2011

# Reformulating R(*,*m*)C with Tree Decomposition

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Acknowledgements

- Experiments conducted at UNL's Holland Computing Center
- NSF Grant No. RI-111795

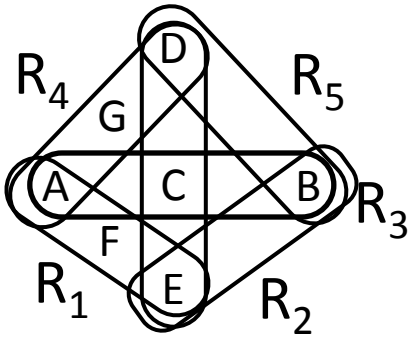
# Outline

- Introduction
- R(*,*m*)C Property & Algorithm
- Exploit Tree Decomposition to
  - Avoid useless update & reduce propagation effort
    - ↪ Update queue: PROCESSQ ⇝ PROCESSMQ
    - ↪ The two algorithms *yield the same filtering*
  - Synthesize & add new constraints to improve propagation
    - ↪ Property enforced: R(*,*m*)C ⇝ T-R(*,*m*,*z*)C
    - ↪ The same algorithm *yields stronger filtering*
- Experimental Results
- Conclusion

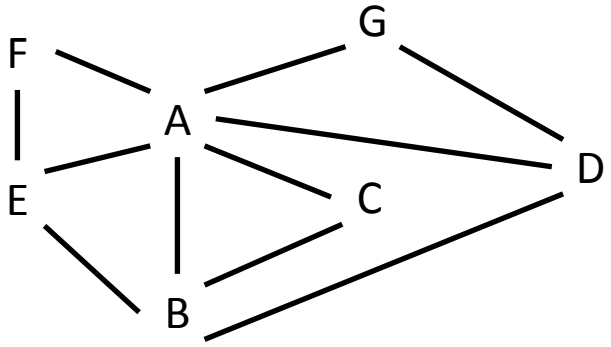
# Constraint Satisfaction Problem

- CSP
  - Variables ($\mathcal{V}$), domains
  - Constraints: relations ($\mathcal{R}$), scope
- Representation
  - Hypergraph
  - Primal graph
  - Dual graph
- Solved with
  - Search
  - Enforcing consistency
- Warning
  - Consistency property vs. algorithms

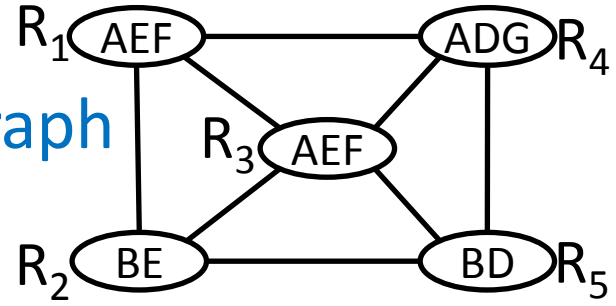
Hypergraph




Primal graph

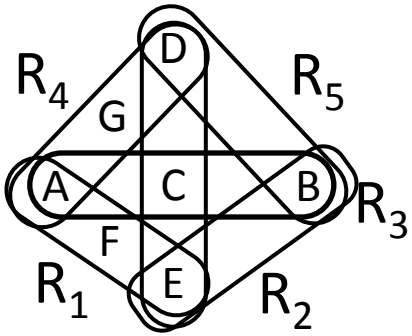


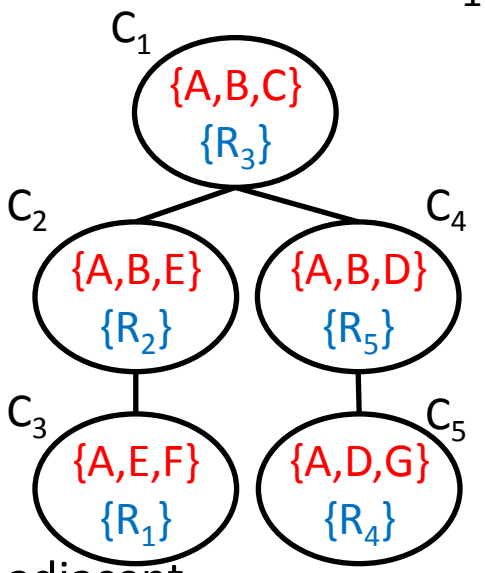

Dual graph

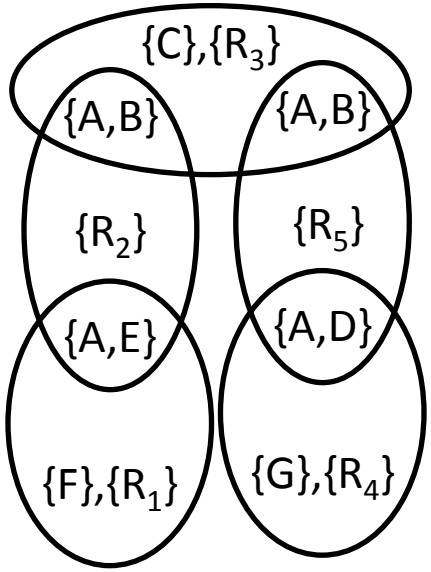


# Tree Decomposition

- Tree: Vertices/clusters, edges
- Each cluster is labeled with
  - A set of variables $\subseteq \mathcal{V}$
  - A set of relations $\subseteq \mathcal{R}$
- Two conditions
  1. For each relation R, $\exists$cluster $c_i$
     - R appears $c_i$
     - Scope(R) is also in $c_i$
  2. Every variable
     - Induces a connected subtree
- Separators
  - Variables & relations common to 2 adjacent clusters
  - channel communications between clusters

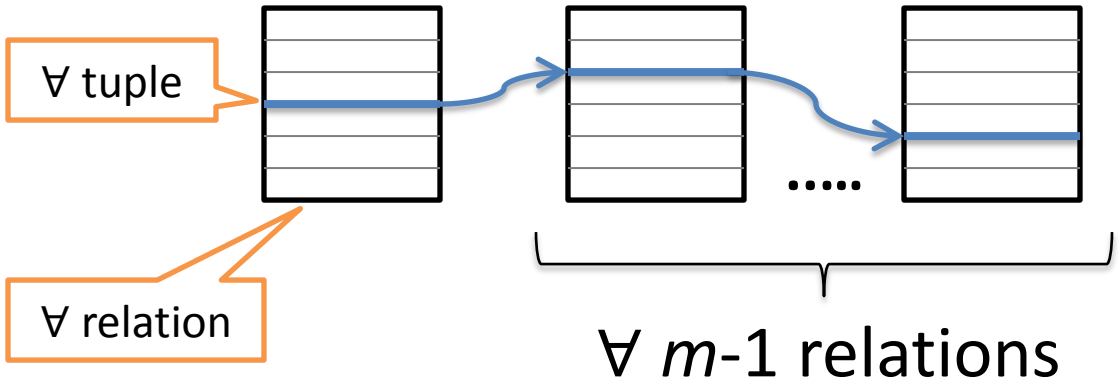


# R(*,*m*)C Property [Karakashian+ 10]

- A CSP is R(*,*m*)C iff
  - Every ***tuple*** in a relation can be extended to the variables in the scope of any (*m*-1) other relations in an assignment satisfying all *m* relations simultaneously



# ProcessQ: Algorithm for R(*,*m*)C

- Φ: combination of *m* connected relations in the dual graph

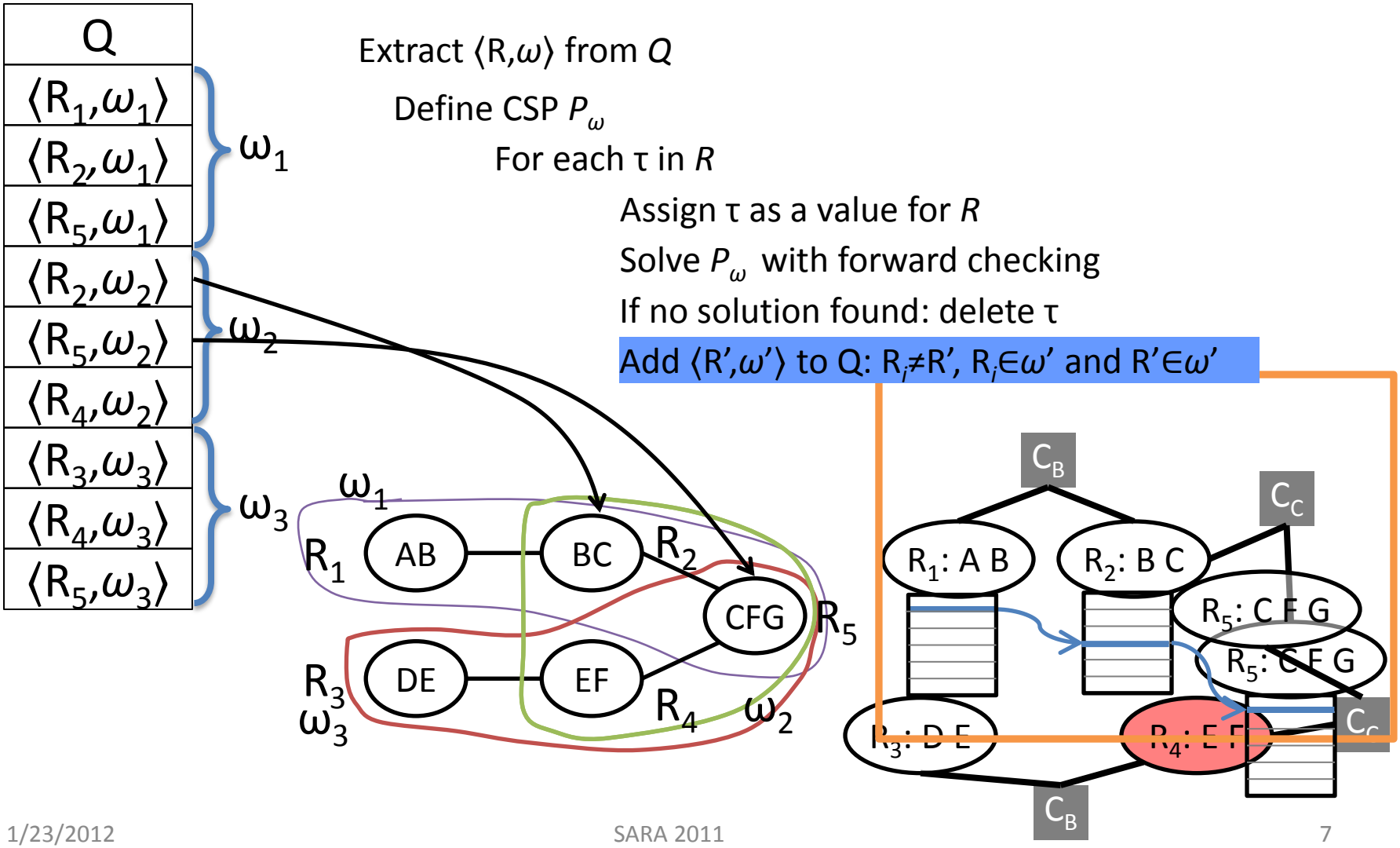
$$\Phi = \{ \omega_1=\{R_1,R_2,\ldots,R_m\}, \omega_2, \omega_3,\ldots, \omega_k\}$$

- Q propagation queue

$$Q=\{\langle R_1,\omega_1\rangle,\langle R_1,\omega_2\rangle,\langle R_1,\omega_3\rangle,\ldots. ,\langle R_n,\omega_{k-1}\rangle,\langle R_n,\omega_k\rangle\}$$

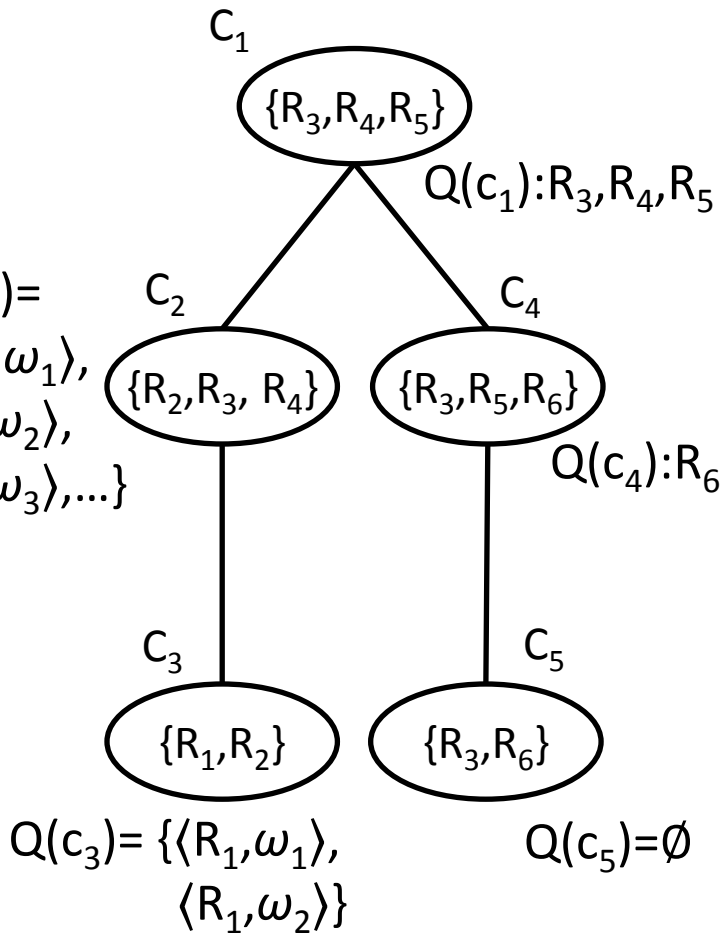
- For each $\langle R_i,\omega_j\rangle$ in Q, ProcessQ
  - Deletes from $R_i$ tuples that cannot extended to relations in $\omega_j$
  - As some tuples of relations $R_x \in \omega_j$ may lose support, it requeues $\{\langle R_x,\omega_y\rangle\}$ for every threatened relation

# ProcessQ: Animation

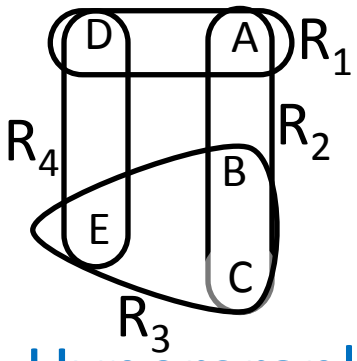


# ProcessMQ: Intelligent update scheduling

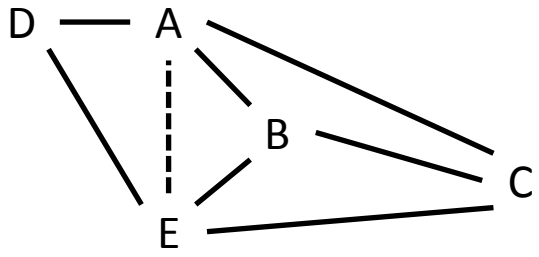
- Cluster $c_i$ has a local queue $Q(c_i)=\{\langle R_i,\omega\rangle\}$ for relations $R_i$ in cluster but not in parent
- Using the tree decomposition
  - As an ordering heuristic for checking consistency of $\langle R_i,\omega\rangle$
  - Repeat "leaves up to root, down to leaves," until quiescence
  - Update relations in only local queue
  - Example: $R_3$ is updated only when root is reached
- Advantage fewer updates, same filtering
  - In previous example, $R_3$ is updated once although it appears in 3 clusters



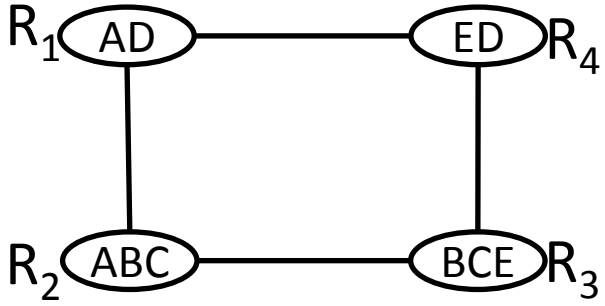
# T-R(*,*m*,*z*)C [Rollon+ 10]



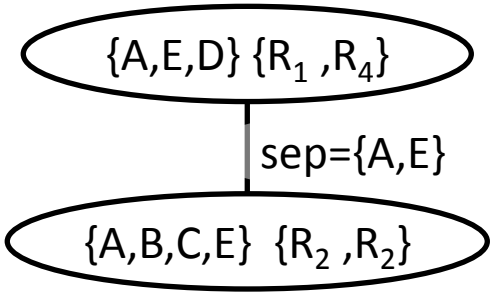

Hypergraph

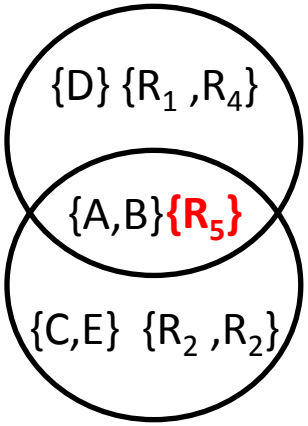


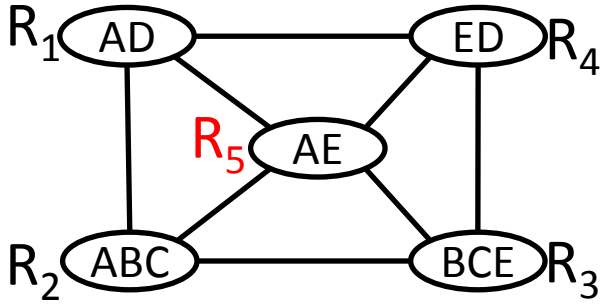

Primal graph




Dual graph




Tree decomposition Adding $R_5$

# T-R(*,*m*,*z*)C Strictly Stronger than R(*,*m*)C

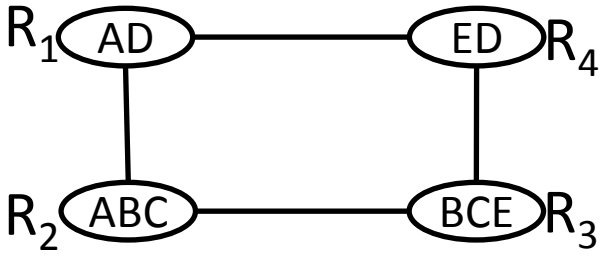
Let A, B, C, D and E be Boolean variables

| $R_1$ | |
|---|---|
| A | D |
| 0 | 0 |
| 1 | 1 |

| $R_2$ | | |
|---|---|---|
| A | B | C |
| 0 | 0 | 0 |
| 1 | 1 | 1 |

| $R_3$ | | |
|---|---|---|
| B | C | E |
| 0 | 0 | 0 |
| 1 | 1 | 1 |

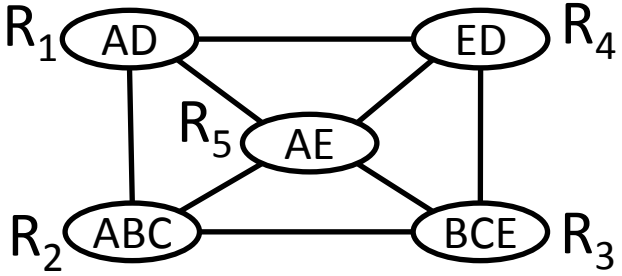
| $R_4$ | |
|---|---|
| E | D |
| 0 | 0 |
| 1 | 1 |




Assignment A=0 & E=1 is valid

Does not violate R(*,2)C

| $R_5$ | |
|---|---|
| A | E |
| 0 | 0 |
| 1 | 1 |




Assignment A=0 & E = 1 is **inconsistent**

# Experimental Results

- Experiments for finding all solutions with BTD maintaining
  wR(*,best(2,3,4))C and T-wR(*,best(2,3,4), best(5,7,9))
- Results shown demonstrate the benefits of ProcessMQ & T-wR(*,m,z)C

| Benchmark | #ins | #vars | tw | | ProcessQ wR(*,best)C | ProcessMQ wR(*,best)C | ProcessQ T-wR(*,b,b)C |
|---|---|---|---|---|---|---|---|
| aim-200 | 24 | 200 | 104.92 | #C | 17 | 17 | **22** |
| | | | | $t_{avg}$ | 246.35 | 252.48 | **238.99** |
| | | | | $t_{max}$ | 3,352.54 | 3,452.98 | **1,540.94** |
| ogdVg | 59 | 134 | 85 | #C | 15 | 15 | 15 |
| | | | | $t_{avg}$ | 283.27 | **242.06** | 266.74 |
| | | | | $t_{max}$ | 1,834.11 | **1,508.27** | 1,720.97 |
| rand-3-20-20 | 50 | 20 | 13 | #C | 13 | **14** | - |
| | | | | $t_{avg}$ | 2,191.56 | **1,949.87** | - |
| | | | | $t_{max}$ | 3,481.04 | **3,145.77** | - |

# Conclusions

- Contributions
  - Reformulated R(*,*m*)C algorithm
  - New relational consistency property T-R(*,*m*,*z*)C
  - Experimental analysis
- Future work
  - Study impact of choice of parameters *z*, *m*
  - Develop strategies for dynamically choosing *z*, *m* as a function of the size of clusters & separators