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A Reformulation Strategy for Multi-Dimensional CSPs: The Case Study of the SET Game

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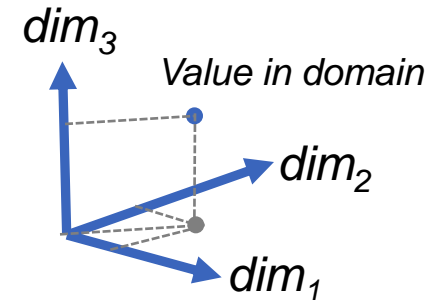
Outline

- **General reformulation strategy for CSPs**
 - Multidimensional CSPs (MD-CSPs)
 - Problem reformulation by value interchangeability
 - A general reformulation strategy for MD-CSPs
- Game of Set: A new toy problem
 - Game, CSP model
 - Problem reformulation
 - Algorithms & Results
- Conclusions

Multi-Dimensional CSPs

[Yoshikawa+ 1992]

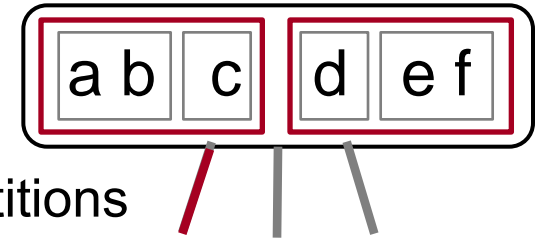
dim/dom	v_1	v_2	v_3	v_4	v_5	v_6	v_7
dim_1	3	l	l	2	3	3	l
dim_2	r	r	g	p	g	r	p
dim_3	f	e	e	f	f	f	f



- All variables have the same domain
- Domain is multi-dimensional
 - A set of dimensions
 - Each domain value is described by a combination of dimensions values
- In MD-CSPs, a constraint can be
 - One-dimensional: defined over a single dimension
 - Multi-dimensional, otherwise
- Typical applications
 - Scheduling, resource allocation, configuration, etc.

Reformulation by value interchangeability

- Value interchangeability [Freuder 91]
 - Domain abstraction: equivalent values
 - ‘Perfect’ equivalence rare, small domain partitions
 - Ignoring some constraints yields larger domain partitions, smaller CSPs, smaller search space [Haselboeck 93, Choueiry+ 94]
- Abstraction in MD-CSPs [Freuder+ 95,97]
 - Abstract domains based on a domain dimension, P_r
 - Solve reformulated CSP
 - Use solution of P_r to guide solving original CSP, P_o
- How to “use solution of P_r to solve P_o ”? Hard to automate



Reformulation Strategy for MD-CSPs

- Process

For each one-dimensional constraint

Abstract domains using interchangeability

Enforce one-dimensional constraint

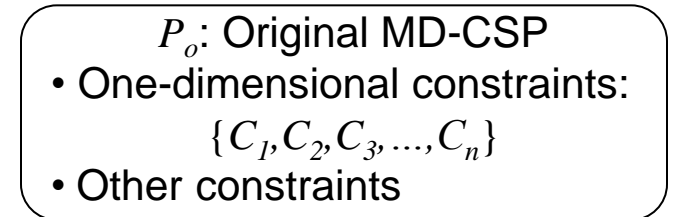
Solve remaining CSPs with some solver

- Questions

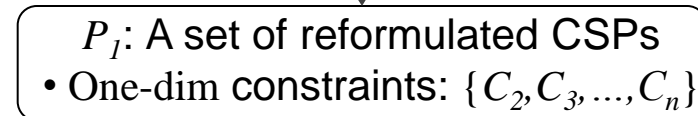
- Which 1-dim constraint to use first?

- How to process reformulated problems?

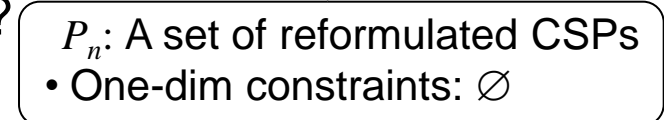
- Case study of the Set game



Exploit approximate symmetries to enforce C_1



Exploit approximate symmetries to enforce C_2



Enforce remaining constraints

Outline

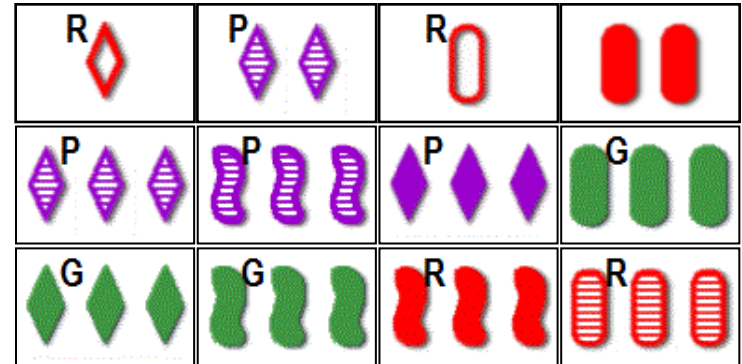
- General reformulation strategy for CSPs
 - Multidimensional CSPs (MD-CSPs)
 - Problem reformulation by value interchangeability
 - A general reformulation strategy for MD-CSPs
- **Game of Set: A new toy problem**
 - Game, CSP model
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Game of Set

[Falco 74]

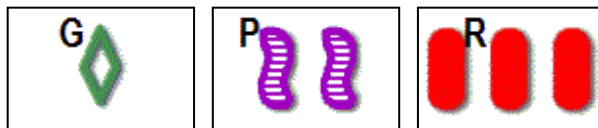
- Deck of 81(=3⁴) cards, each card with a unique combination of 4 attributes values

1. *Number* ∈ {1,2,3}
2. *Color* ∈ {green,purple,red}
3. *Filling* ∈ {empty,stripes,full}
4. *Shape* ∈ {diamond,squiggle,oval}



- Solution set: 3 cards

∀ attribute, the 3 cards have either the same value or all different values

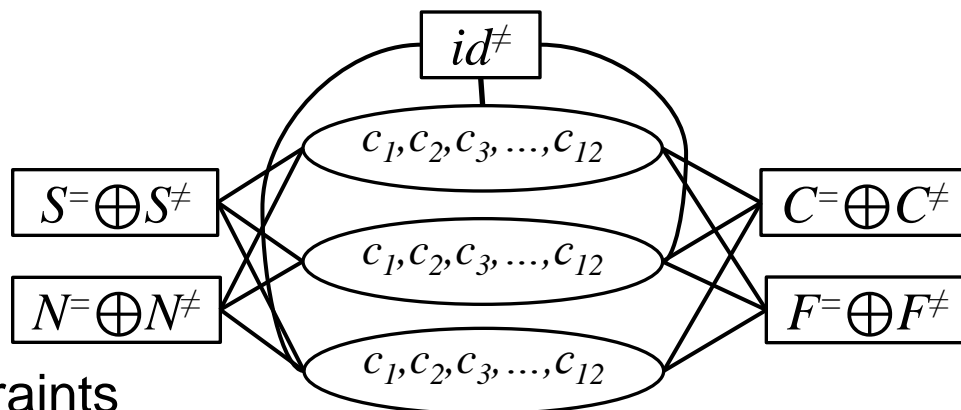


- 12 cards are dealt, on table [3,21]
- Recreational game, favorite of children & CS/Math students
- New toy problem for AI: a typical multi-dimensional CSP

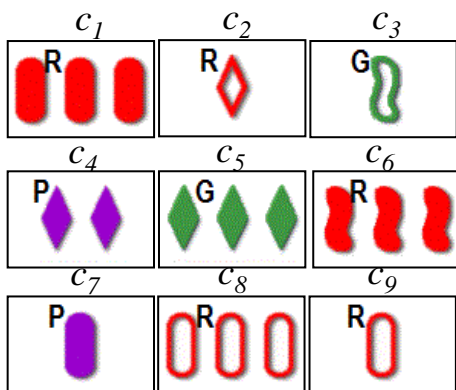
Set as an MD-CSP

- Model

- Three variables
- Same domain (12 cards)
- One ‘physical’ constraints
- Four 1-dimensional constraints



Same domain for all 3 variables

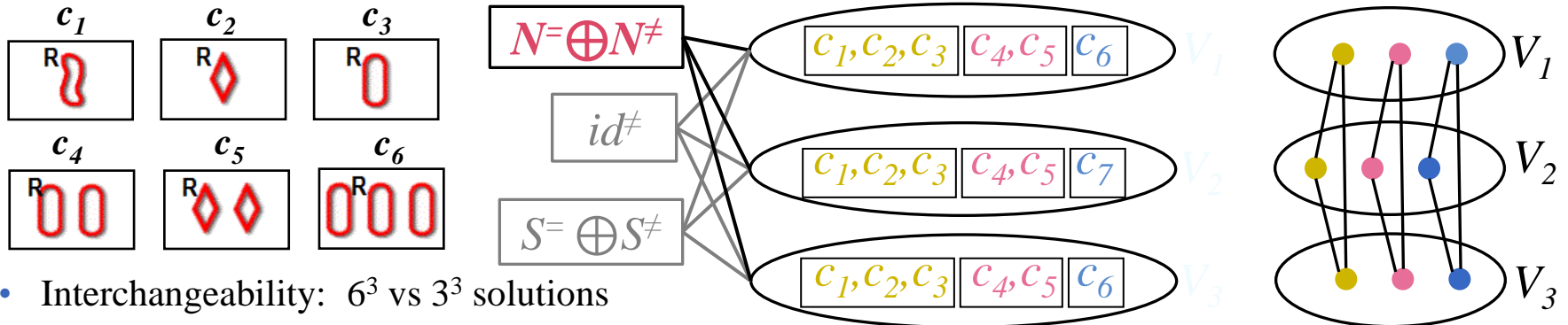


Domain Table

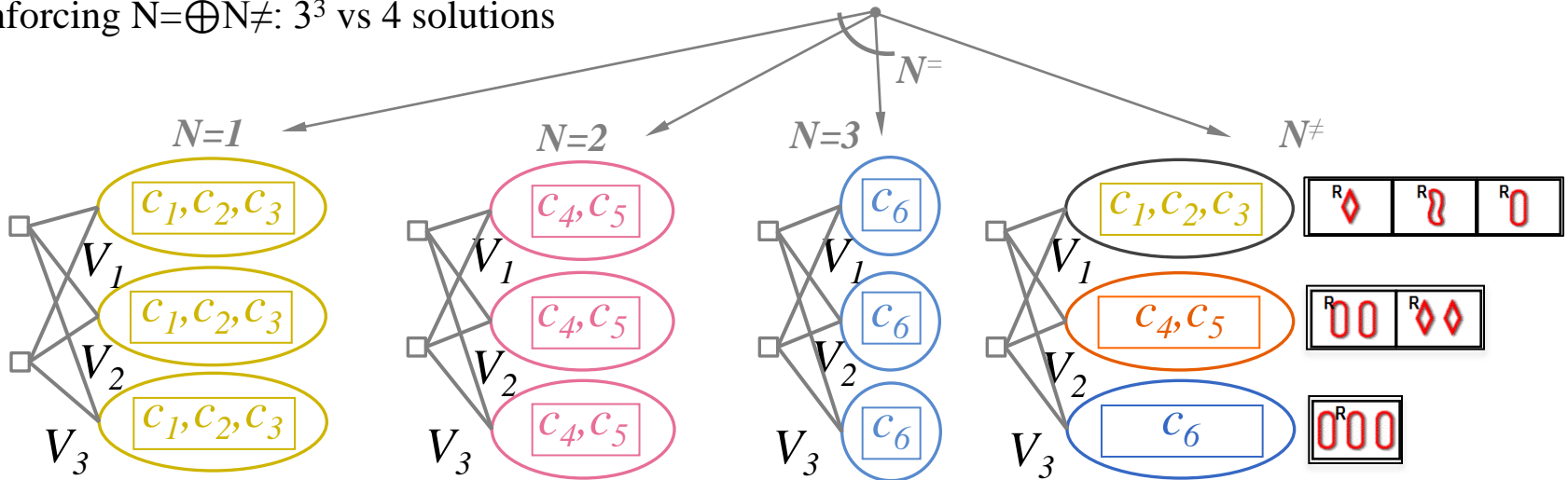
Domain dimensions

		c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9
Number	1	0	1	1	0	0	0	1	0	1
	2	0	0	0	1	0	0	0	0	0
	3	1	0	0	0	1	1	0	1	0
Color	r	1	1	0	0	0	1	0	1	1
	g	0	0	1	0	1	0	0	0	0
	p	0	0	0	1	0	0	1	0	0
Filling	f	1	0	0	1	1	1	1	0	0
	e	0	1	1	0	0	0	0	1	1
	s	0	0	0	0	0	0	0	0	0
Shape	s	0	0	1	0	0	1	0	0	0
	o	1	0	0	0	0	0	1	1	1
	d	0	1	0	1	1	0	0	0	0

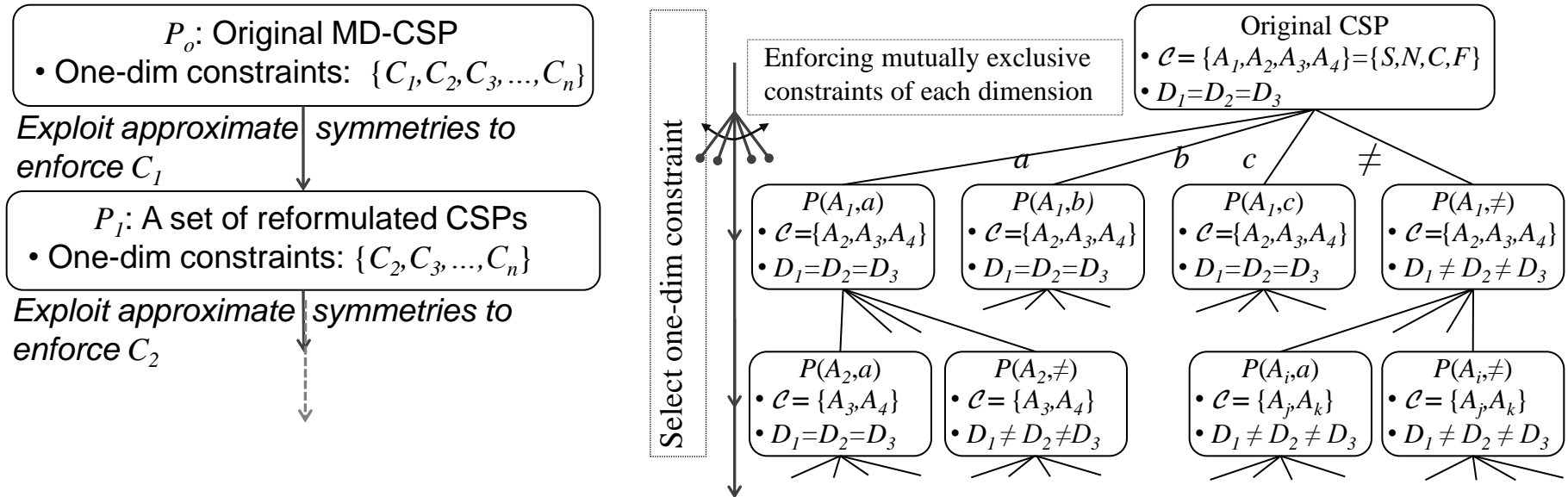
Reformulation by Value Interchangeability



- Interchangeability: 6^3 vs 3^3 solutions
- Enforcing $N = \bigoplus N \neq$: 3^3 vs 4 solutions



Reformulation Strategy for Set

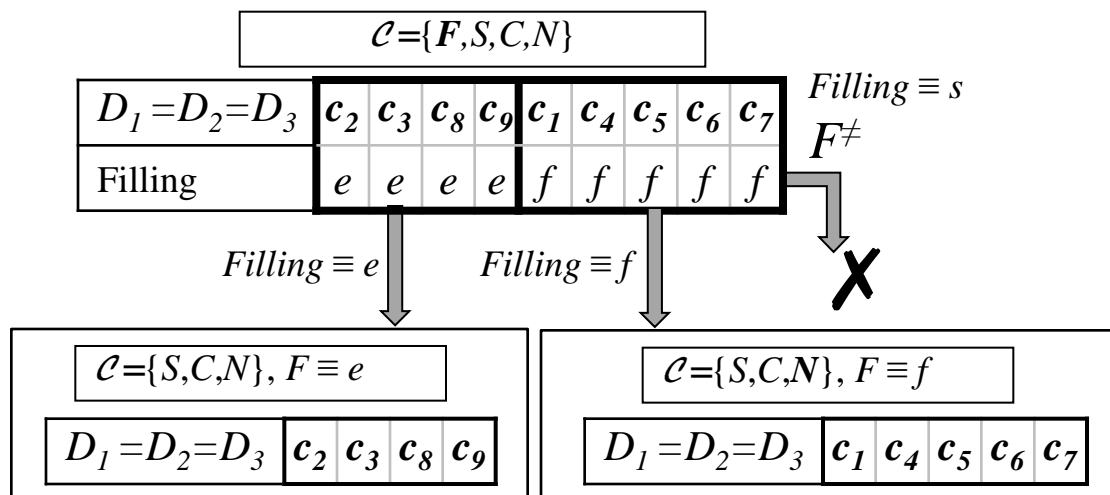


- Which dimension to choose first?
 - ↳ For Set, heuristics based on data in ‘Domain Table:’
 - Fewest subproblems first (infamously, Fail First Principle)

Selecting Domain Dimension

- Goal: Reduce branching factor
 - In example below, no card in domain has a shaded filling, thus, subproblems for $Filling=s$ and $F \neq$ do not exist

Domain		c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	Σ
Number	1	0	1	1	0	0	0	0	0	1	3
	2	0	0	0	1	0	0	0	0	0	1
	3	1	0	0	0	1	1	1	1	0	5
Color	r	1	1	0	0	0	1	0	1	1	5
	g	0	0	1	0	1	0	0	0	0	2
	p	0	0	0	1	0	0	1	0	0	2
Filling	f	1	0	0	1	1	1	1	0	0	5
	e	0	1	1	0	0	0	0	1	1	4
	s	0	0	0	0	0	0	0	0	0	0
Shape	s	0	0	1	0	0	1	0	0	0	2
	o	1	0	0	0	0	0	1	1	1	3
	d	0	1	0	1	1	0	0	0	0	4



- Our reformulation algorithm for Set
 - Uses ‘Domain Table’ & ‘Summary of Domain Table’
 - Has 4 tests & 5 heuristics

Algorithms: Finding all Solutions

1. Brute-force search (BF)

- 3-nested *for*-loops generate all combinations, then test for solutions
- Contradicts 40+ years of CP research & experience ☹
- Does not scale ($12^3 \sim d^n$)

2. Backtrack search (Basic Solver)

- Symmetry breaking (lexicographic ordering)
- Both forward-checking (equality) & back-checking (All-diff constraints)

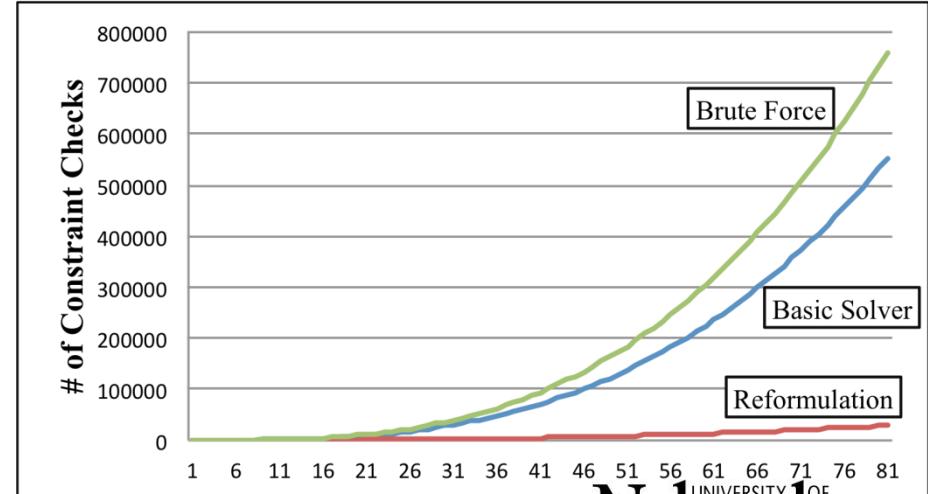
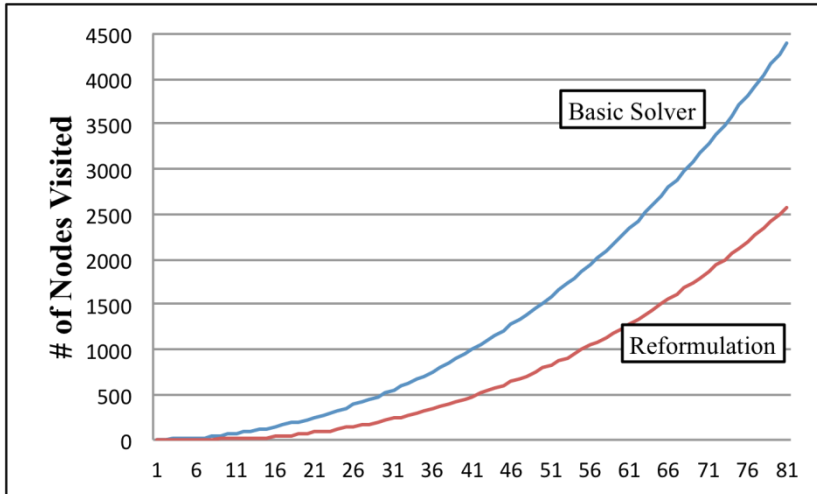
3. Reformulation-based algorithm

- Uses 2 data structures: ‘Domain Table’ & ‘Summary of Domain Table’
- Includes 5 selection heuristics
- Open subproblems maintained in an agenda: room for heuristics (1Sol)
- Empirical tests: randomly selected ‘hands’ of 3 to 81 cards, results averaged over of 1,000 runs

Results

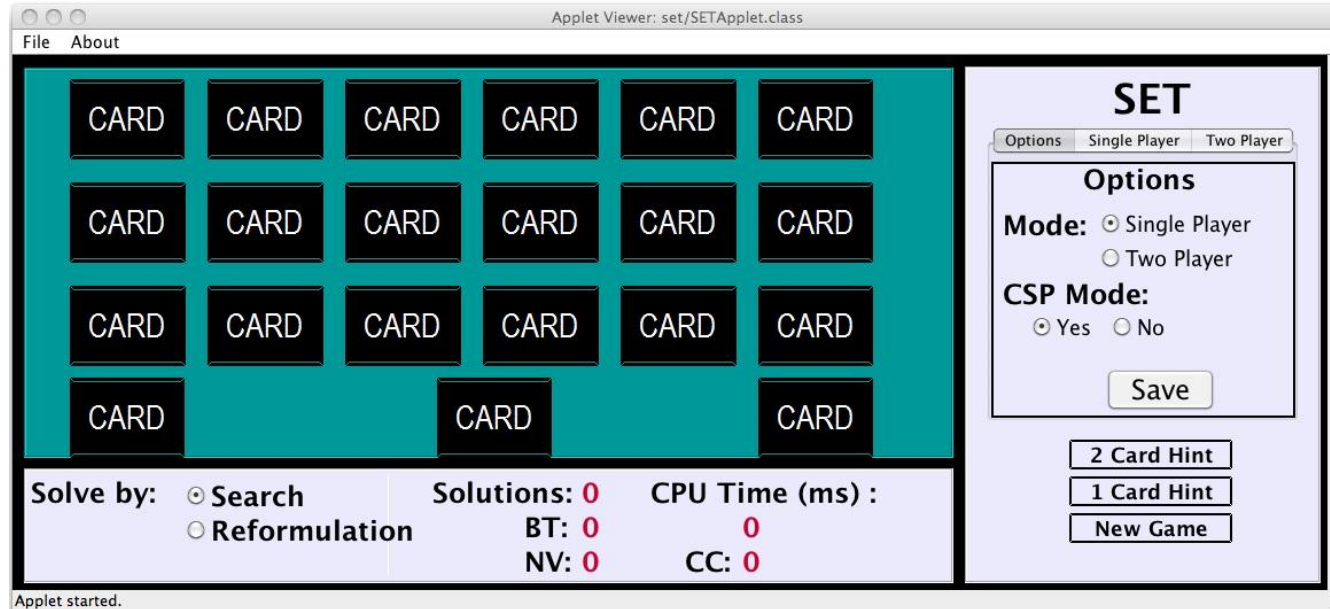
- **#CC,#NV**: Reformulation dramatically reduces # of combinations tested
- **CPU time** reflects the cost of setting up the data structures for the CSP & search

Algorithm	#Cards	#Sol	#CC	#NV	Time [msec]
Brute Force	12	2.77	1956.8	220	0
BT Search			1726.6	80.77	62.46
Reformulation			85.1	12.65	5.85
Brute Force	81	1080	758808	85320	0
BT Search			553365	4401	101.04
Reformulation			31158	2565	39.44



Online Game gameofset.unl.edu

- Game running online
- Interface explaining the reformulation still in development
- Advertzmt: minesweeper.unl.edu & sudoku.unl.edu (CP-based)



Conclusions

- Contributions
 - A systematic approach to reformulation and ‘conditional’ symmetries
 - Applicability to real-world problems highly promising
 - A new toy problem for AI research & education 😊
- Technical issues
 - Generalize heuristics for dimension selection and problem decomposition
 - Explore other types of interchangeability/symmetries
 - Extend definition of MD-CSP to allow domains that are not all equal
- Modeling lessons
 - Dimensional domain **ANSWER: use tuple representation in Zinc, Essence**
 - CSP variables and values are often ‘objects’ with attributes
 - So far, we have integrated those attributes in the constraint definitions
 - Multi-dimensional CSPs are likely common in practice
 - Let’s not miss out on the opportunity of exploiting them
 - Dynamic reformulation of model during search **ANSWER: Not yet**