A Real-Time Information Based Demand-Side Management System in Smart Grid

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A Real-Time Information Based Demand-Side Management System in Smart Grid

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Abstract—In this paper, we study a real-time information based demand-side management (DSM) system with advanced communication networks in smart grid. DSM can smooth peak-to-average ratio (PAR) of power usage in the grid, which in turn reduces the waste of fuel and the emission of greenhouse gas. We first target to minimize PAR with a centralized scheme. To motivate power suppliers, we further propose another centralized scheme targeting minimum power generation cost. However, customers may not be motivated by a centralized scheme since such a scheme requires total control and privacy from them. A centralized scheme also requires too much real-time data exchange for frequent DSM deployment. To tackle these issues, we propose game theoretical approaches so that most of the computation is performed locally. In the proposed game, all the customers are motivated by extra savings if participating. Moreover, we prove that all parties benefit from the DSM system to the same level because both the centralized schemes and the game theoretical approach minimize global PAR. Such an analysis is further demonstrated by the simulation results and discussions. Additionally, we evaluate the performance of several (partially) distributed approaches in order to find the best way to deploy DSM system.

I. INTRODUCTION

The world has been changing revolutionarily towards a higher efficiency in many aspects due to the fast pace advancements of communication technologies. Among them, traditional power grid is evolving to smart grid lately based on two-way communication networks that connect service providers and customers [1–3]. For example, the advanced metering infrastructure (AMI) [4, 5] equips each customer with a smart meter, whose basic function is to gather the energy consumption status and upload the information to the control center (also known as power distributor or service provider). A smart meter is also capable of receiving control information (e.g., price and tariff bills) from the control center. Such a two-way information exchange is assumed to be near real-time ultimately [4].

A demand-side management (DSM, also known as demand response) system [6–8] further utilizes real-time information in order to let power grid generate and consume energy more efficiently while reducing unnecessary waste. A DSM system is widely agreed to be effective on reducing the peak-to-average ratio (PAR) of energy consumption [7–9]. This improvement helps power suppliers reduce extra fuel cost caused by dramatic and unpredictable margin fluctuations in power generation. Less fuel burning also helps reduce emission of greenhouse gas from those power generators. Moreover, since the control center gets the energy consumption schedule beforehand [10–13], the renewable sources such as photovoltaics (PV) farm and wind turbine field, which are less stable and less controllable compared with the conventional power generators, can support power grid more efficiently. A higher proportion of such renewable sources will further reduce fuel burning from the conventional power generators.

In this paper, we first propose a centralized optimization problem \( P_1 \) in order to reduce PAR to its minimum. Although a minimum PAR is obviously beneficial to the environment, however, it motivates neither power suppliers nor customers. Especially power suppliers must deploy and maintain a more complicated cyber system than what AMI can offer to gather and distribute a huge amount of detailed information in real time. Therefore, a monetary incentive is needed to motivate power suppliers. Another centralized optimization problem \( P_2 \) is then proposed to reduce the total energy generating cost of power suppliers. Although power suppliers may be willing to adopt the DSM system based on \( P_2 \), it is based on direct load control (DLC) [10, 14, 15], which could be defective. In terms of communications, even if the massive centralized problem can be solved efficiently, the transmission overhead will lay a huge burden on the communication network and require more advanced technical upgrade as well as more frequent maintenance. Moreover, the customers could be reluctant to adopt such a DSM system for two reasons. One reason is that the control center takes over the energy consumption scheduling from customers with no clear incentive for them to do so. The other reason is that DLC requires too much privacy from the customers.

To tackle those issues, we must have a DSM system that clearly benefits customers, protects their privacy, and requires much less real-time information exchange compared with \( P_1 \) or \( P_2 \). We formulate a game with two approaches based on smart pricing, which is another major technique applied to the DSM system. In one approach, the customers get to compute the dynamic price based on their own load schedule with the total load of the power grid given. In the other approach, the control center computes the price based on the total load of the power grid and customers get that fixed price schedule that will not be affected by their local scheduling load. In either game theoretical approach, the payoff functions lead customer to a more energy cost saving. Therefore, customers are motivated to adopt this DSM system. In addition, customers reserve the
rights to control their power consumption, and by doing so, they keep the privacy to themselves by submitting only the energy consumption schedule, which is always a request even in traditional power grid. Moreover, since most of the calculation is performed locally at the customer side, the approaches are mostly distributed instead of centralized. The distributed game theoretical approach largely relieves the transmission overhead. More importantly, we prove that all $P_1$, $P_2$ and game theoretical approach with locally computed dynamic smart pricing lead to the same minimum PAR. Therefore, while all parties get enough motivation to participate, the DSM system can be deployed in a distributed way.

We summarize the main contributions in the following.

- In order to benefit the entire society and the environment, we propose a DLC based centralized approach to minimizing the PAR.
- In order to show the benefits to the power suppliers, we propose another DLC based centralized approach to minimizing the power generation cost, and the power generation cost model considers all conventional, non-expanding green energy sources, and expanding renewable energy sources.
- In order to motivate the customers to adopt the DSM system and protect their privacy, we propose smart pricing based game theoretical approaches that can maximize the customer savings by adopting such a DSM system. The game theoretical approaches are mostly distributed and thus they alleviate the communication burden of the network.
- We prove that the proposed DLC based and one of the smart pricing based distributed game theoretical approaches yield to the same optimal solution (minimum PAR), and therefore the distributed game theoretical approach can be applied for real application while all parties observe clear benefits from the DSM system.
- We provide extensive numerical analysis and simulation results to demonstrate our analysis. We also compare several distributed approaches in order to find the best way to deploy the DSM system.

The rest of this paper is organized as follows. In section II, we discuss the related work. In section III, we illustrate the DSM system under study in this paper. In section IV, we formulate and analyze two centralized optimization problems, as well as several game based distributed approaches. In section V, we show the numerical analysis and simulation results. And finally we conclude the work in section VI.

II. RELATED WORK

DSM in smart grid has been studied by many researches recently [6–13, 16, 17]. However, most of the works focus on one of the parties (e.g., power suppliers when applying DLC or customers when adopting smart pricing) in the system only, without clarification on why to overlook others. Game theoretical approach and smart pricing have also been widely adopted in most of the works as efficient approaches. The most related work to this paper includes [7], [8], [10]. The authors of [7] proposed a non-cooperative game played among residential customers, and a two-stage Stackelberg game theoretical approach where power suppliers as the leaders tend to maximize their profit and customers as the followers tend to minimize their cost. Since [7] mostly targeted residential customers, the benefits for other parties in the system were not clearly stated, and some impractical situation where the total load goes negative was carefully avoided. In [8], the authors proposed an efficient game theoretical approach for residential customers without storage unit based on dynamic smart pricing to reduce the PAR. While the computational efficiency was demonstrated, the global optimal PAR was not guaranteed by the distributed approach. The authors of [10] are among the first to minimize PAR by distributed game theoretical approach among customers. Similarly, the storage unit was considered as their future work, and the global optimal PAR was not guaranteed.

III. SYSTEM MODEL

A. The Demand-Side Power Management System

Fig. 1. Demand-side power management system.

The demand-side power management (DSM) system under study mainly consists of three parties, namely control center, power suppliers, and customers, as illustrated in Fig. 1. Power generators include all major types, from fuel consuming conventional power generators to renewable power generators. For simplicity, micro grid that can be attached/detached to the power grid is not considered, and customers do not have power generators. Control center is mainly responsible for power distribution. It also gathers data (e.g., energy consumption) and distributes control information (e.g., price, tariff and emergency control signal). The information is available due to a two-way communication network in the DSM system. At each customer side, there is a smart meter that is responsible for reporting the power consumption and possible scheduling to the control center through the communication network. It is also responsible for receiving price, tariff as well as other control information from the control center.

According to [18], major customers in U.S.A. include residential, business and industrial ones as shown in Fig. 1. Those customers have different characteristics when consuming electric energy. For example, residential customers may consume most of the power from afternoon through midnight, business customers consume most of the energy during office hours, while industrial customers may have a longer peak consumption schedule due to different but continuous
shifts. The deployment of energy storage units and popularity of plug-in hybrid vehicles (PHEV) are increasing rapidly. Such devices/appliances will increase energy consumption and change the current peak time schedule. For example, it is reasonable to assume most of the customers will charge their PHEV during the night time. For simplicity, PHEV is observed as a hybrid of a normal energy consuming appliance and an energy storage unit in the studied DSM system.

B. Mathematical Modeling

Table I lists the key notations of sets and variables we use throughout the paper. Let \( \mathcal{N} = \{1, 2, \ldots, N\} \) be the set of all customers, where \( N \equiv |\mathcal{N}| \) is the total number customers. Although the DSM system can be modeled for any arbitrary time period to satisfy the assumptions, we consider a daily model in this work without loss of generality. Let one day be divided into several uniform time intervals, denoted as \( \mathcal{T} = \{1, 2, \ldots, |\mathcal{T}|\} \).

<table>
<thead>
<tr>
<th>Local (for customer ( n ))</th>
<th>( \mathcal{N} = {a_{n, 1}, a_{n, 2}, \ldots, a_{n, n} } )</th>
<th>set of appliances</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_{n, i} = (x_{n, i}^{1}(t) \in \mathcal{T}) )</td>
<td>daily energy scheduling set of ( a_{n, i} )</td>
<td></td>
</tr>
<tr>
<td>( E_n = {E_n \leq A_n} )</td>
<td>energy requirement set</td>
<td></td>
</tr>
<tr>
<td>( t_n^+ = {t_n^+ \in \mathcal{T}} )</td>
<td>on/off operating scheduling set of ( a_{n, i} )</td>
<td></td>
</tr>
<tr>
<td>( s_n^+ = {s_n^+ \in \mathcal{T}} )</td>
<td>discharging scheduling set of storage unit</td>
<td></td>
</tr>
<tr>
<td>( s_n^- = {s_n^- \in \mathcal{T}} )</td>
<td>charging scheduling set of storage unit</td>
<td></td>
</tr>
<tr>
<td>( I_n = {I_n \in \mathcal{T}} )</td>
<td>daily energy consumption scheduling set</td>
<td></td>
</tr>
</tbody>
</table>

Each customer \( n \) has a set of appliances \( \mathcal{N}_n = \{a_{n, 1}, \ldots, a_{n, n} \} \), where \( A_n \triangleq |\mathcal{N}_n| \). Each appliance (e.g., \( a_{n, i} \)) has a daily energy consumption scheduling set \( x_{n, i} = \{x_{n, i}^+(t) \in \mathcal{T}\} \), which records the needed or consumed energy for each time interval. Moreover, each appliance \( a_{n, i} \) also has a pre-determined energy requirement \( E_{n, i} \) for a time period (assuming one day for simplicity). Therefore, for \( a_{n, i} \), it must satisfy

\[
1^T x_{n, i} = E_{n, i},
\]

where 1 is a column vector of 1s and \((\cdot)^T\) calculates the transposition. Appliance \( a_{n, i} \) has an on/off operating scheduling set \( t_{n, i}^+ = \{0, 1\}^{\mathcal{T}} \), where 1 indicates that \( a_{n, i} \) is allowed to operate whereas 0 indicates off status of \( a_{n, i} \). The on/off operating schedule can model the operating status more precisely than the model using an operating time period, which is more widely adopted [7], [8], [10]. For example, with an on/off operating schedule, it is able to model a 2-hour pause of an air-conditioning (AC). However, if modeled by the operating time period, it must break the time period into three co-related sessions with extra constraints. With the on/off operating scheduling set \( t_{n, i}^+ \), Eq. 1 can be rewritten into

\[
(t_{n, i}^+)^T x_{n, i} = E_{n, i}.
\]

For an appliance which needs to be used for several times daily (e.g., a coffee machine), it can be observed as multiple independent appliances with corresponding energy requirements and on/off schedules. For this reason, we want to emphasize that \( A_n \) may not necessarily be the exact number of appliances of customer \( n \) but the number that counts independent appliances.

When \( a_{n, i} \) is operating, its energy consumption is bounded by \( \gamma_{n, i}^{\min} \) and \( \gamma_{n, i}^{\max} \), and mathematically,

\[
\gamma_{n, i}^{\min} t_{n, i}^+ \leq x_{n, i} \leq \gamma_{n, i}^{\max} t_{n, i}^+.
\]

Besides the appliances, let each customer (e.g., \( n \)) be equipped with an energy storage unit with a design capacity \( s_n \). For simplicity, we assume that the storage unit has 100% discharging/charging efficiency, and the energy can be distributed for all the appliances within the power grid with 100% efficiency. In other words, the storage units can be used to support the appliances of customers themselves as well as to sell the energy to the power grid. Let \( s_n^- = \{s_n^-(t) \in \mathcal{T}\} \) be the discharging scheduling set, and let \( s_n^+ = \{s_n^+(t) \in \mathcal{T}\} \) be the charging scheduling set. Similar to the appliance energy consumption, the discharging/charging energy in each time interval is bounded by the safety thresholds \( s_n^{\min}/s_n^{\max} \), which are expressed as

\[
0 \leq s_n^- \leq s_n^{\max},
\]

\[
0 \leq s_n^+ \leq s_n^{\max}.
\]

To be more precise, the storage unit should not discharge and charge at the same time for efficiency, so we have

\[
s_n^- \circ s_n^+ = 0,
\]

where “\( \circ \)” is the entrywise/Hadamard product, and 0 is the column vector with all 0s. Eq. 6 will help convert the storage unit model into a lossy one easily.

Let \( s_n = \{s_n(t) \in \mathcal{T}\} \) be the set of remaining energy at the beginning of each time interval,

\[
s_n(t) = \begin{cases} s_n, & t = 1 \\ s_n(t-1) - s_n(t-1) + s_n^+(t-1), & t \in \mathcal{T} \setminus \{1\} \end{cases}
\]
where $\tilde{s}_n$ is the initial remaining capacity. Let $\tilde{s}_n$ be the designed capacity of the storage unit, then

$$0 \leq \tilde{s}_n \leq \tilde{s}_n,$$

$$0 \leq s_n - s_n^\prime + s_n^\prime \leq \tilde{s}_n 1. \tag{9}$$

Let the daily energy consumption schedule for customer $n$ be $I_n = \{l_n(t) | t \in T\}$. With the previous modeling, we now have

$$I_n = \sum_{i=1}^{A_n} (x_n^i) - s_n^\prime + s_n^\prime. \tag{10}$$

For the whole DSM power system, the global load schedule $L = \{L(t) | t \in T\}$ is calculated as

$$L = \sum_{i=1}^{N} I_n. \tag{11}$$

C. Energy Cost and Unit Price

Based on [19], power suppliers are categorized into three types, namely fuel (e.g., coal) consuming conventional generators, non-expandable green sources (hydroelectric, nuclear), and expandable renewable sources (PV field, wind farm). The net capacity of those generators/sources is shown in Fig. 3. Conventional generators are still the major energy producers, however their proportion is decreasing. The proportion of non-expandable generators is slowly decreasing since the total energy is increasing. Although the proportion of the expandable renewable energy is low, it is increasing in a faster pace. Therefore, we need to take into consideration all three categories of power suppliers for a more precise modeling.

- **Fuel consuming conventional generators**: a quadratic cost function is widely adopted for this type of generators [6–10] as

$$C_c(l) = a_c l^2 + b_c l + c_c. \tag{12}$$

- **Non-expandable green sources**: assuming the produced energy is predetermined by the fixed facilities, the total cost can be viewed as a fixed cost $c_f$ plus a linear cost w.r.t power transmission capacity as

$$C_f(l) = a_f l + c_f. \tag{13}$$

- **Expandable renewable sources**: since most of the cost comes from the management of the facilities [20], by assuming the facilities can be on/off based on the load requirement, the total cost is increasing w.r.t the load requirement. However it increases slower than the conventional generators [21], especially when carbon tax [22] applies. Therefore we adopt the following cost model for expandable renewable sources.

$$C_f(l) = a_c l \ln(l + 1) + c_r. \tag{14}$$

Taking into consideration the proportions of all the generators/sources, the overall energy cost is modeled as,

$$C(l) = C_c(\beta_c l) + C_f(\beta_f l) + C_r(\beta_r l), \tag{15}$$

where $\beta_c$, $\beta_f$ and $\beta_r$ are the proportions of three power suppliers respectively, and $\beta_c + \beta_f + \beta_r = 1$. Let $C = \{C(L(t)) | t \in T\}$ be the set of total cost for each time interval. At the customer side, the unit price ($\$ per kW$) is more important than the total cost. For simplicity, let the energy be generated uniformly during a time period, the unit price is calculated as

$$p(l) = C(l) / l. \tag{16}$$

Finally let $p = \{p(L(t)) | t \in T\}$ be the set of unit prices for each time interval.

IV. PROBLEM FORMULATION AND ANALYSIS

A. Minimize PAR

One of the ultimate goals of applying DSM is to reduce the peak-to-average ratio, which raises the first problem:

$$\mathcal{P}1 : \min_{L} \sup_{t \in T} \frac{L}{\int_{t}^{t+1} L} \tag{17}$$

s.t. Constraints [11, 5, 6, 7, 8, 9, 10], $\forall n \in N$


**Lemma 1.** Let $L$ be the set of all possible daily load scheduling patterns ($L$ is convex because of the convex, compact and non-empty constraints). Then $\mathcal{P}1$ has a unique optimal solution $L^* = \arg \min_{t \in T} \sup_{\forall L \in L}$

Proof: First, since all appliances (including the storage units) consume energy from the power generators/sources, and each appliance has a daily energy requirement, thus the daily total load $1^T L = 1$ is observed as a constant. Therefore

$$\mathcal{P}1 \triangleq \mathcal{P}1.1 : \min_{t \in T} \sup_{\forall L} L$$

with the same constraints. Let the objective function for $\mathcal{P}1.1$ be $f(L) = \sup_{t \in T} \{L(t) | t \in T\}$, which satisfies, for $0 \leq \theta \leq 1$,

$$f(\theta L_1 + (1 - \theta) L_2) = \sup_{t \in T} \theta L_1(t) + (1 - \theta) L_2(t)) \leq \theta \sup_{t} L_1(t) + (1 - \theta) \sup_{t} L_2(t) \tag{18}$$

$$= \theta f(L_1) + (1 - \theta) f(L_2)$$

Fig. 3. Existing net capacity by energy source and producer type [19].
Thus $f(L)$ is convex. Additionally, the constraint set is compact, convex and non-empty. Therefore, $\mathcal{P}1.1$ has a unique solution $L^*_1$ and does $\mathcal{P}1$.

Note that although the existence of an optimal solution always stands, the uniqueness of the solution only stands when $L$ is considered as the variable set because multiple solutions to Eq. (11) may exist with a given $L$. Although minimizing $\text{PAR}$ is a benevolent objective, it may not be convincing enough for power suppliers or customers to adopt such a DSM system. For this reason, we further formulate another problem with a monetary incentive objective function.

B. Minimize Total Cost

In the electricity energy market, power generators/sources are not fully competitive to each other yet since some of the technologies are still too expensive to apply and they operate based on government subsidy [23]. Moreover, sophisticated regulatory mechanisms are needed to avoid arbitrarily high price led by monopoly and rigid electric energy demand. Therefore, we focus on a cost-oriented instead of profit-oriented objective. In short, $\mathcal{P}2$ minimizes the total cost of the power suppliers, such that

$$\mathcal{P}2: \quad \min_{L} \ p^T L \tag{19}$$

s.t. Constraints $[11, 13, 14, 17, 18, 19, 20, 21, 22]$, $\forall n \in N$.

Lemma 2. $\mathcal{P}2$ has a unique optimal solution $L^*_2 = \arg \min_{L \in L} \ p^T L$, $\forall L \in L$.

Proof: The objective function of $\mathcal{P}2$ is

$$p^T L = \sum_t \left( p(L(t)) \cdot L(t) \right) = \sum_t C(L(t)) \tag{20}$$

For simplicity, let $x \triangleq L(t)$ be the argument in this proof. It is obvious that $C(x)$ is increasing and strict convex. Since Eq. (20) is a composition of $C(x)$, thus $\sum_{t \in T} \left( p(L(t)) \cdot L(t) \right)$ is strict convex w.r.t $L(t)$. With the same compact, convex and non-empty constraint set compared to $\mathcal{P}1$, $\mathcal{P}2$ thus has a unique optimal solution.

Let function $g(L) \triangleq p^T L$. Then the optimal solution to $\mathcal{P}2$ can be found by solving the necessary and sufficient conditions of KKT [24].

Lemma 3. Let $v$ be the Lagrange multiplier of the equality constraint of $\mathcal{P}2$ ($1^T L = 1$), and let $v^*$ minimize the dual problem of $\mathcal{P}2$ over $v$. The optimal solution $L^*_2 = \{L^*_2(t) | t \in T\}$ to $\mathcal{P}2$ is calculated as

$$L^*_2(t) = \max_{v \in \mathcal{N}} \left\{ -\sum_{n \in N} s_{n} \cdot \arg \min_{L(t)} \left( \frac{\partial g(L)}{\partial L(t)} + v \right) \right\}, \forall t \in T$$

$$v^* = \arg \min_{v} \left\{ \sum_{t \in T} L^*_2(t) = 1 \right\}. \tag{21}$$

Note that solution in Eq. (21) is unique w.r.t $L$ and it may have multiple solutions w.r.t the detailed energy consumption scheduling patterns to all the appliances.

Lemma 4. If $L^*$ is the optimal solution to $\mathcal{P}1$, it is also the optimal solution to $\mathcal{P}2$.

Proof: Let $x = \{x_t | t \in T\} \triangleq L^*_1$ be the optimal solution of $\mathcal{P}1$, and let $y = \{y_t | t \in T\} \triangleq L^*_2$ be the optimal solution of $\mathcal{P}2$ in this proof. Also, reorganize $x, y$ to be non-descending sets such that $x_i \leq x_{i+1}$, $y_i \leq y_{i+1}$, $i \in T$ for better illustration. Note that $\sup_{x} x = x_{\gamma(T)}$, and $\sup_{y} y = y_{\gamma(T)}$. We then prove this lemma by contradictory. Assuming $x \neq y$, then it must be

$$x_{|T|} < y_{|T|} \tag{22}$$

Furthermore, inequality (23) indicates that

$$\sum_{i \in T} \left(x_i \cdot p(x_i) - y_i \cdot p(y_i)\right) > y_{|T|} \cdot p(y_{|T|}) - x_{|T|} \cdot p(x_{|T|}) \tag{23}$$

We must then have

$$(\Gamma - x_{|T|}) \cdot p(\Gamma - x_{|T|}) > (\Gamma - y_{|T|}) \cdot p(\Gamma - y_{|T|}) \tag{24}$$

(25)

$$(\Gamma - x_{|T|}) \cdot p(\Gamma - x_{|T|}) \triangleright (\Gamma - y_{|T|}) \cdot p(\Gamma - y_{|T|})$$

(26)

and the fact that

$$\Gamma - x_{|T|} - \Gamma - y_{|T|} = y_{|T|} - x_{|T|} \tag{27}$$

We must then have

$$\sup_{x, y \in L} \sum_{i \in T \setminus \{T\}} \left(x_i \cdot p(x_i) - y_i \cdot p(y_i)\right) \leq (\Gamma - x_{|T|}) \cdot p(\Gamma - x_{|T|}) - (\Gamma - y_{|T|}) \cdot p(\Gamma - y_{|T|}) \tag{28}$$

Note that inequality (28) contradicts inequality (24), and thus $x = y$ or $L^*_2 = L^*_1 = L^*$ after proper ordering, which completes the proof.

Theorem 1. Minimizing the total cost reaches the minimum PAR: $\mathcal{P}2 \triangleq \mathcal{P}1$.

Although power suppliers may be willing to adopt a DSM system according to $\mathcal{P}2$ so that they can reduce their total cost, it still has three major issues to solve for either $\mathcal{P}1$ or $\mathcal{P}2$. Customer privacy is the first issue. Since both $\mathcal{P}1$ and $\mathcal{P}2$ are exclusively executed at the control center side (often regarded as DLC schemes), all the customers must submit their detailed information to the control center and willingly let the
control center schedule their power usage. The incentive for the customers to adopt a DSM system is the second issue. Although a DSM system smooths PAR and reduces the total cost, it is not clear whether customers can benefit from \( \mathcal{P}1 \) or \( \mathcal{P}2 \) or not. Third, both \( \mathcal{P}1 \) and \( \mathcal{P}2 \) are centralized optimization problems, which can get quite complicated and computation-consuming to solve. Even if the problems can be solved efficiently, the huge overhead of raw data gathering lays too much burden on the communication network. Therefore, we need a distributed DSM system that also protects the customers by letting them control their own appliances.

C. Game Theoretical Approaches

Smart pricing is another widely adopted cost strategy in order to attract customers’ interests. Moreover, a game theoretical approach is an efficient way to solve a problem in a distributed fashion with some limited shared information. Formally, we define a non-cooperative game \( \mathcal{G} = [\mathcal{N},\{\mathcal{L}_i\},\{h_i(\cdot)\}] \), where \( \mathcal{L}_i \) is the strategy set (all possible load scheduling patterns) of player (customer hereafter for consistency) \( i \) (the notation of a customer is changed from \( n \) to \( i \), which is more generic for game theoretical approach), and \( h_i(\cdot) \) is the payoff function of customer \( i \), which is

\[
h_i(l_i) = (\Omega^T - p^T)l_i,
\]

where \( \Omega \) is a flat rate price vector if smart pricing strategy is not applied. This payoff function shows the saving of a customer from adopting this particular DSM system. Intuitively, if adopting a DSM system reduces their cost, customers are willing to participate. In this game, each customer calculates a best response \( l_i^* \) with a given \( l_{-i} = \{l_1,\ldots,l_{i-1},l_{i+1},\ldots,l_N\} \) such that,

\[
h_i(l_i^*, l_{-i}) \geq h_i(l_i, l_{-i}), \quad \forall l_i \in \mathcal{L}_i.
\]

The best response in Eq. 30 is the same as calculating

\[
l_i^* = \arg\max_{l_i \in \mathcal{L}_i} h_i(l_i, l_{-i}),
\]

s.t. Constraints \( 4, 5, 6, 7, 8, 9, 10 \).

Note that Eq. 31 has an incentive physical meaning, which is to find the set of load scheduling patterns that maximize the energy cost saving for customer \( i \) by adopting a DSM system. Since each customer has a constant total daily energy consumption, i.e., \( \sum_l l_i(t) = \Gamma_i, \forall i \in \mathcal{N} \), the flat rate price yields a constant cost for each customer, therefore, the problem in Eq. 31 equals the following one.

\[
l_i^* = \arg\min_{l_i \in \mathcal{L}_i} p^Tl_i,
\]

s.t. Constraints \( 4, 5, 6, 7, 8, 9, 10 \).

1) GA1: locally computed smart pricing.

In this approach, we assume that each customer submits an initial load schedule \( l_i^0 = \{l_i(t)|t \in T\}, \forall i \in \mathcal{N} \) to the control center, and then the control center broadcasts the initial total load schedule \( L^0 = \{l_i(t)|t \in T\} = \sum_{i \in \mathcal{L}} l_i^0 \) to the customers. In this approach, let the price computing function in Eq. 15 be known to all customers. Then each customer will be able to compute the smart pricing schedule as

\[
p_i(l_i(t), l_i^0, l_i^*) = p_i(L_i^0(t) - l_i(t) + l_i^0), \quad \forall t \in T
\]

Let \( p_i = \{p_i(L(t) - l_i(t) + l_i^0)|t \in T\} \). Customer \( i \) will need to solve the following problem to find \( l_i^* \),

\[
P_3: \min_{l_i \in \mathcal{L}_i} p_i^Tl_i,
\]

s.t. Constraints \( 4, 5, 6, 7, 8, 9, 10 \).

Lemma 5. With given \( L_i^0 \) and \( L_i^0 \), \( P_3 \) has a unique optimal solution w.r.t \( l_i = \{l_i(t)|t \in T\} \).

Proof: The objective function of \( P_3 \) is analogous to that of \( P_2 \) and thus is monotonically increasing and strictly convex. The constraint set is also compact, convex and non-empty. Thus \( P_3 \) has a unique optimal solution.

Let function \( g(l_i) = p_i^Tl_i \), and \( \Gamma_i \) be the total energy requirement for customer \( i \). Then the optimal solution to \( P_3 \) can be found by solving the necessary and sufficient conditions of KKT [24].

Lemma 6. Let \( v_i \) be the Lagrange multiplier of the equality constraint, and \( v_i^\star \) minimizes the dual problem of \( P_3 \) over \( v_i \).

The optimal solution \( l_i^\star = \{l_i(t)|t \in T\} \) to \( P_3 \) is

\[
l_i^\star(t) = \max \left\{-s_i^\max, \arg\max_{l_i(t)} \left( \frac{\partial g(l_i)}{\partial l_i} + v_i^\star \right) \right\}, \quad \forall t \in T
\]

\[
v_i^\star = \arg\min_{v_i} \left\{ \sum_{t \in T} l_i^\star(t) = \Gamma_i \right\}
\]

Definition 1. (Nash equilibrium (NE)): a scheduling set \( l_i^\star = \{l_i^\star, l_2^\star,\ldots,l_N^\star\} \) is an NE of \( \mathcal{G} = [\mathcal{N},\{\mathcal{L}_i\},\{h_i(\cdot)\}] \) if, \( \forall i \in \mathcal{N}, \forall l_i \in \mathcal{L}_i, h_i(l_i^\star, l_{-i}^\star) \geq h_i(l_i, l_{-i}^\star) \), where \( l_{-i} = \{l_1^\star,\ldots,l_{i-1}^\star,l_{i+1}^\star,\ldots,l_N^\star\} \).

Lemma 7. \( \mathcal{G} \) has a unique NE by GA1.

Proof: First, the payoff function (Eq. 29) is strictly concave, and the constraint set for this approach is compact, convex and non-empty, thus NE exists [22], [23]. Second, Lemma 5 guarantees that the best response of each player can be found uniquely. Therefore NE exists uniquely by GA1. The NE of GA1 can be found by Alg. 1.

Lemma 8. Alg. 1 converges to NE.

Proof: According to Eq. 30 the payoff of each customer increases after each iteration. Because the payoff is bounded, the algorithm will converge to an equilibrium, which is the NE.

Theorem 2. The NE of \( \mathcal{G} \) by GA1 is also the optimal solution to \( P_2 \).
Algorithm 1 Algorithm to find NE by GA1

\textbf{Input:} \( \gamma_i, t_i, E_i, s_i, s_i^{\text{max}}, \ s_i^{\text{max}}, \ \forall i \in \mathcal{N}, \ T \)

\textbf{Output:} \( l_i^*, \ \forall i \in \mathcal{N} \)

1. Each player computes a feasible \( l_i^0 \)
2. while NE is not achieved do
3. \( l_i^0 \leftarrow \sum_{i \in \mathcal{N}} l_i^0 \); // Computed and distributed by the control center
4. \( l_i^* \leftarrow l_i^* = \arg \min P_i \mathcal{L}, \ \forall i \in \mathcal{N} \); // Computed by customer
5. end while
6. \( l_i^* \leftarrow l_i^0, \ \forall i \in \mathcal{N} \); // Output NE

\textbf{Proof:} According to the definition of the best response, we have

\[ \sum_{i \in \mathcal{N}} h_i(l_i^*, l_{-i}) \geq \sum_{i \in \mathcal{N}} h_i(l_i, l_{-i}), \ \forall l_i \in \mathcal{L}_{-i} \]
\[ \Rightarrow \sum_{i \in \mathcal{N}} g(l_i^*, l_{-i}) \leq \sum_{i \in \mathcal{N}} g(l_i, l_{-i}), \ \forall l_i \in \mathcal{L}_i \]
\[ \Rightarrow \sum_{i \in \mathcal{N}} g(l_i^*, l_{-i}) \leq \mathcal{P}^T \mathcal{L}, \ \forall \mathcal{L} \in \mathcal{L} \]
\[ \Rightarrow \mathbf{p}(l_i^*, l_{-i}) = \arg \min_{\mathcal{L} \in \mathcal{L}} \mathcal{P}^T \mathcal{L} \]
\[ \Rightarrow \sum_{i \in \mathcal{N}} (l_i^*) = \arg \min_{\mathcal{L} \in \mathcal{L}} \mathcal{P}^T \mathcal{L} \]

Theorem 1 demonstrates that GA1 not only favors the customers, but also minimizes the total cost and thus favors power suppliers. So GA1 minimizes PAR according to theorem 1. However, control center may not want to release the price calculating function to customers. We then propose another approach based on pre-calculated fixed pricing schedule.

2) GA2: semi-fixed smart pricing.

In this approach, each customer \( i \) also submits an initial load schedule \( l_i^0 \), however the price function is hidden from customers. Only control center is able to calculate the fixed price vector with each \( L_0 \) as,

\[ \mathbf{p} = (p(L_i^0(t)) | t \in T). \]

Then each customer \( i \) will solve the following problem to find \( l_i^* \),

\[ \mathcal{P}^4: \ \min_{l_i \in \mathcal{L}_i} \mathbf{p}^T l_i, \]
\[ \text{s.t. Constraints } 4, 5, 6, 7, 8, 9, 10. \]

Note that \( \mathcal{P}^4 \) is a linear optimization problem, which can be solved with a unique solution. However the NE for \( \mathcal{G} \) is not guaranteed since the objective function is no longer strictly concave.

3) Mixed approach: adopting GA1 and GA2 based on the property of customers.

As mentioned earlier, GA1 reveals the price function to the customers, which may not favor the control center. The total load is also given to all customers. However, customers who use a significant proportion of the energy may not want to reveal such privacy. GA2 does not have those issues but it has no guarantee to the optimal solution of \( \mathcal{P}^1 \) or \( \mathcal{P}^2 \). In the mixed approach, large energy consumption customers adopt GA1 (e.g., business and industrial customers) and regular energy consumption customers (e.g., residential customers) adopt GA2. In the numerical analysis and simulation section we will show that the mixed approach converges to the minimum PAR in practice.

D. Precision and Truthfulness of the Proposed DSM Systems

Because customers could have exceptional energy consumption, one time scheduling approach \( [7], [8], [10] \) can hardly be followed strictly. In order to increase the precision of the proposed DSM system, the system should run at the beginning of each time interval for the rest of the day. The control center and the computing device of each customer (e.g., the smart meter) should save the previous status and subtract it from the constraints when approaching the load scheduling for the rest of the day. With the DSM system following schedule, and since the payment is collected after each time interval based on the real energy usage in that interval, for \( \mathcal{P}^1 \), \( \mathcal{P}^2 \) and GA1, the minimum total cost and the maximum local saving will only be achieved when customers report the load schedules truthfully. The conclusion is quite intuitive because lying about the load will deviate customers from the optimal solution. Therefore, the truthfulness of the proposed DSM systems should be guaranteed.

V. NUMERICAL ANALYSIS AND SIMULATION RESULTS

A. Setting for the numerical analysis

Assume that power suppliers are available to support customers with any energy requirement. The control center is able to gather/distribute information from/to the power suppliers and the customers in real-time (e.g., 100 ms). The customers are categorized into residential, business and industrial types. Without loss of generality, we assume that the daily energy consumption of the customers follows the proportions obtained from the data in Fig. 2. Specifically, as shown in Fig. 4, residential customers consist of three types (i.e., families in big houses, families in townhouses, families in apartments), each with 55 \( \text{kWh} \), 41 \( \text{kWh} \) and 33 \( \text{kWh} \) daily average energy consumption respectively. Each residential customer type has 50 customers. Industrial customers have two types (i.e., day-time based business and shopping malls), each with 2400 \( \text{kWh} \) and 2700 \( \text{kWh} \) daily average energy consumption respectively, and each type has 1 customer. Industrial customers have two types (i.e., non-stop shift-based manufacturers and day-time based industry), each with 2100 \( \text{kWh} \) and 2500 \( \text{kWh} \) daily average energy consumption respectively, and each type has 1 customer. Note that the settings for the customers are flexible as long as the total energy consumption of each category follows the practical data.

The granularity of time intervals is important to the DSM system. As shown in Fig. 5 when \( |T| = 24 \), the total load of the power grid is constant throughout the day while it fluctuates when \( |T| = 8 \).

Part of the detailed initial settings for residential customers are shown in Table III. We assume that each storage unit is able...
three DSM systems. Fig. 7 shows the results for residential customers. Note that GA1 leads to a negative load for some customers as shown in Fig. 7(c). It indicates that those customers tend to sell extra energy from their storage unit to maximize the savings.

![Daily energy consumption](image)

Fig. 4. Daily consumption of the customers.

![Solution to P1 with different T](image)

Fig. 5. Solution to P1 with different T.

to hold 1/6 of daily power usage, each family has 2 PHEVs for type 1 and type 2 residential customers, the customers living in apartment have 1 PHEV each. Each PHEV holds 9 kWh electricity. The power usage and rough schedules for other appliances are estimations based on day-to-day experiences. For simplicity, the on/off schedule is shown as start/end time. Note that some of the appliances are observed as multiple ones so that the modeling is more precise, for example AC for residential users is observed as an all-day ventilation and a fully operating one in the afternoon.

![Load schedules for P1, P2 and GA1](image)

Fig. 6. Load schedules by P1, P2 and GA1.

Load schedules for business customers and industrial customers are shown in Fig. 8 and Fig. 9 respectively.

![Average residential load schedules by different approaches](image)

Fig. 7. Average residential load schedules by different approaches.

![Different business load schedules by different approaches](image)

Fig. 8. Different business load schedules by different approaches.

C. Comparison of Different Distributed Approaches

Fig. 10 shows the load scheduling results of GA1, GA2, mixed GA and a distributed approach when all customers intend to minimize their local PAR. From the simulation results we can see that both GA1 and mixed GA converge to the optimal PAR while neither GA2 nor min local PAR minimizes PAR.

<table>
<thead>
<tr>
<th>App</th>
<th>E_n^a (kWh)</th>
<th>Start</th>
<th>End</th>
</tr>
</thead>
<tbody>
<tr>
<td>Storage</td>
<td>1/6 \sum E_n^a - E_n^a/3</td>
<td>E_n^a/3</td>
<td>0</td>
</tr>
<tr>
<td>PHEV</td>
<td>18, 18, 9</td>
<td>0</td>
<td>E_n^a/3</td>
</tr>
<tr>
<td>AC1</td>
<td>5, 3, 2</td>
<td>E_n^a/48</td>
<td>E_n^a/16</td>
</tr>
<tr>
<td>AC2</td>
<td>5, 3, 2</td>
<td>E_n^a/4</td>
<td>E_n^a/4</td>
</tr>
<tr>
<td>Cooking</td>
<td>2, 1, 1</td>
<td>0</td>
<td>E_n^a/1</td>
</tr>
<tr>
<td>Dish washer</td>
<td>2, 1, 1</td>
<td>0</td>
<td>E_n^a/2</td>
</tr>
<tr>
<td>Washing/dryer</td>
<td>5, 3, 2</td>
<td>0</td>
<td>E_n^a/2</td>
</tr>
<tr>
<td>Electronics</td>
<td>2, 2, 2</td>
<td>E_n^a/8</td>
<td>E_n^a/3</td>
</tr>
<tr>
<td>Others</td>
<td>4, 2, 1</td>
<td>E_n^a/48</td>
<td>E_n^a/12</td>
</tr>
</tbody>
</table>
Fig. 9. Different industrial load schedules by different approaches.

In fact, GA2 performs the worst in the simulation when all customers are considered. Because of the fixed given smart price, customers will shift their load to the low price time intervals without considering the consequence of doing so. When all customers do so, the time intervals with a relatively low previous load will be scheduled with a higher load and the price will go up. Then customers will shift back based on the updated price schedule. The simulation also indicates that GA2 alone fluctuates between these two states without converging to the NE. Fig. 11 shows the two states of the total load schedule by GA2 for the first 8 time intervals.

Fig. 12 shows the average load schedules by GA1 and mixed GA for the first type of the customers in each category. It appears that GA1 schedules a smoother load for each customer because residential customers have better assessments of the cost changes based on their updated schedule by GA1. When applying mixed GA, although each customer does not affect the price much, they together still cause a big impact. Therefore, both business and industrial customers must schedule their loads in a more fluctuating way to adapt to the residential load.

Fig. 13 shows that both GA1 and mixed GA converge quickly to the NE. In game theoretical approaches, the uplink of the network transmits the load schedule of each customer to the control center, and the control center broadcasts the total load schedule to all customers. Although it requires multiple iterations, it requires much less data transmission compared with the centralized approach, which needs much more detailed data from the customers and the scheduled appliance load must be delivered to each customer individually. Moreover, it is worth mentioning that the energy providers may not want to declare the true cost function to all the customers in practice. By adopting mixed GA, only a few customers are required to know the cost function and some confidential agreements can be made.

D. The Impact from Storage Unit

From Fig. 14 we can see that the total load schedule is no longer a straight line without the assistance of storage units. And this causes a higher PAR for the power grid, as shown in Fig. 15. However, a storage unit may not necessarily help reduce the PAR for each customer individually.
E. The Impact from Increasing Renewable Energy

In Fig. 16(a) we show the estimated proportion of different power suppliers based on the data in Fig. 3 up to year 2020. Based on the estimations, the total energy cost of the customers in 2014, 2018 and 2020 is shown in Fig. 16(b). Clearly, the expanding renewable energy helps reducing the energy cost.

VI. CONCLUSION AND FUTURE WORK

In this paper, we studied DSM in smart grid. In order to motivate and benefit all parties including the whole society (environment), power suppliers and customers, we proposed several approaches for the DSM system. First, $P_1$ directly minimizes PAR. Second, a DLC-based cost minimization approach $P_2$ motivates the power suppliers. However both $P_1$ and $P_2$ fail to protect the customer privacy and the communication overhead is too high to be deployed for a real-time system. We further proposed two smart pricing-based game theoretical approaches $GA_1$ and $GA_2$ to address the shortcomings of $P_1$ and $P_2$. We successfully proved that both $P_2$ and $GA_1$, where each customer calculates the dynamic price locally, reach the solution to $P_1$ (min PAR). In the numerical analysis and the simulations, we further demonstrated the results, and compared several distributed approaches. In the future work, detachable micro grids and customer-side power generator will be considered for a more precise modeling of the DSM system. Other schemes where the true cost function is hidden from the customers will also be studied.

### Reference


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