# Fractal Analysis of Temporal Yield Variability of Crop Sequences: Implications for Site-Specific Management

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## STATISTICS

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## ABSTRACT

**Characterizing spatial and temporal variability is important in site-specific or long-term studies to evaluate the effects of different management systems on crop performance. Long-term experiments offer unique possibilities to study the effects of management practices on crops and soils over time. The objective of this study was to characterize temporal grain yield variability of seven crop sequences using fractal analysis and to determine whether temporal or spatial variability dominated the grain yield variability. Three crops of corn (*Zea mays* L.), soybean [*Glycine max* (L.) Merr.], and sorghum [*Sorghum bicolor* (L.) Moench] were studied from 1975 to 1995 in various sequences. Semivariograms were estimated for the standardized crop yield. The slopes of the regression lines of log semivariogram vs. log lag (year) were used to estimate and compare fractal dimensions, which are indications of variability patterns. The intercepts of the log–log lines, which indicate extent of yield variability, were also compared between crop sequences. A small *D*-value indicates dominance of long-term variation, while a large *D*-value (near 2) indicates dominance of short-term (year-to-year) variation. Corn had significantly less temporal yield variability than soybean or sorghum. Continuous corn had less yield variability than corn following soybean. Soybean had the greatest yield variability, regardless of crop sequence. Temporal variability was much more dominant than spatial variability in this study. Temporal variability may greatly influence how spatial variability is expressed in a given field. Yield maps, which are used as an indication of past management in site-specific cases, may not be useful in making future management decisions when temporal variability is great. In a less productive year, spatial variability of any nutrient may not make much difference in crop yield of a given field.**

Temporal and spatial variability of soil and plant parameters have been difficult to characterize and quantify. Temporal variability is an important consideration when evaluating the performance of long-term experiments for sustainability. Soil spatial variability can result in differing crop performance in different parts of a field. Fractal analysis can be used to distinguish between short- and long-term variations for parameters collected in time or space. Fractal analysis, which is based on self-similarity (the manner in which a pattern at one scale is repeated at other scales), has been useful in characterizing plant and soil parameters in several studies (Burrough, 1981; Eghball et al., 1993a; Perfect and Kay, 1991). Perfect and Kay (1995) reviewed application of fractals in soil and tillage research. In fractal analysis, the fractal dimension $D$ (which, as the name implies, can be fractional) need not be an integer, and is scale independent. Fractal dimension is an indicator of the shape (geometry) of the fractal parameter being studied. Eghball et al. (1993b) used the fractal dimension to statistically compare treatments that influenced the morphology of corn roots. Eghball et al. (1993a) found that no-till had a smaller fractal dimension of soil fragmentation than three other tillage systems, indicating a better soil structure for no-till than chisel, disk, or plow.

For spatial and temporal variability, $D$ can range from 1 (values within spatial and temporal range of analysis fall on a line) to 2 (which indicates so much variation that an entire two-dimensional surface is covered by the extent of variation). Large $D$-values indicate the importance of short-range variation, while small $D$-values reflect the importance of long-range variation (Burrough, 1983). Eghball and Power (1995) used fractal analysis to characterize temporal variability for average yield in the United States of 10 crops with a wide range of yield levels, and found that crops were significantly different in terms of temporal variability. They observed less year-to-year grain yield variability for rice (*Oryza sativa* L.) than for other grain crops, which was judged to be due in part to management practices commonly used for this crop. Eghball et al. (1995) found temporal variability of corn grain yield in a long-term manure and fertilizer experiment under irrigation to be due to environmental factors, and the management practices did not change this variability.

Because of natural soil variability in any field, site-specific application of fertilizer and pesticides are becoming more common. Patterns of nutrients and organic

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carbon in soil are spatially correlated (Cahn et al., 1994; Pierce et al., 1995). Managing variability with variable rate application of fertilizer and pesticide has the potential of being economically and environmentally sound. By applying inputs where needed instead of to the entire field, a farmer can increase the yield potential of low-productivity areas within the field and maintain high productivity in productive areas. This may not only reduce fertilizer input, but can also reduce adverse effects on the environment.

In recent years, with the integration of computer and sensor technology, it has become possible to monitor crop yield for different sites within a field (Birrell et al., 1995). Yield maps can illustrate the location of problem sites within a field, which can be used to guide or identify management practices for the next growing season. Data collected from yield maps can be analyzed for grain yield variability across space or time.

Characterizing temporal variability in long-term studies where different cropping systems are compared is important, because it provides an indication of whether site-specific management will be beneficial. Our objective was to characterize and compare temporal yield variability of seven crop sequences in a long-term study.

## MATERIALS AND METHODS

### Experimental Procedure

The data we worked with are from selected treatments of a large long-term study that has been conducted at the University of Nebraska Agricultural Research and Development Center near Mead, NE, since 1974. The average yearly rainfall for this part of Nebraska is 710 mm. The experimental results reported in this paper are from the following seven cropping systems: corn–corn, sorghum–sorghum, soybean–soybean, corn–soybean, sorghum–soybean, soybean–corn, and soybean–sorghum under rainfed conditions from 1974 to 1995. The experimental design was a randomized complete block with four blocks. Each block of the larger study was 30.5 m wide and 137.2 m long, with a 7.6-m alley between blocks. Individual crops in each sequence were assigned to experimental units 9.1 m wide and 30.5 m long. Results from two 4-yr cropping sequences included in the overall study are not reported here, because these sequences had a fallow treatment as a component for the first 10 yr of the study.

Nitrogen rate subplots were added to the study in 1984, but results reported on in this paper are from the 90 kg ha$^{-1}$ treatments for corn and sorghum and the 0 kg ha$^{-1}$ treatment for soybean. These N treatments have been present for those crops throughout the study. Nitrogen was sidedressed as liquid urea–ammonium nitrate solution (32–0–0) from 1974 to 1984, and has been broadcast as granular ammonium nitrate (34–0–0) in succeeding years. Nitrogen applications were made in early to mid-June for corn, sorghum, and soybean.

Most of the soil at the site is mapped as a Sharpsburg silty clay loam (fine, smectitic, mesic Typic Argiudoll), but there is a small inclusion of Butler silty clay loam (fine, smectitic, mesic Abruptic Argiaquoll) in the first block. Depth to sand in the study ranged from 0.75 m to >8 m. Blocks 1, 3, and 4 were located on sideslope and Block 2 was on upland. The hill in the experimental area had slopes ranging from 4 to 6%.

Corn hybrids and sorghum and soybean varieties were selected for their suitability to eastern Nebraska growing conditions. Throughout the duration of the study, hybrids and varieties have been changed as deemed appropriate to reflect improvements in each of the crops. Corn was planted in rows 76 cm wide at a rate of 47 000 seeds ha$^{-1}$ during the first 2 wk of May in all years. Soybean and sorghum were also seeded in 76-cm-rows, at rates of 370 000 and 173 000 seeds ha$^{-1}$, respectively. All plots were disked, field cultivated, and harrowed just prior to seeding. Weed control was accomplished using combinations of broad-spectrum herbicides applied preemergence and hand hoeing. Grain yields used in the fractal analysis were determined by combining two to four of the inner rows of each plot.

### Fractal Analysis

The yield data for each block was standardized to a mean of zero and unit variance based on the following equation (Eghball and Power, 1995):

$$SV = (Y - \mu)/s$$

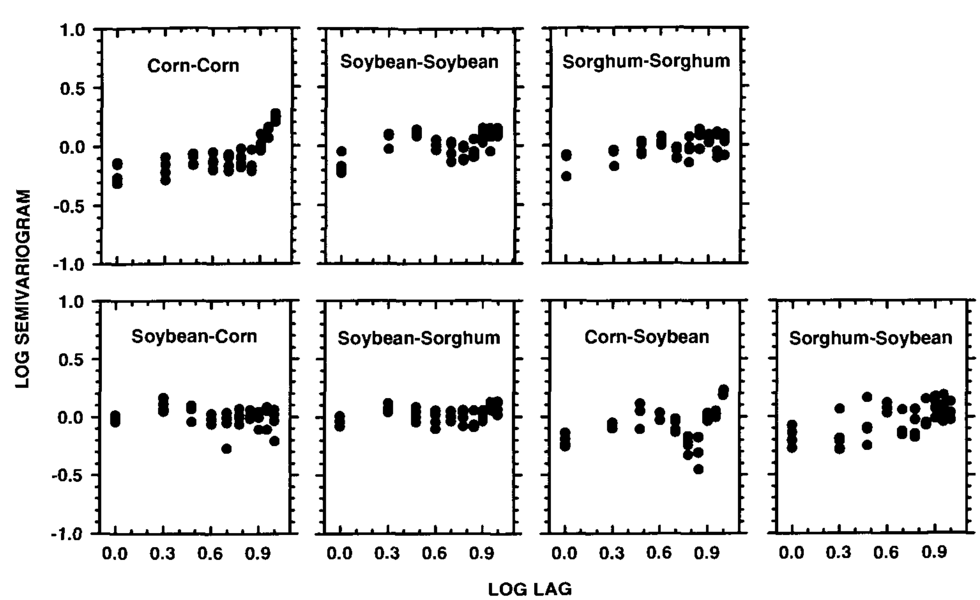


**Fig. 1. Semivariograms for the seven crop sequences at Mead, NE.**

**Table 1. Homogeneity test for the slopes of log semivariogram vs. log *h* (lag, year) for seven crop sequences.**

| | | Homogeneity test | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Variable | df | Corn–Corn | Sorghum–Sorghum | Soybean–Soybean | Corn–Soybean | Soybean–Soybean | Soybean–Corn | Soybean–Sorghum |
| | | probability level | | | | | | |
| Block | 3 | 0.56 | 0.06 | 0.96 | 0.94 | 0.65 | 0.88 | 0.56 |
| Log *h* | 1 | 0.01 | 0.01 | 0.01 | 0.06 | 0.01 | 0.33 | 0.13 |
| Log *h* × Block | 3 | 0.49 | 0.24 | 0.88 | 0.87 | 0.65 | 0.74 | 0.49 |

where SV is the standardized value, $Y$ is the yield level, $\mu$ is the mean, and $s$ is the standard deviation. Standardization was necessary to remove gross yield level differences among crops, so that they can be compared for variability on the same scale. Fractal analysis was performed on the standardized grain yields of the crop sequences based on the method described by Eghball and Power (1995). Briefly, semivariograms were estimated for standardized grain yield of each plot from 1975 to 1995 based on the method described by Clark (1979). Regression of log semivariogram vs. log lag (year) for each treatment provided an estimation of fractal dimension [$D = (4 - \text{slope})/2$], since variance of increments of a Weierstrass–Mandelbrot fractal function varies as $h^{4-2D}$ (Berry and Lewis, 1980). The intercept of this line (log $K$), which is the log semivariogram at lag (year) = 1, is an indication of the extent of variation and can be compared between crop sequences. Since the slopes and $D$-values are related by constants, the differences between slopes also reflected differences between $D$-values. The differences between intercepts reflected differences between $K$-values. Homogeneity of variability between blocks was determined using covariance analysis. Since no differences between blocks were observed for $D$ for any of the crop sequences, analysis of covariance was performed on the data to estimate and compare the slopes and $K$-values between the crop sequences using SAS (SAS Inst., 1985). Semivariograms were estimated using SAS. Semivariograms from lags (years) 1 to 10 (out of 19) were used for determination of $D$- and $K$-values to ensure an adequate number of squared differences and use of linear portion of log semivariogram vs. log lag. Semivariograms for the seven crop sequences are given in Fig. 1. A probability level of $P < 0.05$ was considered significant.

## RESULTS AND DISCUSSION

The first step in comparing the slopes (or $D$, since slopes and $D$ are related by constants) of log semivariogram vs. log lag (year) between crop sequences is to determine whether the slopes are homogeneous across blocks of each treatment. Test of homogeneity indicated that the slopes were not significantly different between blocks of each crop sequence, as indicated by nonsignificant log $h$ × block interactions (Table 1). A significant log $h$ × block interaction indicates that different variability patterns exist between blocks, with the implication of dominance of spatial over temporal variability. In this and another (Eghball et al., 1995) long-term study, temporal variability was far more dominating than the spatial variability that was present. This can have implications for site-specific management where yield maps are used to determine the effects of the previous year's management and for making future management decisions. Yield maps can be greatly influenced by temporal variability, and the smaller influence of spatial variability may not be reflected in a less productive year. For example, grain yield variability due to year-to-year variation in environmental factors can be as high as two to three orders of magnitude, while variability due to spatial variability is rarely more than one order of magnitude. In a less productive year, yield level may not be influenced by spatial variability of any nutrient in a given field. For example, if the yield level is reduced one- or twofold because of year-to-year variation, the effect of N availability on crop yield will not be as great as in a high-yielding year.

Analysis of covariance indicated significant differences between crop sequences for $D$ as well as $K$ (Table 2). In this analysis, the variable crop sequence indicates differences between the intercepts of log semivariogram vs. log lag for the crop sequences. Each intercept indicates the extent of temporal variability for that crop sequence. The variable log $h$ × crop sequence interaction indicates differences between slopes of log semivariogram vs. log lag for the crop sequences. Since slopes and $D$-values are related by constants, the differences between slopes also reflect differences between $D$-values. Fractal dimension was significantly smaller for corn–corn than for other crop sequences (Tables 2 and 3), suggesting less short-term (year-to-year) variation for corn–corn. The extent of variation, as indicated by the $K$-values, was also lowest for corn–corn. Corn–corn also had a lower $D$- or $K$-value than corn after soybean. Overall, corn had a lower $K$-value than sorghum or soybean. This was surprising, as one might expect less year-to-year variation in yield of soybean or sorghum than in corn under rainfed conditions, since these crops

**Table 2. Analysis of covariance for semivariograms of standardized crop yields from 1975 to 1995 for seven crop sequences.**

| Variable | df | Probability level |
|---|---|---|
| Block | 3 | 0.27 |
| Crop sequence† | 6 | 0.01 |
| Corn–Corn vs. Corn–Soybean | 1 | 0.01 |
| Soybean–Soybean vs. Soybean–Corn & Soybean–Sorghum | 1 | 0.09 |
| Sorghum–Sorghum vs. Sorghum–Soybean | 1 | 0.18 |
| Corn vs. Sorghum | 1 | 0.03 |
| Corn & Sorghum vs. Soybean | 1 | 0.01 |
| Log *h*‡ | 1 | 0.01 |
| Log *h* × Crop sequence§ | 6 | 0.01 |
| Corn–Corn vs. Corn–Soybean | 1 | 0.01 |
| Soybean–Soybean vs. Soybean–Corn & Soybean–Sorghum | 1 | 0.04 |
| Sorghum–Sorghum vs. Sorghum–Soybean | 1 | 0.27 |
| Corn vs. Sorghum | 1 | 0.24 |
| Corn & Sorghum vs. Soybean | 1 | 0.01 |

† The contrasts that follow compare intercepts.
‡ Log $h$ is the log of lag (year).
§ The contrasts that follow compare slopes ($4 - 2D$) where $D$ is fractal dimension.

**Table 3. Fractal dimension (*D*) and log *K* values as determined by regression of log semivariograms of standardized grain yields vs. log lag (year) for the crop sequences.†**

| Crop sequence | D | log K |
|---|---|---|
| Corn–Corn | 1.82 ± 0.03 | −0.313 ± 0.041 |
| Corn–Soybean | 1.92 ± 0.04 | −0.164 ± 0.055 |
| Soybean–Corn | 2.00 ± 0.02 | 0.030 ± 0.031 |
| Soybean–Soybean | 1.93 ± 0.02 | −0.073 ± 0.036 |
| Soybean–Sorghum | 1.97 ± 0.02 | −0.021 ± 0.024 |
| Sorghum–Soybean | 1.88 ± 0.03 | −0.191 ± 0.041 |
| Sorghum–Sorghum | 1.92 ± 0.02 | −0.119 ± 0.028 |

† For analysis of covariance, see Table 2.

have lower water requirements than corn. Soybean had higher *D*- and *K*-values than corn or sorghum, indicating greater sensitivity of soybean yield to environmental factors (Tables 2 and 3). Eghball and Power (1995) also showed that soybean had the greatest variation in average crop yield in the USA from 1930 to 1990, compared with nine other crops. Continuous soybean had a lower *K*-value than soybean–corn or soybean–sorghum, indicating less year-to-year variation for continuous soybean than soybean in rotation (Tables 2 and 3). Continuous sorghum had *D*- and *K*-values similar to those of sorghum–soybean, indicating similar variability for both systems.

Coefficient of variation can be used to provide an indication of the variability from the mean of a treatment or a crop. The coefficient of variation for corn–corn, which had the lowest *D*- and *K*-values in the fractal analysis (indicating the least variation), was the highest among the crop sequences (Fig. 2). Using standard statistics may result in reaching a wrong conclusion regarding temporal or spatial variability. In spatial and temporal cases, variability from the mean value does not reflect variability in distance or lag. Fractal analysis and yield standardization (Fig. 3) are better methods of characterizing and comparing variability. Standardized grain yield levels reflect temporal variability for the crop sequences on the same scale, which is essential when comparing crop sequences for the extent of variation. In fractal analysis, since *D*-values are scale independent and do not depend on the yield but rather on variability pattern, they can be statistically compared between crops or treatments with different yield levels. Since actual or standardized yield levels are related by constants, they both will result in similar *D*-values. Also, dominance of short-term vs. long-term variation can be determined for each crop sequence in fractal analysis.

For evaluating variability, other methods can be compared with fractal analysis. For a discussion of comparison between fractal analysis and standard statistics, see Eghball and Power (1995). Another method of evaluating variability is stability analysis. Stability analysis, which is based on regression of mean yield of a genotype or a treatment on mean yield of the environment or the study, has been used to evaluate treatments or genotypes for stability over time or location (Finlay and Wilkinson, 1963; Mead et al., 1986; Raun et al., 1993). The relationship is usually linear with lower yields in less productive environments and higher yields in favorable conditions. This analysis should actually be called a *consistency test*, since a genotype or a treatment is compared with others for consistency. A genotype should not be considered stable when its yield can differ by several orders of magnitude in different environments. The pattern of year-to-year variation is ignored in stability analysis and the environment or study mean yield depends on what genotypes or treatments are included. It is not clear in a stability analysis whether less productive years followed each other, or if there was a pattern of alternate poor and favorable years. In stability analysis (a mean-based procedure), dissimilar crops should not be compared for consistency, because of the scale dependency of the analysis.

## CONCLUSION

Fractal analysis provided an indication of the pattern and extent of variability in data collected in a long-term study. In this study, which included cropping systems, spatial variability was not reflected in grain yields, be-

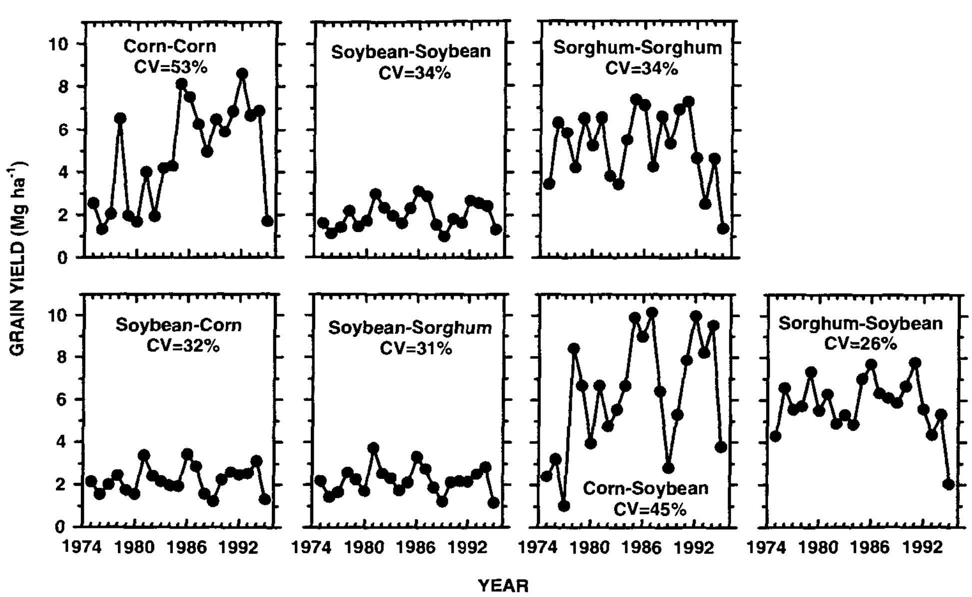


Fig. 2. Actual grain yield from 1975 to 1995 for seven crop sequences at Mead, NE. The yield is for the first crop in the sequence.

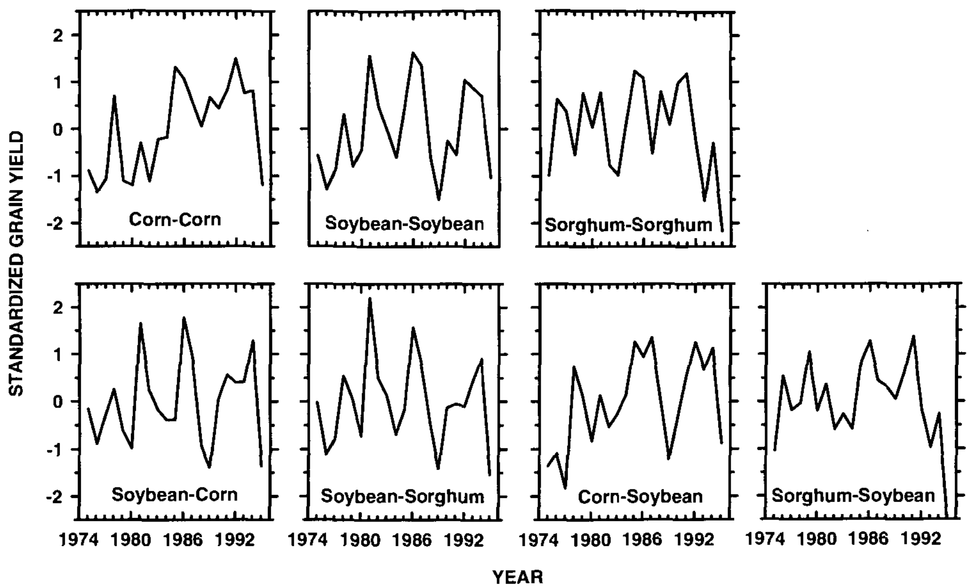


**Fig. 3. Standardized grain yield levels from 1975 to 1995 for seven crop sequences at Mead, NE. The yield level is for the first crop in the sequence.**

cause temporal variability was the overriding factor. Spatial differences across blocks had little effect on variability of grain yield. The corn–corn cropping system had the least short-term (year-to-year) variation, but this temporal variation was still great enough to dominate the spatial variability in the field.

The results from this study have implications for site-specific studies where yield maps are used both as indicators of past management practices and for making future management decisions. Spatial variability may not be reflected in grain yield if temporal (year-to-year) variability is great. For example, variability of soil nitrate or any other nutrient can be great in a given field, but this variability may have little effect on grain yield of a crop in a less productive year. Our results suggest that this may indeed be the case, because, even though we exerted long-term best management practices in this study, environmental conditions from year to year had a greater effect on the resulting yields than did our management practices. It may also imply that, under rainfed conditions, site-specific management practices are likely to produce highly variable results from year to year, which would still cause problems for interpretation of yield maps.

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