienced and hence the number of subchannels N increases with increasing bandwidth B while B_c stays fixed, the wideband regime analysis is the same as the low-power regime analysis. Therefore, as $B \rightarrow \infty$ in the rich multipath fading scenario, we have $\left. \frac{E_b}{N_0} \right|_{R=0} = \infty$ for all $\theta \geq 0$.

We now focus on the scenario of sparse multipath fading. In particular, we consider the case in which the number of subchannels N remains bounded as B increases. Under these assumptions, we have the following formulation. We denote $\zeta = 1/B$. Note that as $B \rightarrow \infty$, $\zeta \rightarrow 0$. With this notation, we can express the normalized effective capacity as

$$R_E(\text{SNR}) = -\frac{\zeta}{\theta T} \log_e \left(1 - P\{|w|^2 > \alpha_{\text{opt}}\} (1 - e^{-\theta T r_{\text{opt}}}) \right).$$

Theorem 3: For sparse multipath fading channel with bounded number of subchannels, the minimum bit energy and wideband slope in the wideband regime are given by

$$\frac{E_b}{N_0 \min} = \frac{-\delta \log_e 2}{\log_e \xi} \quad \text{and} \quad (27)$$

$$S_0 = \frac{\xi \log_e^2 \xi \log_e 2}{\theta T \alpha_{\text{opt}}^* (1 - \xi) \left(\frac{1}{T} \left(\sqrt{1 + \frac{\gamma \bar{P} T}{N_0}} - 1 \right) + \frac{\varphi \alpha_{\text{opt}}^*}{2} \right)}, \quad (28)$$

respectively, where $\delta = \frac{\theta T \bar{P}}{N_0 \log_e 2}$, $\xi = 1 - e^{-\alpha_{\text{opt}}^*} (1 - e^{-\frac{\theta T \varphi \alpha_{\text{opt}}^*}{\log_e 2}})$,

and $\varphi = \frac{\gamma \bar{P}}{N_0} \left(\sqrt{1 + \frac{N_0}{\gamma \bar{P} T}} - \sqrt{\frac{N_0}{\gamma \bar{P} T}} \right)^2$. α_{opt}^* is defined as $\alpha_{\text{opt}}^* = \lim_{\zeta \rightarrow 0} \alpha_{\text{opt}}$ and α_{opt}^* satisfies

$$\alpha_{\text{opt}}^* = \frac{\log_e 2}{\theta T \varphi} \log_e \left(1 + \frac{\theta T \varphi}{\log_e 2} \right). \quad (29)$$

Above, \bar{P} denotes the power allocated to each subchannel.

Proof: We first derive the following result for optimal fraction of power on training expressed in (21)

$$\rho_{\text{opt}} = \rho_{\text{opt}}^* + \dot{\rho}_{\text{opt}}(0) \zeta + o(\zeta) \quad (30)$$

where ρ_{opt}^* is a real value achieved as $\zeta \rightarrow 0$, and $\dot{\rho}_{\text{opt}}(0)$ is the first derivative of ρ_{opt} evaluated at $\zeta = 0$. We have

$$\rho_{\text{opt}}^* = \sqrt{\frac{N_0}{\gamma \bar{P} T}} \left(1 + \frac{N_0}{\gamma \bar{P} T} \right) - \frac{N_0}{\gamma \bar{P} T} \quad (31)$$

and

$$\dot{\rho}_{\text{opt}}(0) = \frac{1}{2T} \sqrt{1 + \frac{\gamma \bar{P} T}{N_0}} \left(\sqrt{1 + \frac{N_0}{\gamma \bar{P} T}} - \sqrt{\frac{N_0}{\gamma \bar{P} T}} \right)^2. \quad (32)$$

Furthermore, $\text{SNR}_{\text{eff, opt}}$ defined below equation (25) can be simplified to

$$\text{SNR}_{\text{eff, opt}} = \varphi \zeta + \omega \zeta^2 + o(\zeta^2) \quad (33)$$

where

$$\varphi = \frac{\gamma \bar{P}}{N_0} \left(\sqrt{1 + \frac{N_0}{\gamma \bar{P} T}} - \sqrt{\frac{N_0}{\gamma \bar{P} T}} \right)^2 \quad (34)$$

and

$$\omega = -\frac{\gamma \bar{P}}{N_0 T} \left(\sqrt{1 + \frac{N_0}{\gamma \bar{P} T}} - \sqrt{\frac{N_0}{\gamma \bar{P} T}} \right)^2 \left(\sqrt{1 + \frac{\gamma \bar{P} T}{N_0}} - 2 \right). \quad (35)$$

Assume that the Taylor series expansion of r_{opt} with respect to small ζ is

$$r_{\text{opt}} = r_{\text{opt}}^* + \dot{r}_{\text{opt}}(0) \zeta + o(\zeta) \quad (36)$$

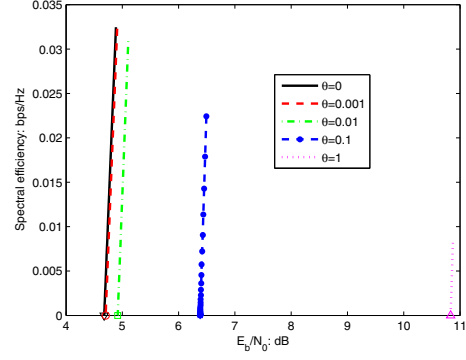


Fig. 3. Spectral efficiency vs. E_b/N_0 in the Rayleigh channel with $E\{|h|^2\} = \gamma = 1$. $\bar{P}/N_0 = 10^4$.

where $r_{\text{opt}}^* = \lim_{\zeta \rightarrow 0} r_{\text{opt}}$ and $\dot{r}_{\text{opt}}(0)$ is the first derivative with respect to ζ of r_{opt} evaluated at $\zeta = 0$. From (9), we can find that

$$\alpha_{\text{opt}} = \frac{2^{\frac{r_{\text{opt}} \zeta}{1 - \zeta/T}} - 1}{\text{SNR}_{\text{eff, opt}}} \quad (37)$$

$$= \frac{r_{\text{opt}}^* \log_e 2}{\varphi} + \left(\frac{\dot{r}_{\text{opt}}(0) \log_e 2}{\varphi} + \frac{r_{\text{opt}}^* \log_e 2}{\varphi} \left(\frac{1}{T} - \frac{\omega}{\varphi} \right) + \frac{(r_{\text{opt}}^* \log_e 2)^2}{2\varphi} \right) \zeta + o(\zeta) \quad (38)$$

from which we have as $\zeta \rightarrow 0$ that

$$\alpha_{\text{opt}}^* = \frac{r_{\text{opt}}^* \log_e 2}{\varphi} \quad (39)$$

and that

$$\dot{\alpha}_{\text{opt}}(0) = \frac{\dot{r}_{\text{opt}}(0) \log_e 2}{\varphi} + \frac{r_{\text{opt}}^* \log_e 2}{\varphi} \left(\frac{1}{T} - \frac{\omega}{\varphi} \right) + \frac{(r_{\text{opt}}^* \log_e 2)^2}{2\varphi} \quad (40)$$

where $\dot{\alpha}_{\text{opt}}(0)$ is the first derivative with respect to ζ of α_{opt} evaluated at $\zeta = 0$. According to (39), $r_{\text{opt}}^* = \frac{\varphi \alpha_{\text{opt}}^*}{\log_e 2}$.

Combining with (33) and (39), we can obtain from (17) as $\zeta \rightarrow 0$

$$\frac{\log_e 2}{\varphi} \left(1 - e^{-\frac{\theta T \varphi \alpha_{\text{opt}}^*}{\log_e 2}} \right) - \theta T e^{-\theta T r_{\text{opt}}^*} = 0 \quad (41)$$

from which we get

$$\alpha_{\text{opt}}^* = \frac{\log_e 2}{\theta T \varphi} \log_e \left(1 + \frac{\theta T \varphi}{\log_e 2} \right). \quad (42)$$

We now have

$$\begin{aligned} \frac{E_b}{N_0 \min} &= \lim_{\zeta \rightarrow 0} \frac{\frac{\bar{P}}{N_0} \zeta}{R_E(\zeta)} \\ &= \frac{-\frac{\theta T \bar{P}}{N_0}}{\log_e (1 - P\{|w|^2 \geq \alpha_{\text{opt}}^*\} (1 - e^{-\theta T r_{\text{opt}}^*}))} \\ &= \frac{-\delta \log_e 2}{\log_e \xi} = \frac{\frac{\bar{P}}{N_0}}{\dot{R}_E(0)} \end{aligned} \quad (44)$$

where $\dot{R}_E(0)$ is the derivative of R_E with respect to ζ at $\zeta = 0$, $\delta = \frac{\theta T \bar{P}}{N_0 \log_e 2}$, and $\xi = 1 - P\{|w|^2 \geq \alpha_{\text{opt}}^*\} (1 - e^{-\frac{\theta T \varphi \alpha_{\text{opt}}^*}{\log_e 2}})$. Obviously, (44) provides (27).

Note that the second derivative $\ddot{R}_E(0)$, required in the computation of the wideband slope \mathcal{S}_0 , can be obtained from

$$\begin{aligned} \ddot{R}_E(0) &= \lim_{\zeta \rightarrow 0} 2 \frac{R_E(\zeta) - \dot{R}_E(0)\zeta}{\zeta^2} \\ &= \lim_{\zeta \rightarrow 0} 2 \frac{1}{\zeta} \left(-\frac{1}{\theta T} \log_e (1 - P\{|w|^2 \geq \alpha_{\text{opt}}\}) (1 - e^{-\theta T r_{\text{opt}}}) \right) \\ &\quad + \frac{1}{\theta T} \log_e \left(1 - P\{|w|^2 \geq \alpha_{\text{opt}}^*\} (1 - e^{-\theta T r_{\text{opt}}^*}) \right) \\ &= \lim_{\zeta \rightarrow 0} -\frac{2e^{-\alpha_{\text{opt}}}}{\theta T (1 - P\{|w|^2 \geq \alpha_{\text{opt}}\}) (1 - e^{-\theta T r_{\text{opt}}})} \\ &\quad \times \left(\dot{\alpha}_{\text{opt}}(\zeta) (1 - e^{-\theta T r_{\text{opt}}}) - \theta T e^{-\theta T r_{\text{opt}}} \dot{r}_{\text{opt}}(\zeta) \right) \end{aligned} \quad (45)$$

$$\begin{aligned} &= -\frac{2e^{-\alpha_{\text{opt}}^*}}{\theta T (1 - P\{|w|^2 \geq \alpha_{\text{opt}}^*\}) (1 - e^{-\theta T r_{\text{opt}}^*})} \\ &\quad \times \left(\dot{\alpha}_{\text{opt}}(0) (1 - e^{-\theta T r_{\text{opt}}^*}) - \theta T e^{-\theta T r_{\text{opt}}^*} \dot{r}_{\text{opt}}(0) \right) \end{aligned} \quad (46)$$

where $r_{\text{opt}}^* = \frac{\bar{P}\alpha_{\text{opt}}^*}{N_0 \log_e 2}$. Above, (45) and (46) follow by using L'Hospital's Rule and applying Leibniz Integral Rule.

Meanwhile, substituting (41) and (40) into (46) gives us

$$\begin{aligned} \ddot{R}_E(0) &= -\frac{2e^{-\alpha_{\text{opt}}^*}}{\theta T (1 - P\{|w|^2 \geq \alpha_{\text{opt}}^*\}) (1 - e^{-\theta T r_{\text{opt}}^*})} \\ &\quad \times \alpha_{\text{opt}}^* (1 - e^{-\theta T r_{\text{opt}}^*}) \left(\frac{1}{T} - \frac{\omega}{\varphi} + \frac{\varphi \alpha_{\text{opt}}^*}{2} \right) \\ &= -\frac{2(1 - \xi) \alpha_{\text{opt}}^*}{\theta T \xi} \left(\frac{1}{T} - \frac{\omega}{\varphi} + \frac{\varphi \alpha_{\text{opt}}^*}{2} \right) \\ &= -\frac{2(1 - \xi) \alpha_{\text{opt}}^*}{\theta T \xi} \left(\frac{1}{T} \left(\sqrt{1 + \frac{\gamma \bar{P} T}{N_0}} - 1 \right) + \frac{\varphi \alpha_{\text{opt}}^*}{2} \right) \end{aligned} \quad (47)$$

Combining (47) and (44), we can prove (28). \square

We note that the minimum bit energy in the sparse multipath case with bounded number of subchannels is achieved as $B \rightarrow \infty$ and hence as $\text{SNR} \rightarrow 0$. This is in stark contrast to the results in the low-power regime and rich multipath cases in which the bit energy requirements grow without bound as SNR vanishes. This is due to the fact that in sparse fading with bounded number of independent resolvable paths, uncertainty does not grow without bound because the number of subchannels N is kept fixed as $B \rightarrow \infty$.

Fig. 3 plots the spectral efficiency–bit energy curve in the Rayleigh channel for different θ values. In the figure, we assume that $\bar{P}/N_0 = 10^4$. As predicted, the minimum bit energies are obtained as SNR and hence the spectral efficiency approach zero. $\frac{E_b}{N_0 \min}$ are computed to be equal to $\{4.6776, 4.7029, 4.9177, 6.3828, 10.8333\}$ dB for $\theta = \{0, 0.001, 0.01, 0.1, 1\}$, respectively. Moreover, the wideband slopes are $\mathcal{S}_0 = \{0.4720, 0.4749, 0.4978, 0.6151, 0.6061\}$ for the same set of θ values. As can also be seen in the result of Theorem 3, the minimum bit energy and wideband slope in general depend on θ . In Fig. 3, we note that the bit energy requirements (including the minimum bit energy) increase with

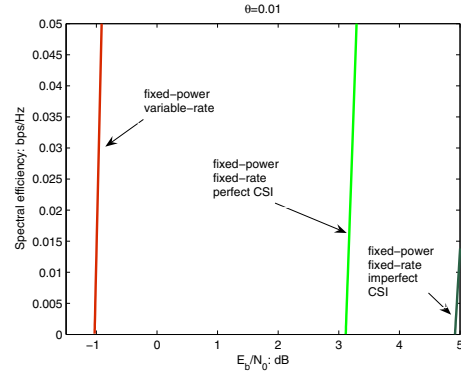


Fig. 4. Comparison of spectral efficiency; $\bar{P}/N_0 = 10^4$, $\theta = 0.01$, and $E\{|h|^2\} = \gamma = 1$.

increasing θ , illustrating the energy costs of stringent queueing constraints.

In this paper, we have considered fixed-rate/fixed-power transmissions over imperfectly-known channels. In Fig. 4, we compare the performance of this system with those in which the channel is perfectly-known and fixed- or variable-rate transmission is employed. The latter models have been studied in [8] and [9]. This figure demonstrates the energy costs of not knowing the channel and sending the information at fixed-rate.

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