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Comment on "Beach Water Table Fluctuations Due to Wave Run-Up: Capillarity Effects" by L. Li et al.

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1. Introduction

Li et al. [1997] published an analysis of capillary effects on beach water table fluctuations for different wave frequencies. Toward this goal the authors used a coupled system of groundwater flow and shallow water equations. While the latter is standard, the former includes a more thorough exploration of a water exchange term in the boundary condition on the moving free surface. The authors should be commended for finding a new operational way to account for the interaction between the saturated and unsaturated zones. A kinematic boundary condition for the water table is combined with a one-dimensional approximation of the recharge term as suggested by *Parlange and Brutsaert* [1987], who indirectly employed the Richards model. This condition was applied to the one-dimensional Boussinesq equation under Dupuit-Forchheimer assumptions. Li et al. extend the use of this term to quantify the effect of capillarity on the propagation inland of high-frequency sea level fluctuations.

Unfortunately, this analysis makes no mention of extensive new developments in the well hydraulics of unconfined aquifers, supported by theoretical, laboratory, and, more importantly, field studies that explore the role of capillarity using Boulton's model [*Moench*, 1995]. The goals of this comment are (1) to show a link between the two approaches and (2) to quantify Boulton's parameter with parameters derived by *Li et al.*'s [1997] boundary condition. The results of Li et al. shed new light on the physical meaning of the parameter α called the reciprocal of the "delay index" by *Boulton* [1954, 1955].

2. Background

Historically, *Boulton* [1954, 1955] was the first to investigate separately two- and three-dimensional groundwater flow, a moving water table, and slow drainage (delayed yield) from the unsaturated zone for interpretation of pumping tests in unconfined aquifers. *Boulton* [1955] presented an extension of the traditional, by that time, *Theis* [1935] two-dimensional approach. An axisymmetric model based on the Dupuit assumption was supplemented by a recharge term. This integral term was an attempt to describe flow to the aquifer from the unsaturated zone; however, the model did not use an equation for the water table in explicit form. *Boulton* [1954] used a radically different model: an equation for the moving boundary was explicitly introduced, and a fully three-dimensional groundwater flow model was adopted to account for vertical flow in the unconfined aquifer. However, two important physical properties of the unconfined aquifer were omitted by *Boulton* [1954]: aquifer compressibility and delayed yield. *Boulton and Pontin* [1971] improved the *Boulton* [1954] model by considering a delayed yield term in the condition of the moving boundary. Yet aquifer compressibility was not included in the more general model.

Efforts by *Prickett* [1965] to classify characteristic values of the Boulton parameter for different formations using the first *Boulton* [1955] model were unsuccessful because there was no complete theoretical basis for pumping tests in unconfined aquifers at the time. At the same time, *Dagan* [1967] improved technical aspects of the *Boulton* [1954] model by considering partial penetration of the pumping and observation wells. The model captured basic features of many practical problems but produced unrealistic drawdown behavior at short times.

Neuman [1972, 1974, 1975, 1979] and *Streltsova* [1972] demonstrated that the drawdown pattern stemming from the delayed yield model can be reproduced by considering aquifer compressibility in the three-dimensional axisymmetrical groundwater flow equation and aquifer specific yield in the condition at the moving boundary. Also, *Streltsova* [1972, p. 1065], *Neuman* [1975, p. 336], and *Gambolati* [1976] suggested that the Boulton index can be simply expressed from parameters of the unconfined aquifer. Importantly, *Neuman* [1975] developed a computer algorithm and a method for parameter identification which were widely used later by practicing hydrologists. The *Neuman* [1974] model can be considered as a direct extension of the *Dagan* [1967] model.

Since no good method was proposed for identification of the elusive Boulton parameter α, the actual hydrodynamic problem of delineation of the effect of capillarity turned out to be more challenging than it seemed initially. Unfortunately, additional investigations on the relative importance of compressibility and partial saturation in unconfined aquifers did not resolve this problem. *Brutsaert and El-Kady* [1984, p. 400], *El-Kady and Brutsaert* [1986, pp. 1361, 1365], and *Parlange and Brutsaert* [1987] attacked this problem by applying the one-dimensional Boussinesq models neglecting the compressibility and vertical flow component under the water table. *El-Kady and Brutsaert* [1986] analyzed a laboratory outflow experiment [*Higgins*, 1980] by combining the Boussinesq equation with the *Boulton* [1955] term for delayed yield. Naturally, such "partial" use of the Boulton model was unsuccessful, and they concluded that the Boulton model was therefore invalid.

Meanwhile, *Neuman* [1981] explained the *Higgins* [1980] experiments by using a proper two-dimensional problem statement with a proper kinematic condition at the water table, and he included compressible properties of the aquifer. The resulting agreement between theory and experiment tends to support the point of view that the role of delayed yield (or flow from the unsaturated zone) is secondary to effects of aquifer compressibility and vertical component of groundwater velocity.

El-Kady and Brutsaert [1986, p. 1361] properly summarized the situation that occurred as follows:

> ... numerous papers dealing with the delayed yield problem have been published (see *Neuman* [1979] for a review). The delayed

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yield observed in pumping tests has been attributed to various factors such as unsaturated flow above the water table . . . By any objective criterion the matter remains unclear, but it would seem that any or all of these effects could play a role depending on the flow situation under consideration.

While all of the above mentioned studies dealt primarily with theoretical matters, the real breakthrough on the significance of delayed yield occurred when actual field data on pumping tests in unconfined aquifers were investigated by *Moench* [1995]. In a series of papers, *Moench* [1993, 1994, 1997] also provided a review of recent developments on various aspects of delayed yield, and he proposed an effective methodology of interpretation of pumping tests in unconfined aquifers including computer program and curve-matching algorithm. A significant improvement of the *Neuman* [1974] model has been achieved in simulation of shallow piezometer responses during pumping tests [e.g., *Nwankwor et al.*, 1992] by combining the Neuman and Boulton models [*Moench*, 1995], which could not be explained otherwise. One should notice that this has been achieved by adding a "delayed yield" term to a kinematic boundary condition for the moving surface, which is hereinafter called the Boulton-Neuman-Moench (BNM) condition.

3. Comparison of Boundary Conditions

I will now demonstrate the equivalence of the BNM and *Li et al.* [1997] boundary conditions. I start from the operational linearized boundary equation used by *Moench* [1995, equation (8), p. 380],

$$K_z \frac{\partial h}{\partial z} = -\alpha S_y \int_0^t \frac{\partial h}{\partial t'} e^{-\alpha(t-t')}\, dt' \qquad z = \eta(x, y, t) \approx b \tag{1}$$

and the boundary condition derived by *Li et al.* [1997, equation (4), p. 936],

$$n_e \cos \beta \frac{\partial \phi}{\partial t} = -K \frac{\partial \phi}{\partial n} - B \frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial n}\right) \qquad z = \eta(x, y, t) \tag{2}$$

Since both cited papers use different notation and nonoverlapping lists of references, the notation has to be reconciled. Here h and ϕ denote the hydraulic head with some horizontal reference level (below the initial horizontal static water level), S_y and n_e denote the specific yield (effective porosity), K and K_z denote the vertical hydraulic conductivity, α is the Boulton parameter, B is the thickness of the capillary fringe, $\eta(x, y, t)$ is the equation of the water table surface in Cartesian coordinates (x, y), z is the vertical coordinate taken as elevation above the base of the aquifer, n is the local coordinate on the boundary in the normal direction outward from the flow domain bounded by the water table, $\partial/\partial n$ is the gradient normal to the water table, b is thickness of the unconfined aquifer for a static water table, β is the angle between the free surface and x axis, and t is time.

Boundary condition (2) is nonlinear because function $\eta(x, y, t)$ is unknown a priori. A linearized form is obtained without loss of accuracy [*Dagan*, 1967] by assuming negligible slope and variations of a free surface ($\cos \beta \approx 1$, $\eta(x, y, t) \approx b$, and $\partial/\partial n \approx \partial/\partial z$). Thus (2) becomes

$$S_y \frac{\partial h}{\partial t} = -K_z \frac{\partial h}{\partial z} - B \frac{\partial}{\partial t}\left(\frac{\partial h}{\partial z}\right) \qquad z \approx b \tag{3}$$

Both conditions assume that initial (static) conditions are taken for a flat horizontal water table:

$$h(x, y, z, 0) = 0 \qquad \frac{\partial h(x, y, z, 0)}{\partial z} = 0 \qquad z \approx b \tag{4}$$

Now one can demonstrate the equivalency of boundary conditions (1) and (3) assuming that all other components of the boundary value problem with initial conditions are identical. The procedure involves the Laplace transformation with parameter p of these equations together with conditions (4). The transform of (1) subject to (4) for the Laplace transform of hydraulic head $\bar{h}(x, y, z, p)$ is

$$K_z \frac{\partial \bar{h}}{\partial z} = -\frac{\alpha S_y p}{p + \alpha} \bar{h} \qquad z \approx b \tag{5}$$

and the transformed condition (3) subject to (4) is

$$(K_z + Bp) \frac{\partial \bar{h}}{\partial z} = -S_y p \bar{h} \qquad z \approx b \tag{6}$$

These boundary conditions in Laplace transform space are equivalent when

$$\alpha = K_z/B \tag{7}$$

The importance of this relationship between the Boulton parameter and the thickness of capillary fringe is twofold. First, it shows the link (or equivalency) between the Boulton and *Li et al.* [1997] approaches. Second, it allows for expression of the Boulton parameter in physical terms. Of course, the rigor of such a relationship is only as good as the physical meaning of the capillary fringe thickness. It should be noted that (6) becomes the same as the condition used by Neuman when $B = 0$ (or $\alpha \to \infty$), which implies instantaneous drainage of the unsaturated zone in response to a decline in the water table.

This equivalency readily explains why the water table is sensitive to wave run-up with high frequency in sandy beaches found by *Li et al.* [1997]. It is obvious that the Boulton parameter has "large" values for sands. The Boulton integral indicates that a high value of the parameter α makes capillarity important to free-surface movement only for relatively short times, on the order of value $1/\alpha$. This means that high-frequency perturbations of flow under such conditions must have a period T_w comparable to or less than a characteristic time $1/\alpha$. Substituting the expression for α from (7), one obtains the criterion

$$T_w < 1/\alpha = B/K_z \tag{8}$$

which is the alternative derivation of the criterion for the relative importance of capillary effects on water table movement by *Li et al.* [1997, p. 938]. Interestingly, the parameter B/K_z was previously introduced by *Parlange and Brutsaert* [1987, p. 808].

4. Summary

Li et al.'s [1997] boundary condition for description of water table response under capillary effects is equivalent to Boulton's condition although in a different form. This leads one to address the area of potential hydrological applications of the model. In pumping tests the free surface is generally exposed to a monotonic head change, and the BNM boundary condition, considering monotonic time drawdown, is quite legitimate. However, simulation of periodic fluctuations in the water table would require an analysis that includes hysteresis, an effect that appears to have been neglected by *Li et al.* [1977]. Nevertheless, *Li et al.*'s [1997] results are important because they help to establish a link between the Boulton "effective" parameter and "physical" aquifer parameters for falling water table conditions, as it follows from the given explanation.

Several problems for future research arise from the discussion:

1. Is the *Li et al.* [1997] condition on a moving boundary applicable for simulation of pumping tests in water table aquifers?
2. Are hysteretic effects in a capillary fringe negligible in modeling of oscillatory water table movement?
3. Is the Boulton integral term in the kinematic condition on a moving boundary (together with the introduction of aquifer compressibility) an alternative (or improvement) to the *Li et al.* [1997] boundary condition?

Analysis of field data on pumping tests in unconfined aquifers can be a benchmark problem to clarify these issues.

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