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Beyond Formulas: A Collaboration between Liberal Arts Honors Underclassmen and Senior Math Majors

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> "Congratulations! Sony Pictures producers have hired you as a scientific consultant to assist them in producing a remake of the classic film *Gone With The Seabreeze*, which chronicles the founding of the Westchester campus of Loyola University in 1929. The original film features shots of various old buildings that can still be found across campus today. The producers wish to recreate these shots, but need to know where to position the camera, which they have tasked you with finding."

Thus began the month-long, collaborative project at Loyola Marymount University between the honors underclassmen in HNRS 140, On Motion and Mechanics, taught by Alissa S. Crans, and the senior applied mathematics majors in MATH 495, Mathematical Modeling, taught by Robert Rovetti. During a period of four weeks, six teams of freshman and sophomore liberal arts honors students, each led by a senior math major, set out to reconstruct an old photograph using a mathematical technique based on straightforward geometry. Along the way they would run into inaccessible landscapes, blocked views, and busy schedules, but ultimately they emerged with both a finished product and a clearer understanding of what it means to apply a theoretical method to a real-world problem. We begin by describing the courses and the assigned project itself, and then we reflect on the pedagogical goals of the project and various observations made by both students and instructors.

DESCRIPTION OF COURSES

On Motion and Mechanics (HNRS 140) offers freshman and sophomore honors students outside of LMU's Seaver College of Science and Engineering the opportunity to experience the methods, practices, and techniques common to science, mathematics, and engineering disciplines. While the name of the course does not change, the content each year depends entirely on the instructor. In the past few years, this course has been taught by faculty members in the fields of engineering and natural science who focused on comparing and contrasting scientific and engineering methods and then climate change and environmental issues.

This past fall, however, the course was structured around the dual questions "What is mathematics?" and "What do mathematicians do?" Throughout the semester we discussed questions such as "What does it mean to perform research in mathematics? Is mathematics created or discovered? Where, and by whom, is mathematics used and practiced?" and "How is mathematics similar to and different from the humanities, sciences, and engineering?" Our reading list was extensive and included Steven Strogatz's "Elements of Math" column from the *New York Times* website, *How Mathematicians Think* by William Byers, biographies of mathematicians from the undergraduate mathematics journal *Math Horizons* (published by the Mathematical Association of America), essays from Campbell and Higgins' *Mathematics: People, Problems, Results*, entries on the website *weusemath.org*, and *A Mathematician's Apology*, by G. H. Hardy. We also watched video presentations by mathematical speakers on the website *TED.com* and attended the Einstein Public Lecture in Mathematics given by Fields Medalist Terrance Tao at UCLA.

In addition to the project described in this paper, the students completed group projects on graph complexity, voting paradoxes, and statistics. The major assignment for the course consisted of a paper about a mathematical topic together with an in-class presentation. Topics included the Monty Hall Problem, knot theory, game theory, Lewis Carroll logic problems, the golden ratio, check digits, and infinity, among others.

In contrast, *Mathematical Modeling* (MATH 495) is a course designed specifically for students in the College of Science and Engineering. It provides senior applied mathematics majors with a capstone experience through activities that bring together the material and skills they have acquired during their undergraduate training. Students are exposed to classic problems in modeling and are encouraged to use a variety of techniques, both analytical and computational, to solve them. During fall 2010, particular focus was placed on modeling as an *interpretive process*, in which one begins with a verbal statement of a real-world problem, narrows the scope of the problem to a realistic size by creating a list of assumptions and needed facts, and then proceeds to cast the problem within a mathematical framework that leads to a formal solution (either exact or approximate). Consistent with our department-wide learning outcomes, the course heavily emphasizes writing as a central means of communicating the students' learning and discovery. In the previous year, students in this course were also asked to read and comment on essays by writers such as Douglas Hofstadter, Isaac Asimov, and Stanislaw Lem on topics related to the creative process in science and mathematics. The course ends with a significant research paper in which students compare and contrast multiple approaches to modeling a topic of their choosing.

DESCRIPTION OF CORE PROJECT CONCEPT

For the core task of the project, we gave each team an old picture of one of several buildings that still stand on the campus of LMU and asked them to (1) identify the building, (2) physically measure (or estimate) various lengths on the building and on the photograph, (3) use the provided formulas to calculate the location of the photographer, and (4) with a camera reproduce the original photo from that location. This project was based on the method described by Byers and Henle in their article "Where the Camera Was." The main result of their method is given as a single mathematical proposition:

Proposition: If a picture of a rectangular solid taken by a vertically-held pinhole camera has measurements (on the photograph) of *a, b, c, d,* and *e* (Figure 1A), then the camera was positioned a distance *L* to the left of point **B** (in the direction from **C** to **B**) and a distance *F* in front of point **B** (in the direction from **A** to **B**), where $\overline{\mathbf{BC}}$ and $\overline{\mathbf{AB}}$ are measured lengths from the actual building, and

$$L = \left(\frac{dc}{d(b-c)+e(b-a)}\right)\overline{\mathbf{BC}}$$

$$F = \left(\frac{ae}{d(b-c)+e(b-a)}\right)\overline{\mathbf{AB}}$$

The lowercase italicized letters represent lengths as measured on the photograph itself (using a hand-held ruler) while the uppercase bold letters represent locations on the building and the quantities $\overline{\mathbf{BC}}$ and $\overline{\mathbf{AB}}$ represent actual lengths of the building itself (measured using, for instance, a long tape measure).

Figure 1A

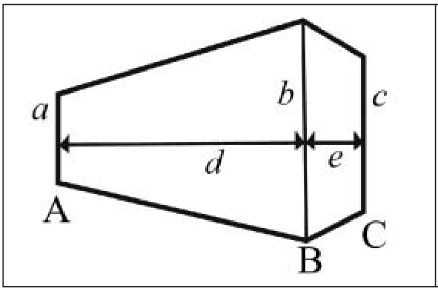

Figure 1B

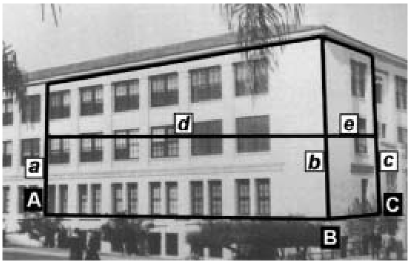

Figure 1A: An idealized rectangular prism with three points A, B, and C, and five lengths *a* through *e*. Figure 1B: A schematic drawing of a student-generated prism superimposed onto a photograph of St. Robert's Hall. Note that the prism does not necessarily cover the entire building or have the same shape as the idealized prism in Figure 1A. Lengths $\overline{\mathbf{AB}}$ and $\overline{\mathbf{BC}}$ must be measured (or estimated) on the building itself while lengths *a* through *e* are measured on the photograph.

In order to successfully solve the problem, each team needed to:

1. Draw an appropriate rectangular prism on their photo, as shown in Figure 1B;
2. Measure the lengths *a, b, c, d,* and *e* on their photograph;
3. Locate the points **A, B,** and **C** on the actual building and determine (using a method of their choosing) the on-site lengths of $\overline{\mathbf{AB}}$ and $\overline{\mathbf{BC}}$;
4. Compute the quantities *L* and *F*; and
5. Find the location of the photographer by moving distance *L* to the left of **B** in the direction from **C** to **B,** and distance *L* in front of point **B** in the direction from **A** to **B**.

The proof of the proposition relies on a multi-step geometric argument involving comparisons of similar triangles and principles of projective geometry (e.g. vanishing points and horizon lines), the details of which can be found in the original article (Byers and Henle 255). Both the method and its proof were discussed prior to the beginning of the project with the senior math majors, who were later asked to explain the method to their teams.

DESCRIPTION OF PROJECT PROCESS

We formed six teams, each consisting of two or three honors students and one senior math major as the team leader. Enough honors students from the School of Film and Television were enrolled so that we could allocate one to each team. Each team was given a unique photo of an LMU building, chosen from the LMU Hannon Library Digital Collection, including Sacred Heart Chapel, St. Robert's Hall, Xavier Hall, and a building from the old Loyola College, now Loyola High School (about 15 miles from LMU). After an initial orientation meeting in which the classes were introduced to each other and the teams were formed, the students were given approximately four weeks to analyze their original photo, reproduce the photo, create their multimedia presentation, and write their report.

MEASUREMENTS AND CALCULATIONS

Comprising the "meat" of the assignment, the process of measurement and calculation required students to take measurements from both the photograph and the building and then use the formulas to calculate a resulting location of the original photographer (and where they would stand to reconstruct the photo). Although the method is simple in theory, many groups encountered various challenges along the way, including inaccessible building areas, blocked views, and variability caused by the imprecision of their method of measurement. We reminded the teams that real-world problems rarely work out as nicely as the familiar "textbook problems" and that problem solving requires creativity and independent thought. As an example, one group found they were unable to physically measure their building as newer buildings had been subsequently erected around it during the intervening years; their solution was to use Google Earth to access an overhead satellite photo of the building and estimate the dimensions of their building using other known objects in the satellite photo.

Multimedia Presentation

In order for the students to demonstrate their success, the assignment asked them to use a digital photo-editing platform to create a visual comparison of the original and reconstructed photos. Under the guidance of Glenn Gebhard from the School of Film and Television, the class met in LMU's Film Editing Lab to use the software *Final Cut Pro* and produce a twenty-second "cinematic dissolve" from the original photograph to the reproduction. The resulting movies enabled all of us to clearly gauge the accuracy of their measurements, calculations, and approximations made in adjusting for physical obstacles. Some groups found they had to move, zoom, or otherwise adjust their reproduced photo, indicating the presence of some error or overly generous approximation perhaps stemming from the estimation of on-site lengths and distances. Nonetheless, the final products from most teams were quite impressive.

The students also produced a presentation of their work, using whatever medium they preferred, that outlined their process from beginning to end and that could be showcased in the mathematics department and at LMU Honors Program events. Affirming their innate familiarity with computers and digital devices, most teams chose to extend their initial twenty-second "dissolve" movies into full-fledged mini-documentaries ranging from three to five minutes and including snapshots of individual students "in action" as well as voice-over narration.

Written Report

In the written report, teams described their methods, including steps taken to overcome any obstacles. Additionally, teams were asked for brief commentary on the history of the photo, some of which was obtained via personal interviews with a long-serving LMU professor, Father Rich Robin. In order to emphasize the goal of understanding the origin of a mathematical formula, we also asked students to fill in several small-sized "gaps" in the mathematical proof as presented in the original article (Byers and Henle 255). (It is not uncommon for a mathematical proof to omit one or two steps that are assumed obvious to the intended readership or amount to straightforward computation; asking students to provide these missing steps enables them to work through the details and demonstrate an understanding of the proof.) Finally, we asked students to comment on the differences between the original and reconstructed photo and to propose how those differences might have been reduced by modifying their approach.

REFLECTIONS

Finally, we asked the students to reflect on their experiences working on this project by providing them with prompts, including "What were the most enjoyable parts of the experience? What were the most difficult aspects? What were the hardest parts about managing your group? Were there any unexpected discoveries or surprises?" and "What was it like working with someone in a field completely different from your own?"

In addition, the honors students considered and responded to the following questions: "Did this project change your perception of what it means to solve a 'real world' problem using mathematics? Is this the type of activity you imagined mathematicians doing? Did you feel like you were 'doing mathematics'?" and "Did you feel that this project still required analytical thought, even though the 'math' wasn't difficult?"

PROJECT GOALS

In assigning this joint project, our primary goal was to engage both the honors students and the senior math majors in experiencing several aspects of applied work that are familiar to any practicing scientist:

1. *Communication is vital to intellectual work*. A large portion of this project required the students to produce a clear record of their work that could be understood by a general audience without sacrificing fidelity to the technical details. In addition, by design the math majors were challenged to lead a team of younger, non-science students who did not speak the "language of mathematics." This challenge presented a learning opportunity not only for the honors students (who likely benefited from the peer-level tutoring) but also for the math majors. As noted by the National Council of Teachers of Mathematics (NCTM), "when students are challenged to think and reason about mathematics and to communicate the results of their thinking to others orally or in writing, they learn to be clear and convincing . . . students who have opportunities, encouragement, and support for speaking, writing, reading, and listening in mathematics classes reap dual benefits: they communicate to learn mathematics, and they learn to communicate mathematically" ("Guiding Principles" 3).
2. *Applied problems rarely follow a straight line from problem to solution.* In any field, problem-solving skills go hand in hand with theoretical understanding. Although the problem is initially stated in an idealized form, real-world problems quickly emerge that must be dealt with by dynamically and creatively adjusting the method and observing the modified results. The framework of assumptions and definitions that defines the end result is often not completely understood until the end. "Effective problem solvers constantly monitor and adjust what they are doing. Effective problem solvers plan frequently. They periodically take stock of their progress to see whether

they seem to be on the right track. If they decide they are not making progress, they stop to consider alternatives and do not hesitate to take a completely different approach" (NCTM, "Principles and Standards" 54).

3. *Creative and/or investigative projects increasingly involve interdisciplinary teamwork.* Generating lasting solutions to today's complicated problems requires insight and techniques from a wide variety of disciplines. Once our students leave LMU, regardless of their major they will undoubtedly be working with others having different backgrounds. This project provided a glimpse into their future as the teams consisted of three or four students, no two having the same major. The importance of teamwork is acknowledged in multiple disciplines; for example, Julius Jackson, in *Math and Bio 2010: Linking Undergraduate Disciplines*, writes that the "undergraduate curriculum must change to raise expectations and meet the challenge to educate new biological scientists capable of joining in research collaboration with mathematicians, computer scientists, physical scientists, and engineers to solve biological questions" (56).

4. An additional goal, meant primarily for the mostly liberal-arts honors students, was to *draw a connection between the use and the understanding of a mathematical method*. Most of us can relate to the experience of being given a formula, recipe, or other list of steps and then being asked to follow it blindly in order to produce a desired output. The two formulas needed to complete the computations in this project involve only simple high-school algebra, yet their simplicity obscures their origins, and a surface reading of the method offers little in the way of understanding why it works. Here the honors students had to rely primarily on their math major consultants rather than their professors (again providing an opportunity for the math majors to communicate their knowledge at an appropriate level) for an explanation of the *why*. Again from the NCTM: "Listening to others' explanations gives students opportunities to develop their own understandings. Conversations in which mathematical ideas are explored from multiple perspectives help the participants sharpen their thinking and make connections" ("Principles and Standards" 60).

SAMPLE OUTCOMES

We now present the work of two student teams. The first example features St. Robert's Hall (Figure 2), which was originally built in 1929 as one of only two buildings on the Westchester campus of Loyola Marymount University. Caroline Barlett, Oneika Barrett, and Luke Vetter were assisted by senior Jessica Young and charged with reconstructing a photograph of this building taken in the 1940s. The final multimedia presentation by this team, including a description of the process and a cinematic dissolve of the original photograph into the new one can, be found at: <http://myweb.lmu.edu/acrans/StRoberts.m4v>.

Figure 2: St. Robert's Hall in 1940s (original photo) St. Robert's Hall in 2010 (reconstructed photo)

Our second example is a photograph of Sacred Heart Chapel, which was finally completed in 1953 after being delayed for a year by the Hughes Aircraft Company, who claimed that the bell tower might interfere with their radar system. Angelica Cadiente, Monica Rosales, and Jack Shain, in consultation with mathematics major (and honors student) Kayla Pietruszka, reproduced a photograph of the 1953 Loyola Marymount University commencement ceremony (Figure 3). The final presentation of this team, including a wonderful history of the chapel, a description of the process, and a cinematic dissolve of the original photograph into the new one can be found at: <http://myweb.lmu.edu/acrans/SacredHeart.mov>.

Figure 3: Sacred Heart Chapel in 1953 (original photo) Sacred Heart Chapel in 2010 (reconstructed photo)

OBSERVATIONS AND REFLECTIONS

The most frequently stated challenge in the students' reflections was the difficulty in finding time to meet as a group, given their differing majors and numerous commitments. Although we anticipated this problem and gave the students a month to complete their work along with a fairly detailed outline of the necessary steps, we did not micromanage their progress and usually intervened only when asked. As we hoped, this assignment provided the senior math majors with invaluable (although sometimes frustrating) experience in leadership of a diverse group, time management, and delegation of tasks.

The "imperfection" of the real world appeared as the second most common challenge. One honors student stated, "the source of my greatest joy during the project was also the source of my greatest frustration. In finding where the original photo was taken and realizing that due to the change in landscape we would not be able to directly recreate the photo was in a way perfectly imperfect. In a field where the answer is either right or wrong . . . our pure math was right, but a means to make it applicable to the task at hand did not exist." This comment is revealing in that it highlights a common preconception (due, undoubtedly, to how the subject is taught) that mathematics is purely computational, driven by preexisting formulas, and that these computations must have binary outcomes (right or wrong) with no room for "fuzzy thinking." Another student commented, "the most unexpected discovery during this project was to how much of an extent mathematics cannot account for all the imperfections of the real world. For instance, there is no formula for getting rid of physical obstacles in your way." This student's comment sums up the opinion of many non-science students in thinking that, if they only just use the formula appropriately, their answer will undoubtedly be correct. The realization of this student that such clear answers do not occur in "real world" problems is one we hope many students shared. Indeed, a burgeoning recognition in the students that mathematics involves a coupling of creativity, initiative, and analytical thought seemed to be present: "the math in some sense felt the same as I had always known it, plugging in numbers and doing basic arithmetic, but in [another sense] it was all brand new and not math at all." Another wrote that "applied math is not sitting in a room . . . in this case, it was actually going out into the field and solving a problem . . ."

The senior math majors had a wide range of experiences, but most agreed that having to play a leadership role, some for their first time, was the most challenging part. Reflecting on this component of the project, one senior wrote that "this was a good way to familiarize me with, hopefully, what I will be doing in my future career." Many of the honors students expressed a strong dislike (or even phobia) of mathematics, and, despite the fact that the formulas themselves did not require the technical expertise of someone trained in advanced mathematics, some seniors found it easier simply to perform the calculations themselves rather than guide the honors students through them. One senior wrote, "[a]s someone who loves math and finds the proofs fascinating, it was somewhat difficult to explain it to people who did not care for it to be explained in the first place." However, another senior rose to the challenge of mentoring the honors students, writing that she "explained [to her team] that sometimes people get a mental block toward doing math because they believe they cannot do it, and this becomes a self-fulfilling prophecy."

Carrying out the measurements and calculations without having any understanding of why the method works is obviously possible. One of the goals of the project was to encourage the students to move beyond plugging numbers into a formula. To this end, we instructed the seniors to explain the proof of the procedure to their group members, a definite challenge. Some honors students appreciated this explanation, including the students who stated that "it led to our understanding of why a formula works because we were required to understand how the formula came to be through the proof. I found this very helpful"; another wrote, "I realized that this was doable, and not only was it doable, it was something that I'd learned before. I remember learning about similar triangles in my geometry class in high school, and I was extremely surprised to find that I still remembered why certain angles had to be equal." Coming from students who openly and actively expressed their fear and frustration with mathematics, these comments were encouraging.

Yet there remained a disconnect between understanding and implementation in some students who found it difficult to see the importance of the proof when they could simply use the formulas without a deep comprehension of it. Finding a means through which to make this connection more explicitly will be important in future versions of this project. Nevertheless, we feel strongly that simply having the honors students exposed to the proof of the process held enormous value (especially considering that this course may be one of the only undergraduate math classes they ever take). The majority of these students' mathematical experiences up to this point had consisted of being handed formulas by their teachers or textbooks with little or no explanation or justification for why they hold true. They, like the majority of K–12 students, were likely taught how to "plug and chug" using the formulas provided in the chapter to solve the problems of that chapter. As observed in the NCTM Curriculum and Evaluation Standards: "[F]or many non-mathematicians, arithmetic operations, algebraic manipulations, and geometric terms and theorems constitute the elements of the discipline to be taught in grades K–12. This may reflect the mathematics they studied in school or college rather than a clear insight into the discipline itself" (6). Thus, merely seeing that the formulas they were using had justification (and a lengthy one at that), together with the verbal reinforcement of the proof provided by the seniors, was a small step toward having the students achieve the NCTM standard of recognizing "reasoning and proof as fundamental aspects of mathematics" ("Principles and Standards" 187).

FINAL THOUGHTS

This joint project was a self-contained module that fit neatly into the existing structure of two separate courses and allowed for a meaningful interaction between two disparate groups of students in a reasonable amount of time. Our hope was to expand the honors students' perception, understanding, and appreciation of what constitutes the art and practice of mathematics; give our senior math majors a taste of what it is like to lead a diverse team on a technical project (a task with which they may very well be faced upon graduation); and provide all the students the opportunity to exercise their creative and critical-thinking skills. We were encouraged to see evidence of changed perceptions, with one student remarking, "after all the discussion about applied math vs. pure math, it was nice to finally see what applied/'real world' math was. This was not the type of activity I would imagine mathematicians doing . . . I really did feel as though I was 'doing mathematics.' Although the mathematical formulas in this project were relatively simple, finding the location did require some amount of analytical thought."

One of the most rewarding outcomes was seeing that the seniors, perhaps unknowingly, changed some of the honors students' impressions of the type of student who chooses to major in mathematics; as two honors students remarked, "[our senior] broke the stereotype of the 'typical' math major," and "it was even nicer that my math major was a woman—it once again helped to dispel the notion that mathematics is a male field."

In future implementations of this project, we will likely draw more attention to the imprecise nature of the method and emphasize that errors and approximations are expected to occur. One possible modification of the assignment is asking the teams to carefully analyze the process they used in producing their first photo and to identify any major obstacles, come up with a revised method (perhaps using another mathematical or technical solution), and take a second photo, thus highlighting the iterative nature of problem solving. Another possible mathematical angle to pursue is an "error analysis," in which students examine the connection between errors in their measurements and errors in the final product. For example, if a student measures one of the photographs with a ruler that has only eighth-inch increments, by how many feet might the final photographer location be miscalculated?

Overall we felt this project was a success. It certainly paved the way for future collaborations between two separate entities at our university (the College of Science and Engineering and the Honors Program). Despite the various challenges noted above, the student teams followed the assignment with enthusiasm and produced remarkably impressive final presentations. We look forward to creating additional cross-course thematic experiences with the LMU Honors Program.

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