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Workshop on Physics Teaching and the Development of Reasoning: Complete Set of Modules

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WORKSHOP ON PHYSICS TEACHING AND THE DEVELOPMENT OF REASONING

BY

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Preface

Are physics teachers in high schools, colleges, and universities knowledgeable concerning the reasoning patterns their students use? The personal experiences of many instructors and research carried out during the last few years indicate that a substantial fraction of physics students have difficulty applying functional relationships among variables, considering all necessary combinations of experimental and theoretical conditions in a problem, and examining their own reasoning critically to locate possible errors. The theory of intellectual development formulated by the Swiss psychologist and epistemologist Jean Piaget deals with these matters and can therefore be of help to physics teachers.

We have prepared these individualized workshop materials to present the two principal concepts of Piaget's theory, stages of development and self-regulation, with background and illustrations that will make clear their relevance for physics teaching. The complete workshop includes audio-visual materials, laboratory activities, and discussions among groups of participants and workshop leaders, as described more fully in the "Guide for Workshop Leaders" also available from AAPT.

The titles of the eleven workshop modules are as follows:

1. How Students Think
2. Concrete and Formal Thought
3. Proportional Reasoning of College Students (Videotape)
4. "Formal Thought" (Film)
5. Analysis of Physics Problems
6. Analysis of Instructional Materials
7. Self-Regulation
8. Learning Activities for Self-Regulation
9. Analysis of Physics Concepts
10. Teaching Goals and Strategies
11. Suggested Reading

You will begin your workshop experience by studying the Orientation Module prepared by your workshop leader to describe the procedures and schedule that will be followed in your workshop.

We are grateful for many thoughtful comments and suggestions to the more than one hundred participants in the workshop held at Anaheim, CA using the trial edition of these materials. We are also indebted to Arnold A. Strassenburg, Warren Wollman, and Anton E. Lawson for reviewing our drafts and providing extensive assistance in the preparation of these materials.

Module 1 How Students Think

Introduction

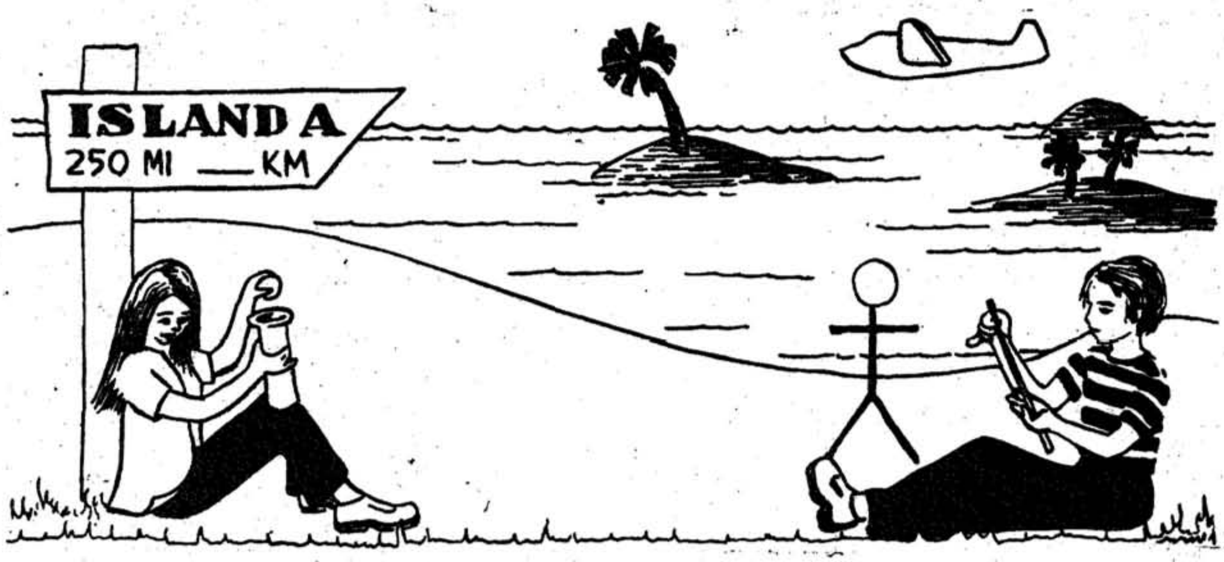
You have probably been curious at various times in your teaching career about the thinking strategies that students enrolled in physics appear to use to solve problems. It is difficult for most of us to understand that many students do not use reasoning patterns that seem to be obvious. Many students substitute numbers into a formula they remember, even though the formula may not be applicable to the problem at hand. This situation quite naturally leads us to wonder about the reasoning that students utilize when we would employ mental operations such as separating variables, excluding an irrelevant factor, or applying a mathematical relationship such as ratios.

Objectives

To assist you in distinguishing among various patterns of thought used to solve simple problems in physics and mathematics.

Procedure

This module includes three puzzles: the Volume Puzzle (page 1-2), the Ratio Puzzle (page 1-6), and the Islands Puzzle (page 1-10). Each puzzle is followed by several typical student responses to the puzzle. Please complete at least two of the puzzle activities by writing out your own solution to the puzzle and then comparing your ideas with those of the students. The puzzles may be used in any order. Then please answer the review questions before going on to the next module.



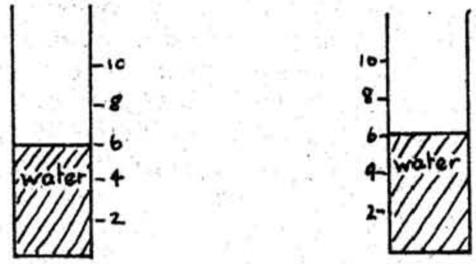
Module 1 Instructional Materials

Volume Puzzle Activity

1. Please write your answers in the spaces below.

VOLUME PUZZLE

Here are drawings of two vertical tubes (cylinders) which are filled to the same mark with water: the cylinders are identical in size and shape.



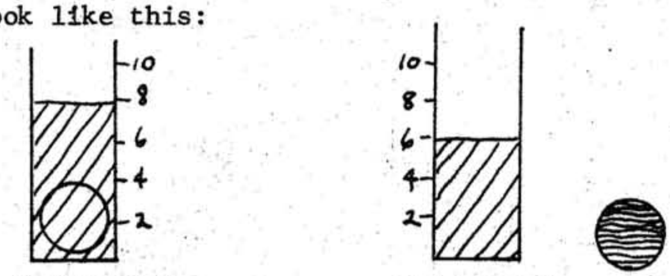
Cylinder 1 Cylinder 2

Here are two marbles, one made of steel and one made of glass. Both marbles have the same volume (that is, they are the same size). The steel marble is heavier.

Glass   Steel

The steel marble is heavier than the glass one, but both marbles will sink if placed in one of the cylinders. We are going to put one marble into each of the cylinders.

After we have put the glass marble into cylinder 1, both cylinders and their contents look like this:



Cylinder 1 Cylinder 2

If we now put the steel marble into cylinder 2, what will happen to the water level in that cylinder? (Tell whether it will rise, fall, or stay the same; if it rises or falls, tell what the final water level will be in cylinder 2.)

Explain why you predicted the result above.

2. This puzzle is accompanied by a videotape showing three girls working on the Volume Puzzle with the equipment described on the preceding page. The first girl (interviewed by Robert Karplus) is about fourteen years old*, the second (interviewed by John W. Renner) is seventeen, and the third (interviewed by Robert Karplus) is nineteen. Each girl's approach has a unique feature. Try to identify it while you are watching the videotape.

   Please view the videotape now, and then describe the unique feature of each girl's thinking in the spaces below.

   Fourteen-year-old:

   Seventeen-year-old:

   Nineteen-year-old:

3. The next pages present written student responses to the Volume Puzzle presented as a paper-and-pencil task. Study the responses and compare Students A with Students B.

<u>Student A_1 (High School Junior)</u>

Prediction: "Rise to 8."
Explanation: "Equal volume spheres displace the same volume of water."

<u>Student A_2 (John Blake - Age 16)</u>

Prediction: "The water level in cylinder 2 will rise to the same height as in cylinder 1 after the glass marble is put in."
Explanation: "Both marbles had the same volume, therefore the water level, after the marbles were put in, was the same in each cylinder. The weight in no way affected the degree to which the water rose."

<u>Student A_3 (Barbara Downing - Age 21)</u>

Prediction: "Cylinder 2's water level will rise to the number 8.
Explanation: "Since the 2 marbles have the same volume they will displace the same amount of water. Eureka! (weight has nothing to do with it)."

<u>Student A_4 (Harold O'Keefe - Age 20)</u>

Prediction: "The level in cylinder 2 will also be 8; the same as in cylinder 1."
Explanation: "If both marbles are heavy enough to sink and are the same size they will therefore displace the same amount of water thus raising the water level to the same numbers."

*Scene from PIAGET'S DEVELOPMENTAL THEORY: CONSERVATION. Compliments of Davidson Films, Inc., 3701 Buchanan Street, San Francisco, CA 94123

Student A_5 (College Junior)

Prediction: "The level of the water will be 8."
Explanation: "The reason that the water levels were both the same was because the objects both had the same volume and the cylinders which they were placed in were the same size with the same water level, therefore the objects displaced the same amount of water in both cylinders and their water level remained the same."

Student B_1 (High School Junior)

Prediction: "I think cylinder 2 would be higher to about the number 10 mark."
Explanation: "Because it put more pressure onto the water. This means it would push it upward. The steel ball seems to me like it would be heavier."

Student B_2 (High School Junior)

Prediction: "It will rise. The final water level in cylinder 2 will be 7."
Explanation: "The steel marble is heavier therefore the water will not rise as much."

Student B_3 (College Junior)

Prediction: "The water level in that cylinder will rise. I estimate the metal marble is twice as heavy so the water level will be at 10."
Explanation: "If you put an object that has the slightest weight into water, the level of the water will rise. The result would be the same as if you added water to the cylinder. Add something to something and you get more."

Student B_4 (College Junior)

Prediction: "The level of H_2O in cylinder 2 will rise to higher than 8 - probably 10."
Explanation: "Because the marble in cylinder 2 is heavier than the marble in cylinder 1. It's just like scales, the more weight the higher it goes up."

Student B_5 (David Kenting - Age 19)

Prediction: "The water in cylinder 2 will rise but not as much as in cylinder 1 because the glass marble has more volume."
Explanation: "Since the steel marble is heavier and smaller, it will sink faster but not have as much volume. Therefore the water level would rise, but not as much as the glass marble."

Student B_6 (Norma Kuhn - Age 20)

Prediction: "The steel marble will make it rise to a level of ten or more."
Explanation: "The reason for the increase in rise on the steel marble was because the steel marble is twice as heavy if not more than the glass marble."

Student B_7 (Deloris Johnson - Age 19)

Prediction: "Cylinder level will rise because the marble is heavy. Final water level will be 10."

Explanation: "Because the steel marble is heavier than the glass marble -- it took up more space than the glass marble."

Student B_8

Prediction: "I think it will stay the same."

Explanation: "I don't really know why. But it would seem the steel marble might have the weight to hold it down. The glass marble is lighter so it pushes the water up."

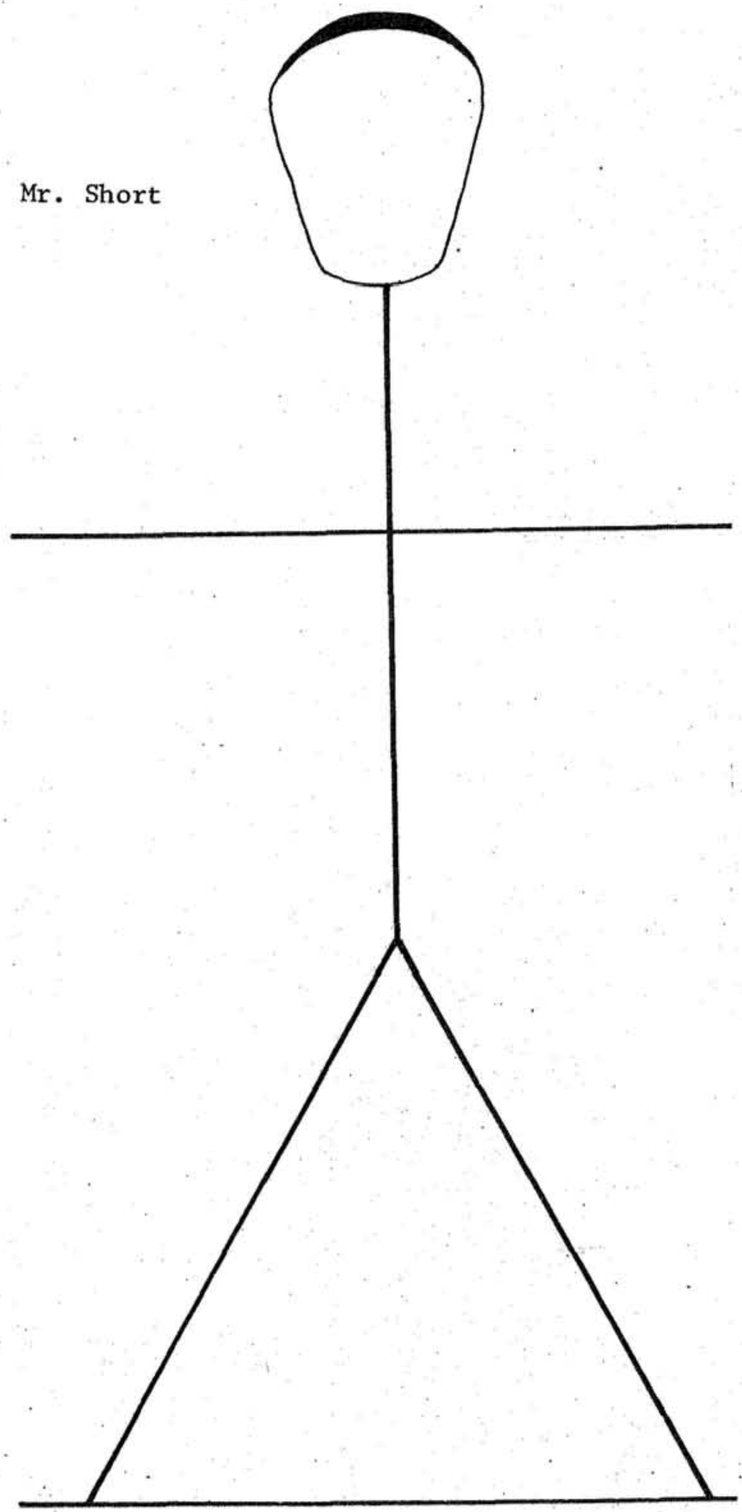
4. What similarities did you find among the responses of Students A? Please record your analysis here.

5. What similarities did you find among the responses of Students B? Please record your analysis here.

6. Please look at the responses again briefly and add any comments you may have about the differences between the two types.

Now proceed to another puzzle or to the Review Questions on page 1-14.

Mr. Short



THE RATIO PUZZLE

The figure at the left is called Mr. Short. We used large round buttons laid side-by-side to measure Mr. Short's height, starting from the floor between his feet and going to the top of his head. His height was <u>four</u> buttons. Then we took a similar figure called Mr. Tall, and measured it in the same way with the same buttons. Mr. Tall was <u>six</u> buttons high.

Now please do these things:

1. Measure the height of Mr. Short using paper clips in a chain provided. The height is ______
2. Predict the height of Mr. Tall if he were measured with the same paper clips.__________
3. Explain how you figured out your prediction. (You may use diagrams, words, or calculations. Please explain your steps carefully.)

Ratio Puzzle Activity

1. Please write your answer to the Ratio Puzzle on this page.

2. The next pages present written student responses to the Ratio Puzzle. Read these responses and compare them with your own.

Student A_1 (Age 16)

Prediction for Mr. Tall: 9 3/6
Explanation: "Figured it out by seeing that Mr. Tall is half again as tall as Mr. Short, so I took half of Mr. Short's height in clips and added it on to his present height in clips and came up with my prediction."

Student A_2 (Age 16)

Prediction for Mr. Tall: 9 1/2 paperclips
Explanation: "I figured that the ratio of paper clips to buttons to be approximately 1 1/2:1 so two more buttons would make approximately 3 more clips. Since it's a little more than 1 1/2:1 he is approximately 9 1/2 clips tall."

Student A_3 (Age 16)

Prediction for Mr. Tall: 9.49 clips.
Explanation: "I took the relationship of the clips to the buttons on Mr. Short and the unknown clips to buttons of Mr. Tall and found the unknown, algebraically."

Student A_4 (John Blake - Age 16)

Prediction for Mr. Tall: 9 1/2 clips
Explanation: "Mr. Tall is 1.5 times the height of Mr. Short, as measured with buttons, and if the measurement techniques were identical would be 1.5 times Mr. Short's height with any measurement medium. Assuming that the measurement techniques are identical, Mr. Tall's height in clips is 1.5 x 6 1/3, which is 9 1/2 (I think)."

Student A_5 (Barbara Downing - Age 21)

Prediction for Mr. Tall: 9.2 paperclips.
Explanation: "The ratio using buttons of height of Mr. Short and Mr. Tall is 2:3. Figuring out algebraically and solving for x:

$2/3 = \frac{6\ 1/6}{x}$ gives you 9.2 as the height in paper clips."

Student A_6 (Deloris Johnson - Age 19)

Prediction for Mr. Tall: 9 paperclips tall.
Explanation: "I figured this out by figuring that Mr. Small is 2/3 as tall as Mr. Tall."

Student B_1 (Age 16)

Prediction for Mr. Tall: 8 1/2 clips.
Explanation: "If he is 2 buttons taller I guess he is 2 clips bigger which would make it 8 1/2."

Student B_2 (Age 18)

Prediction for Mr. Tall: 8 clips
Explanation: "Because he is two times as high as Mr. Short."

Student B_3 (David Kenting - Age 19)

Prediction for Mr. Tall: 8 1/2 clips.
Explanation: "I figured the buttons the same size as the clips."

Student B_4 (Age 14)

Prediction for Mr. Tall: 9 clips (pencil marks along Mr. Short)
Explanation: "I estimated the middle and then one fourth of Mr. Short. That's about the size of one button. I measured the button with my clips and found one-and-a-half. So then I counted out six times one-and-a-half buttons and got nine."

Student B_5 (Age 16)

Prediction for Mr. Tall: 12 clips
Explanation: "Mr. Tall was 2 buttons taller than Mr. Short. The buttons must be larger than the paper clips. So I doubled Mr. Short's height in paper clips for Mr. Tall's height."

Student B_6 (Norma Kuhn - Age 20)

Prediction for Mr. Tall: 8 paper clips
Explanation: "Mr. Tall is 8 paper clips tall because when using buttons as a unit of measure he is 2 units taller. When Mr. Short is measured with paper clips as a unit of measurement he is 6 paper clips. Therefore, Mr. Tall is 2 units taller in comparison which totals 8."

Student B_7 (Harold O'Keefe - Age 20)

Prediction for Mr. Tall: 8 paper clips tall
Explanation: "If Mr. Short measures 4 buttons or 6 paper clips (2 pieces more than buttons), then Mr. Tall should be 2 paper clips more than buttons."

Student B_8 (Age 25)

Prediction for Mr. Tall: 8 paper clips tall.
Explanation: "4 buttons reached top of Mr. Short's head. Mr. Tall is 6 buttons tall. 6 paper clips Mr. Short. Mr. Tall is 8 paper clips tall. Paper clips are approximately 1 inch long and the buttons were probably the same."

3. Center your attention on several of those responses which were different from yours. See if you can detect any common elements among them. Record those common elements here.

4. Center your attention on those responses which agree with yours and see if you can detect any common elements among the student responses and your response.

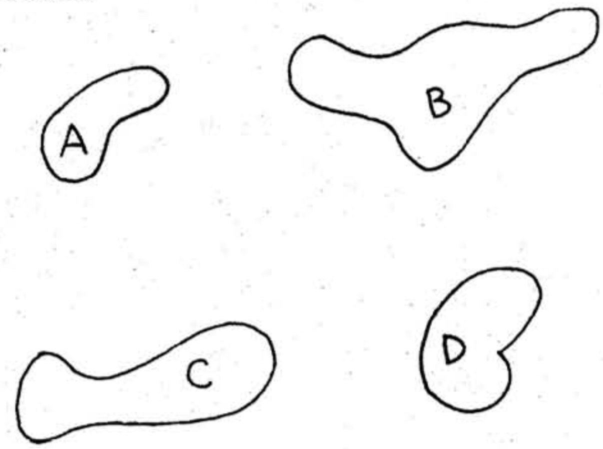
Now proceed to another puzzle or to the review questions on page 1-14.

Islands Puzzle Activity

1. Please write out your answers to the **islands** puzzle in the spaces below.

THE ISLANDS PUZZLE

The puzzle is about Islands A, B, C, and D in the ocean. People have been traveling among these islands by boat for many years, but recently an airline started in business. Carefully read the clues about possible plane trips at present. The trips may be direct or include stops and plane changes on an Island. When a trip is possible, it can be made in either direction between the islands. You may make notes or marks on the map to help use the clues.



First Clue: People can go by plane between Islands C and D.

Second Clue: People cannot go by plane between Islands A and B.

Use these two clues to answer Question 1. Do not read the next clue yet.

Question 1: Can people go by plane between Island B and D?
Yes_____No_____ Can't tell from the two clues____
Please explain your answer.

Third Clue (do not change your answer to Question 1 now!): People can go by plane between Island B and D.

Use all three clues to answer Question 2 and 3.

Question 2: Can people go by plane between Island B and C?
Yes_____No_____ Can't tell from the three clues_____
Please explain your answer.

Question 3: Can people go by plane between Islands A and C?
Yes_____No_____ Can't tell from the three clues_____
Please explain your answer.

2. What types of thinking did you do while completing the Islands Puzzle? For example, did you need to recall or review some of the initial instructions while answering the questions? Did you use combinations of information? Did you exclude any information as irrelevant? Did you make any hypotheses and then test them? Please record your observations of your own procedure here.

3. Did you notice any similarity between the Islands Puzzle and four-terminal "mystery" circuit boxes sometimes used in general physics laboratories?

   If you did not, turn back to the puzzle, try to construct an electric analogue, and compare the results obtained from this analogue with your answers to the puzzle. Did you find the comparison helpful? Please record your comments.

4. The next pages present written student responses to the Islands Puzzle. Please read the responses and compare them with yours.

<u>Student A_1 (John Blake - Age 16)</u>

1. Answer: Can't tell from the two clues.
   Explanation: There is not enough information given.

2. Answer: Yes
   Explanation: "They can go to Island B from Island C, then on to Island D."

3. Answer: No
   Explanation: There is no information on a direct flight from A to C, but if you could get to C from A then you could also get to B. Since you cannot get to B from A you cannot get to C from A.

<u>Student A_2 (Deloris Johnson - Age 19)</u>

1. Answer: Can't tell from the two clues.
   Explanation: "There was no information given concerning the two."

2. Answer: Yes
   Explanation: "They can fly from C to D, have a lay over and catch the plane from D to B."

3. Answer: No
   Explanation: "It was said you <u>cannot</u> go from A to B. There is no information about a flight direct from C to A - only C to D."

Student A_3 (College Student - Age 17)

1. Answer: Can't tell from the clues given.
   Explanation: "The two clues don't relate the upper islands to the lower ones.

2. Answer: Yes
   Explanation: They can go from B to D and then to C, even if there are no direct flights.

3. Answer: No
   Explanation: If they could go from C to A, then the people on B could go first to D, then to C, and then on to A. But this contradicts the second clue, that they can't go by plane between B and A.

Student A_4 (David Kenting - Age 19)

1. Answer: Can't tell from the two clues.
   Explanation: By information given they could if appropriate landing facilities were on Island B.

2. Answer: Yes
   Explanation: "Yes because planes go from C to D or vice versa and B to D and vice versa. Therefore all have facilities."

3. Answer: No
   Explanation: "No, because Island A has no landing facilities mentioned."

Student A_5 (Norma Kuhn - Age 21)

1. Answer: Yes
   Explanation: "If the trip from C to D includes a stop on B. The clues only state that one <u>cannot</u> go by plane between A and B. The introduction states that the flights need not be direct."

2. Answer: Yes
   Explanation: "Via D"

3. Answer: Can't tell from the three clues.
   Explanation: "The clues do not give any connections to A except via boat."

Student A_6 (Barbara Downing - Age 22)

1. Answer: Can't tell from the two clues.
   Explanation: "The clues tell nothing of the relation of B and D."

2. Answer: Yes
   Explanation: "If you can go from C to D and D to B, C to B should also be possible."

3. Answer: No
   Explanation: "If you can go from C to B, but not B to A, you should not be able to go from C to A."

Student B_1 (College Student - Age 30)

1. Answer: Yes
   Explanation: "You can't go from B to A but you can go from D to B, or go from D to C then to Island B."

2. Answer: Yes
   Explanation: "It doesn't say that you can't go. It says you can't go from A to B islands, you can cut across or go through D."

3. Answer: Yes
   Explanation: "You can as long as you don't go on to Island B."

Student B_2 (Harold O'Keefe - Age 20)

1. Answer: Yes
   Explanation: "Because B is bigger than D and listing is the same sequence."

2. Answer: No
   Explanation: "The sequence is broken."

3. Answer: Yes
   Explanation: "They are listing the same as in Question One."

Student B_3 (High School Student - Age 18)

1. Answer: No
   Explanation: "B and D are not far enough apart."

2. Answer: Yes
   Explanation: "They are a long distance apart."

3. Answer: No
   Explanation: "Not far enough apart."

Student B_4 (College Student - Age 17)

1. Answer: Yes
   Explanation: "Because the people can go north from Island D because in the clue it could be made in both directions."

2. Answer: No
   Explanation: "I am presuming both directions doesn't include a 45^o angle from B to C."

3. Answer: Yes
   Explanation: "Because Island C is right below Island A."

5. Now please identify some features of the thinking used by Students A and B that set them apart from each other and from your thinking.

   Students A:

   Students B:

Please proceed to another puzzle or to the Review Questions on page 1-14.

Module 1 Review Questions

After you have completed two or three puzzle activities in Module 1, read the following responses and classify them as Type A or Type B. Compare your answers with those of other participants sitting near you and with ours (bottom of page).

Student X_1 (College junior) Volume Puzzle

Prediction: Rise above 8 to approximately 10
Explanation: The weight of the object placed in water displaces an amount of water. Thus if the steel marble was heavier, it displaced more water, causing it to rise above the level of water in which the glass marble was in.

Type__________

Student X_2 (age 15) Ratio Puzzle

Prediction for Mr. Tall: 15 paper clips high
Explanation: "Guess. I'm really not sure how to do this."

Type__________

Student X_3 (age 20) Islands Puzzle

1. Answer: Can't tell from the two clues.
   Explanation: No information about flights between B and D

2. Answer: Yes
   Explanation: Go from C to D and then to B.

3. Answer: Can't tell from the three clues
   Explanation: Not possible to fly from B to A, and there is no mention of a direct flight between C and A.

Type__________

Our answers: X_1 - B, X_2 - B, X_3 - A.

Module 2 Concrete and Formal Thought

Introduction

You have just completed several activities in which you examined student responses to various problems involving observation and reasoning. Observations of many children and young people attempting to perform similar tasks have led Jean Piaget and other psychologists to formulate theories concerning the mental processes an individual uses to deal with problem situations. In this module, we shall introduce you briefly to stages of reasoning, a feature of Piaget's theory we consider important for physics teachers. Modules 3 and 4 will give you more details and examples to illustrate what we say here. Modules 5 through 11 will help you to apply Piaget's ideas to physics teaching materials and teaching approaches.

Objectives

To assist you in describing and identifying student behavior that indicates concrete thought and behavior that indicates formal thought.

Procedure

Begin by reading the article, "Piaget's Theory in a Nutshell" included in the attached instructional materials. An audiotape with comments coordinated with the article is available; you may wish to listen to the tape during your first reading or during a review. To follow the article, we have provided two more activities for you in this module - - analyzing the student answers to the puzzles in Module 1, and participating in a group discussion - - each at a designated station arranged by your workshop leader. The order of these **two** activities is optional.

Module 2 Instructional Materials

1. Piaget's Theory in a Nutshell

While you were reading the student responses to the four puzzles in Module 1, you undoubtedly recognized that Type A answers were more satisfactory, more adequate, than Type B answers. In fact, you may have been disturbed to learn that any college students gave Type B answers! We believe that each of the two types of answers is characteristic of a level of reasoning that corresponds to one stage in the intellectual development of children and adolescents as classified by the Swiss psychologist and epistemologist Jean Piaget. We shall therefore give you some background regarding Piaget's theory and then apply it to the problem-solving and reasoning strategies of the students who responded to the puzzles.

The principal concepts of the theory are stages of intellectual development and self-regulation; like concepts in any theory, they are idealizations helpful in analyzing and interpreting observations, and are no more or less real than a point particle or a frictionless plane. A stage of intellectual development is a period when a person's activities and reasoning are characterized by certain distinctive features. We shall give more details below. Self-regulation refers to the process whereby an individual's reasoning advances from one stage to the next. This very important idea is explained in Module 7.

Piaget has described human intellectual development in terms of four stages. The first two, called sensory-motor and preoperational, are usually completed before a child is ten years of age. The last two only are therefore of particular interest to us; they are called concrete thought and formal thought. To give you clues for distinguishing student behavior as falling into one or the other of these stages, we shall now enumerate some of their characteristic patterns of reasoning.

Clues to identify the stage of concrete thought* - affirmative answers to:

(C1) Does the individual make simple classifications and generalizations (e.g., all dogs are animals, only some animals are dogs)?

(C2) Does the individual apply conservation logic (e.g., if nothing is added or removed, the amount remains the same even though the appearance may differ)?

(C3) Does the individual arrange a set of objects or data in serial order and establish one-to-one correspondence between two sets (e.g., the youngest person at dinner gets the most dessert)?

In these respects the individual can reason and solve problems beyond his

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*We have used parenthetical codes with the letters C and F to denote indicators of concrete and formal thought. Numbered items are principal clues, lettered items are illustrative examples.

ability in previous stages. Here are a few examples to illustrate these accomplishments. The individual now:

(Ca) understands concepts and simple theories that make direct reference to familiar actions and examples, and can be explained in terms of simple associations, orderings, or numerical equivalences/differences (e.g., objects that do something to each other are in interaction; the waves are high because there is a strong wind);

(Cb) follows step-by-step instructions as in a recipe, provided each step is correctly specified;

(Cc) relates his/her viewpoint to that of another in a simple situation (e.g., a girl is aware that she is her sister's sister).

Yet the advances in reasoning are limited as compared to those achieved at the stage of formal thought. These limitations may be detected as the individual now:

(Cd) searches for and identifies variables influencing a phenomenon, but does so unsystematically (e.g., investigates the effects of one variable but does not necessarily hold the others constant);

(Ce) relates observations and makes inferences from them, but does not consider all possibilities;

(Cf) responds to difficult problems by applying a related but not necessarily correct algorithm (i.e., relies on analogy or agreement more than on inconsistency or contradiction);

(Cg) processes information but is not spontaneously aware of his own reasoning (i.e., does not check his/her own conclusions against the given data or other experience).

<u>Clues to identify the stage of formal thought</u> - affirmative answers to:

(F1) Does the individual reason with propositions regardless of whether they are factual or hypothesized?

(F2) Does the individual consider all conceivable combinations of experimental or theoretical conditions, even though some may not be realizable in nature?

(F3) Does the individual recognize and interpret functional relationships in situations described by observable or abstract variables (e.g., field strength is inversely proportional to the square of the distance, the volume of a cube varies directly as the third power of the edge length)?

(F4) Is the individual aware and critical of his/her own reasoning (e.g., recognizes options in using various models or approximations, or tests a conclusion to see whether it is based on a fallacious step)?

Here are a few further examples to illustrate these achievements.
The individual now:

(Fa) engages in hypothetico-deductive reasoning (e.g. in the Islands puzzle, he/she would explain, "If there were a plane route between Islands A and C, then people could get by plane also from Island A to Island B.").

(Fb) plans experiments according to an overall design that investigates the effects of one variable while holding the others constant and also allows for unforeseen contingencies;

(Fc) uses theories and idealized models to interpret observations and draw conclusions;

(Fd) understands concepts defined in terms of other concepts or in terms of abstract relationships (e.g., ratios, mathematical limits);

(Fe) solves problems by introducing intermediate variables not given or asked for directly in the original statement.

In all these items it is the reasoning that counts; the answer or conclusions reached may or may not be correct, depending on whether relevant facts were remembered correctly.

The physics teacher who wishes to apply these ideas should know that many theoretical and experimental issues relating to the theory are currently being investigated. Piaget's original notion was that all persons progress through the stages in the same sequence, though not necessarily at the same rate. Yet recent studies suggest strongly that not everyone reaches the stage of formal reasoning. We have, therefore, earlier characterized the stages as idealizations; few advanced high school or beginning college students would fall clearly into the stage of concrete or of formal thought. Rather, we consider their overall behavior as transitional, partially consistent with each stage. Possibly the reasoning patterns of formal thought are only applied actively by individuals in areas in which they are interested and with which they are familiar.

This qualification leads to four additional points that must be kept in mind by the teacher. First, a person may use primarily formal reasoning patterns in relation to ideas with which he is familiar, while using concrete reasoning patterns in other areas. Second, the stage of formal thought is really open-ended, in that an individual may deepen his understandings, broaden the domains, and/or add new intellectual fields within which he can function formally with confidence. Third, one can enter the formal stage in any area only through self-regulation from the concrete stage, which must not be by-passed. Fourth, by applying memorized formulas to familiar problems, a student may appear to use formal thought though the reasoning pattern is actually concrete(Cf).

You may wonder whether Piaget's theory can be used reliably to improve physics teaching, in view of the fact that physics teaching has been taking place for many years without the theory's benefits. In fact, there are some ways in which Piaget's theory contradicts prominent theories of learning, according to

which individuals in the learner's environment shape his behavior through providing suitable stimuli (learning objectives, exercises) and selective reinforcement (grades, social esteem, academic failure). In our opinion, a sound teaching program reconciles these two approaches as follows: (1) all curriculum design and selection of achievement levels are carried out in accordance with Piaget's theory; (2) the interpersonal contacts between teacher and students rely on reinforcement in the sense that the teacher is the "stimulus" by serving primarily as role model for investigative and analytical attitudes and reinforcement is provided by the students' own sense of success, supported by social and verbal signals (smiles, admiration, encouragement) that acknowledge his success.

The theory's implications for physics teaching can be summarized as follows:

1. Be aware that some of your students approach topics in physics with concrete reasoning patterns, while others will approach the same topics using formal reasoning patterns.

2. Provide a teaching program that allows some success through the use of concrete reasoning patterns.

3. When introducing new topics, do so on the level of concrete thought, for two reasons -- (i) to allow students to gain at least a partial understanding through the use of concrete reasoning patterns, and (ii) to permit students to develop and apply formal reasoning patterns gradually through self-regulation.

4. Devote some effort to helping students establish formal reasoning patterns and thereby gradually raise their level of reasoning.

Modules 5 through 11 will expand on these items.

The thought of using Piaget's theory to improve educational programs systematically is relatively recent, having originated in connection with the elementary school science curriculum development projects during the nineteen sixties. In the last few years, researchers have begun to consider the implications for high school and college teaching, and have found in surveys that many students do not use the mental operations of formal thought when answering puzzles such as those included in Module 1. We shall therefore ask you to review these answers more carefully as another activity in this module.

(Note: if you have not yet used the audiotape commentary on "Piaget's Theory in a Nutshell," you may wish to do so, now or later, while reviewing the article.)

This module provides for two more activities:

(1) Analyzing the student answers to the Module 1 puzzles as revealing concrete or formal thought;

(2) Discussing "Piaget's Theory in a Nutshell" with other workshop participants and staff.

Follow your workshop leader's instructions with respect to these activities. At the conclusion, please answer the review questions on page 2-9.

2. Analysis of Student Responses in Module 1

Below is a chart on which we should like you to record your evaluation of the reasoning patterns used by the six students whose responses to the puzzles were given in Module 1. Please use the following more descriptive categories rather than the very superficial A/B designation that we employed:

PC = Preconcrete
C = Concrete
Tr = Transitional from concrete to formal
F = Formal
? = impossible to classify without more information

Category Tr is intended for responses that include several elements, some of which you would call C while others fit the description of F.

Choose first one student and examine his or her responses to each of the three puzzles. Record your evaluation of his/her reasoning patterns, thus making a "profile" of reasoning for this student. Please follow this procedure for at least three students -- more if you have time. Then read our general analysis and summary.

Puzzle Responses

Student	Volume	Ratio	Islands
Deloris Johnson (19)	______	______	______
Barbara Downing (21)	______	______	______
David Kenting (19)	______	______	______
Harold O'Keefe (20)	______	______	______
Norma Kuhn (20)	______	______	______
John Blake (16)	______	______	______

To give you specific illustrations of how the stages of reasoning in Piaget's theory can be applied to student work, we shall now give a general analysis of the responses to the puzzles in Module 1. The parenthetical codes refer to the items listed in "Piaget's Theory in a Nutshell."

Volume Puzzle

FORMAL THOUGHT (TYPE A). Even though the weight is dynamically responsible for lifting the water, the combined volume of water plus marble limits the height to which the water can rise in the container. Since the combined volumes are equal for the two marbles, the water will rise to equal heights if the marbles are fully submerged (F1). Note the intermediate concept of the combined volume, or the alternate formulation that if equal marble volumes are added to equal water volumes, the final volumes will be equal (F1). The combined or final volume is not stressed in the statement of the puzzle, but must be introduced by the student (Fe).

CONCRETE THOUGHT (TYPE B). It is common sense that the weight of an immersed object is responsible for the force that lifts the displaced water (Ca). Hence the direct conclusion, given differing weights, is the greater the weight, the higher the water level (C3). Note that this reasoning leads to the correct conclusion for immersed bodies that float!

Ratio Puzzle

FORMAL THOUGHT (TYPE A). Each button corresponds to a certain number of paper clips, an intermediate quantity not stated in the puzzle nor asked for (Fe). Once this conversion ratio is known, the answer is found by simple calculation. Alternatively, the student might conceptualize the height ratio (Fd), another intermediate abstraction, and then reason that this ratio must be invariant with respect to the units of measurement (F1, F3).

CONCRETE THOUGHT (TYPE B). Since the height of Mr. Short measures more paper clips than buttons, simply add the extra amount to the height of Mr. Tall (C3). Even though the arithmetic difference in units is not stated or asked for; it is a much more direct measure of the qualitative difference than is the ratio, which comes from making a correspondence between each individual button and paper clip. Another concrete approach makes use of the height difference in buttons of the two figures, and associates that directly with the same difference in paper clips (C3). Note that extra buttons are equated to extra paper clips, in contradiction to the fact that the four buttons measuring Mr. Short are equal to six and not to four paper clips. This inconsistency is not noticed at the stage of concrete thought, but would be noticed at the formal stage and would lead the student who had originally made this mistake (self-regulation!) to re-examine his/her procedure (F4).

Islands Puzzle

FORMAL THOUGHT (TYPE A). On Question 2, the trip from Island B to Island C is conceptualized as possibly achieved by a change of planes or stopover

at Island D. In other words, the clues about plane routes are not only evaluated in terms of the direct information they provide, but also in terms of the inferences that are possible by using the general rules about connections that were stated in the introduction of the puzzle (F1, F2). On Question 3, the formal thinker imagines all possible routes from Island A to Island C in order to bring to bear the information available in the clues (F2). In particular, he must hypothesize that air travel is possible and evaluate this hypothesis for consistency with the data (F1, F4, Fa). Note that most of the Type A responses quoted in Module 1 did not make use of the formal approach to Question 3, but did on Question 2. This mixture of procedures is often observed in practice and indicates transitional reasoning, a reflection of the fact that the stages of Piaget's theory are idealizations which help one to classify observed behavior, but should not be used to classify people superficially.

CONCRETE THOUGHT (TYPE B). Since the clues do not give the answers to the questions directly, the concrete thinker either can't tell, selects certain details from the map (geographical placement, island separation) or postulates properties of each island to explain his ideas (C1). The properties of a single island (size, topography) used in this approach are conceptually simpler to manipulate than the plane routes, which represent relationships between islands. This approach also eliminates the need to make use of the rules for combining plane routes.

Summary

Below is a chart in which we have applied the above considerations to the responses of six students who attempted the three puzzles in Module 1. In looking at these responses you can see that only one subject gave all formal responses. This indicates that students are at varying levels in various subject areas. We would not expect college students to think formally in every content area. The transition from concrete to formal thinking depends a great deal on the kinds of experiences that any person has in a particular field of study. If a student is a formal rather than a concrete thinker in one area, however, he is more likely to make the transition to formal thought in another area when he is given suitable intellectual stimulation.

College Students Responses	Volume	Ratio	Islands
Deloris Johnson (19)	C	Tr	Tr
Barbara Downing (21)	F	F	Tr
David Kenting (19)	C	C	Tr
Harold O'Keefe (20)	Tr	C	C
Norma Kuhn (20)	C	C	Tr
John Blake (16)	F	F	F

Please discuss these results with a workshop staff member and other participants at a discussion table. Then complete the Module 2 Review Questions on the next page.

Module 2 Review Questions

Please answer these questions in writing. Then compare your ideas with those of other participants and with our answers below.

1. What are two characteristics of concrete thought?

2. What are two characteristics of formal thought?

3. How would you classify the answers to the following question? Explain in each case. "How many different license plates can be made with letters A, B, and C? Describe how you figured it out."

   Answer X: I made six ABC, CAB, BCA, CBA, BAC, ACB. I tried but can't make any more.

   Answer Y: It depends on whether you reuse the letter. If you use each one once, you have three choices for the first letter and two for the second and one for the third, three times two times one makes six. If you can have each letter more than once, like in ABB, then you have three choices for each of the three spots, that's three times three times three or twenty-seven. I'd hate to write them all down. There aren't any other possibilities because I took all into account.

   Your evaluation of X:

   Your evaluation of Y:

Answers: 1. simple classification, serial ordering, one-to-one correspondence, simple causal association, conservation
2. functional relations, hypothetico-deductive reasoning, systematic manipulation and control of variables, propositional logic
3. X - concrete; enumerates specific instances, concludes by trial and error.
Y - formal; uses general procedure, considers alternative problem formulations.

Module 2 Audiotape "Piaget in a Nutshell"
A discussion by Robert Karplus and Jane Bowyer

Robert Karplus

Hello! This tape offers comments and examples of the use of concrete and formal reasoning patterns in physics. It accompanies Module 2 of the Workshop on Physics Teaching and the Development of Reasoning produced by the American Association of Physics Teachers. I'm Bob Karplus.

Jane Bowyer

And I'm Jane Bowyer. Have you read the article, "Piaget's Theory in a Nutshell" in Module 2? If so, you may find this tape instructive. If not, I'd suggest that you turn off the tape for now and read the article first, because it introduces the ideas on which this tape is based.

A transcript of the tape is included in your study guide beginning on page 2-10. If you'd like to follow the text, turn off the tape until you find the correct page and then turn it on again.

Robert Karplus

Piaget has described human intellectual development in terms of four stages during which individuals use certain patterns of reasoning.

Before continuing, I'd like to explain what I mean by a "pattern of reasoning." A pattern of reasoning is a mental process by which certain data, observations, or ideas are compared, organized, or transformed. For example, recognizing that a pendulum with mechanical energy of 20 joules and potential energy of 6 joules has kinetic energy of 14 joules, is a pattern of reasoning that involved comparing forms and amounts of energy. As another example, consider finding Mr. Ruthgren's telephone number between Rutherford and Ruthie; here one has to make use of the alphabetic order of letters and apply it successively to the first, second, third, fourth, and fifth letters in the names in the directory. A person who cannot conceptualize the alphabetic order of letters and apply it systematically is unlikely to find the listing.

Jane Bowyer

Piaget uses the term OPERATION rather than pattern of reasoning, and describes it in his article reprinted in Module 11. We have avoided the term OPERATION because of its other meanings in physics.

Let's now go back to the four stages. The first two, called sensory motor and pre-operational, are usually completed before a child is ten years of age. Only the last two are therefore of interest to us; they are called concrete operational and formal operational. Bob and I will give examples of some characteristic patterns of reasoning associated with these two stages.

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General clues to identify concrete thought were listed on pages 2-2 and 2-3:

(C1) Does the individual make simple classifications and generalizations?

Robert Karplus — An example is consistently sorting a collection of objects into electrical conductors and electrical insulators after testing them in a circuit.

Jane Bowyer — (C2) Does the individual apply conservation logic?

Robert Karplus — When a rocket of mass M ejects exhaust of mass ΔM, the student concludes that the rocket has remaining mass $M-\Delta M$.

Jane Bowyer — (C3) Does the individual arrange a set of objects or data in serial order and establish one-to-one correspondence between the two sets?

Robert Karplus — Short organ pipes produce high pitched sound waves and long organ pipes produce low pitched sound waves.

Jane Bowyer — In these respects the individual can reason and solve problems beyond his/her ability in the preoperational stage. Items (C1), (C2), and (C3) are called concrete reasoning patterns, because they are applied to concrete objects and directly observable properties--electrical conductors, mass of a rocket, organ pipes, and audible pitch.

For comparison, we'll now describe a physics example that requires reasoning for which concrete patterns are not adequate. The example is an explanation of Archimedes's principle. Why is the bouyant force on body A when immersed in water equal to the weight of the displaced water?

Robert Karplus — First, imagine a hypothetical body B of exactly the same size and shape as A but composed of water. Since this water body is in equilibrium when immersed in water, the bouyant force it experiences is equal to its weight W_B. By the definition of body B, W_B is also the weight of the displaced water. Furthermore, the bouyant force on body B is the net force exerted by the rest of the water across body B's bounding surface. The bouyant force on body A is the net force exerted by the rest of the water across its bounding surface, which is identical with the bounding surface of B. Hence the bouyant force on A equals the bouyant force on B, and this in turn is equal to the weight of the displaced water.

Jane Bowyer — The reasoning involved here was not limited to concrete patterns because the hypothetical water body B and the "displaced water" were never perceptually distinct. Furthermore, the reasoning

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made use of certain propositions regarding the boundary surfaces and the equality of forces. The required reasoning comprised formal patterns.

Bob and I will now turn to formal reasoning patterns more broadly, with clues as listed on page 2-3:

(F1) Does the individual reason with propositions regardless of whether these are factual or hypothesized?

Robert Karplus

The student who correctly finds the thermodynamic efficiency of an ideal heat engine with black body radiation as working medium uses propositions such as the first law of thermodynamics, the equation of state of the radiation, and hypothesized processes making up the carnot cycle. Similar reasoning was used in our explanation of Archimedes's principle. It is also used when Newtonian mechanics, electrostatics, group theory, or other subjects are derived from definitions and postulates rather than being inferred from concrete examples and observations.

Jane Bowyer

(F2) Does the individual consider all conceivable combinations of experimental and theoretical conditions, even though some may not be realizable in nature?

Robert Karplus

To solve the Islands puzzle, for instance, the individual had to be aware of all possible ways Island C could be reached from Island A. When inferring the construction of an electric network from measurements at its terminals, the student has to consider all possible ways in which resistors, capacitors, and other circuit elements could be assembled.

Jane Bowyer

(F3) Does the individual recognize and interpret functional relationships in situations described by observable or abstract variables?

Robert Karplus

Students who use inverse proportion of weight and distance when equalizing a balance arm apply this formal reasoning pattern. When graphing and interpreting experimental data, they smooth out small irregularities in the measurements and describe the relationship by a simple analytic formula.

Jane Bowyer

(F4) Is the individual aware of and critical of his/her own reasoning?

Robert Karplus

The formal operational student checks an answer by comparing the results of a calculation with other similar calculations. He/she verifies that the solution of a motion problem with friction falls between the solutions to the same problem without friction and with very large friction (no slipping at all).

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Jane Bowyer	On pages 2-3 and 2-4 there are additional examples of concrete and formal reasoning patterns. Unfortunately, we cannot give you a single, simple criterion for distinguishing between these two types of patterns.
Robert Karplus	You have to keep four additional points in mind, as described on page 2-4:
Jane Bowyer	First, a person may use primarily formal reasoning patterns in relation to ideas with which he is familiar, while using concrete reasoning patterns in other areas with which he is unfamiliar.
Robert Karplus	Second, the stage of formal thought is really open-ended, in that an individual may deepen his understandings, broaden the domains, and/or add new intellectual fields within which he can function formally with confidence.
Jane Bowyer	Third, one can enter the formal stage in any area only through self-regulation from the concrete stage, which must not be by-passed.
Robert Karplus	Fourth, by applying memorized formulas to familiar problems, a student may appear to use formal thought though the reasoning pattern is actually concrete.
Jane Bowyer	You may wonder whether you should test your students to identify their developmental stage. In view of what we have just said, and the fact that the stages are idealizations, such a testing effort is likely to give unclear results. I would recommend that you observe your students' work on their physics problems for a period of a week or two and try to identify the reasoning patterns they use.
Robert Karplus	This is the end of our comments. We hope you are finding the workshop interesting. Do discuss these ideas with your fellow participants--they may have a very different point of view from yours. Before turning off the tape player, please rewind the tape so it can be used by other participants. Thank you for listening. Goodbye!

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Module 3 Proportional Reasoning [Videotape]

Introduction

In Piaget's theory, concrete operational thought is characterized by serial ordering, simple classification, and conservation logic applied directly to objects. A concrete thinker doing a Piagetian task must be able to observe objects and/or manipulate them. Formal operational thought involves proportional reasoning, separation of variables, elimination of contradictions, and class inclusion or exclusion operations. A formal thinker is able to work in situations where he does not deal with tangible objects. The formal thinker can apply the operations used by a concrete thinker, but goes beyond these operations when solving problems.

In the video-tape you are about to see, you will observe Francis P. Collea working with college science students who are responding to two Piagetian tasks. The tape clearly demonstrates that a college population includes students who approach certain tasks with concrete reasoning patterns, while others apply formal reasoning patterns. The students' responses indicate a wide range of variation even among science and mathematics majors, a highly selected group.

Objectives

To assist you in describing and/or identifying responses that indicate concrete or formal thought as applied to Piagetian tasks.

Procedure

Begin this module by reading the Overview of the tasks in the attached videotape notes. Then view the videotape. To help you understand the students' remarks in spite of their soft voices, a complete transcript of the dialogue is included at the end of the instructional materials. You may wish to glance at the transcript while you are watching the videotape.

Videotape Notes

Overview of the Tasks

In preparing the videotape, we selected responses of individual students so as to present a variety of approaches. We did not attempt to give an accurate impression of the frequency distribution that might be obtained from college students.

Module 3 Instructional Materials

Equal Arm Balance Task

The Balance Beam Task requires students to apply proportional reasoning and other elements of formal thought to a somewhat difficult problem. The interviewer poses the following four questions (in order) as each student observes the balance beam and attempts to predict the balance conditions.

1. Use a 7-weight and a 3-weight to balance a 10-weight placed 10 units from the fulcrum.
2. Use a 5-weight to balance a 10-weight placed at 10 units from the fulcrum.
3. Use a 7-weight to balance a 10-weight placed at 9 units from the fulcrum.

<u>Dave</u> Our first student is shown responding to Problems 1, 2, and 3. He used proportional reasoning on #2, which involved a 2:1 ratio, but applied an additive process to #3: he placed the 7-weight at 12 units, three further than the 10-weight. He finally balanced the beam by trial and error with the 7-weight at 13 units, not clearly recognizing the relationship between location and weight. Dave appears to have begun the transition from concrete to formal thinking in relation to the balance beam.

<u>Gary</u> The next student answered Problems 1, 2, and 3 quickly, using direct and inverse proportion with ease.

<u>Celia</u> Had no difficulty with #4; she placed the 7-weight at 13 units and the 3-weight at 3 units.

<u>Rosa</u> The fourth student on the tape, she succeeded on #1 (not shown) but did not handle any of the other problems successfully.

<u>Jeff</u> The last student performing the balance beam task approached Problem 2 in a concrete way (direct correspondence of distance and weights), but quickly changed his mind when he observed the tipping of the beam. This is an example of self-regulation, where Jeff re-examined his strategy in the light of new data. Still, Jeff was not able to solve the more difficult Problem 3. Like Dave, he has begun the transition from concrete to formal thought.

Ratio Puzzle

The second task being used here is an extension of the Ratio Puzzle introduced in Module 1. There the student was told that two figures, Mr. Short and Mr. Tall, had heights of four and six buttons, respectively. After measuring Mr. Short with paper clips, the student had to predict the height of Mr. Tall in paper clips. We now include a second question dealing with Mr. Tall's fourteen-paper-clip-wide car; how wide is it in buttons?

Harley — The first student displayed his command of the proportional reasoning operation by determining a ratio of two measurements and then using it to calculate the dimension of Mr. Tall, an object he cannot observe.

Jackie — The next student on the tape quickly set up similar ratios and solved the problem quite easily.

Tracy — The third student working on the ratio puzzle set up the same ratios as Jackie and easily solved the problem.

Eddie — The next student on the videotape did not solve the problem; he thought he could not proceed unless he knew the size of the buttons. His reasoning pattern is concrete or pre-concrete.

Martha — Our last student tried in a very complicated way to establish a ratio between buttons and paper clips. Eventually she arrived at a solution that could be classified as transitional because she did exhibit proportional reasoning in her thinking, but did not apply it simply and consistently. She appeared to have an intuitive notion about establishing ratios.

Transcript of Videotapes

Equal Arm Balance Task

Frank: Now here is what I would like you to do. Here is a 7 weight and a 3 weight, put them on the other side so the beam will balance.

Dave: Okay. Put them together?

Frank: Sure you can. Okay, Dave, why do you think they will balance?

Dave: Because they are both equal distances.

Frank: Because they are both equal distances. Okay, shall we try it? (Tests it.) Very good. Let's try another one, Dave. This time, Dave, I'm going to give you a 5 weight. Could you place the 5 weight on the other side so the beam will balance again?

Dave: Okay. It will work.

Frank: Okay, why do you think it will work out there?

Dave: Because it's twice the distance and only half the weight.

Frank: Okay. Shall we try it?

Dave: Yes.

Frank: Very good. Very good. Let's try one more, Dave. This time, Dave, I'm going to put the weight, I'm going to move the ten weight to the nine slot. Okay. Now what I would like you to do is to take the 7 weight and balance the beam. (PAUSE) Why did you put it there, Dave?

Dave: Because there is three less than that, and you moved that in one and I moved this one out two.

Frank: Okay, you think it will balance?

Dave: I hope so.

Frank: Want to try it again?

Dave: Okay. I'll use one more.

Frank: Okay, now explain why you did it - how you did it.

Dave: Well, this one just went down and there's just the heavier weight moved that over one.

Frank: So you couldn't figure it out. You just did it by observation.

Dave: Yes.

Frank: Okay, shall we try another one. Shall we try it. Good observation. That's good, thanks a lot Dave.

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Gary: Seven

Frank: A 7 and a 3 weight. You think it will work now?

Gary: I think so.

Frank: Why?

Gary: You put 10 and 10 an equal distance apart.

Frank: (Tests balance arm) Okay, it works. Let me change it just a little bit. Let me ask you to put a 5 weight, where do you think you will put the 5 weight on the beam balance?

Gary: Twenty notches away.

Frank: Why do you think that it will work?

Gary: Because it is twice as far away and half the weight.

Frank: One more, okay? Let me move this to -- let's move it to 9, okay?

Gary: Okay.

Frank: Now I'll give you a 7 weight. Where would you put the 7 weight so that the beam will balance?

Gary: Oh, you couldn't put it on any notch.

Frank: Put it on the closest one.

Gary: Oh, okay. Could I lay it down?

Frank: Sure, if you want to. But leave it on the closest notch.

Gary: Okay.

Frank: Put it on the nearest one. Let's see, did that work? Okay, how did you figure that out?

Gary: Ninety pulling down against 90 on the other side (gestures), and 13 times 7 is about ninety.

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Frank: Want to try one more?

Celia: Okay.

Frank: Let me take this one off. Let me take this now and let me put it back at 10. Let me give you the 7 and 3. Okay. Now can you put the 7 and 3 on the other side so that the beam will balance? But you can't put them both at the same place.

Celia: Can't put them at the same place. (LONG PAUSE)

Frank: What did you do now?

Celia: Put 7 times 13 and 3 times 3, 9.

Frank: Think it will work now?

Celia: I hope.

Frank: Do you think it's a ratio of some kind?

Celia: Nope.

Frank: How did you figure it out?

Celia: Well, there's going to be a 100, so I had to match it over here and a combination here.

Frank: Okay, want to try it?

Celia: Okay, it works.

Frank: Very good. Thank you very much.

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Frank: Want to try another one now.

Rosa: Okay.

Frank: Let me take these off and let me give you a 5 weight. Now where would you put the 5 weight so that the beam will balance. (PAUSE) Okay, can you explain why you put it there?

Celia: Um, I'm not sure if I'm supposed to put it there or over here. It's a lighter weight, so I'm going to put it here it will go up, but over here close to center it will balance.

Frank: So you think it will balance then. It didn't work. Want to try it again? Want to put it some other place? Why did you put it there, Rosa? What number is it?

Rosa: Fifteen.

Frank: Why did you put it there?

Rosa: Because it tops the weight off, the 10 weight. It has to be half the distance over here more.

Frank: Shall we try it?

Rosa: Yes.

Frank: Didn't work again. Let's try another one. Suppose I put this at 9, okay and I gave you a 7 weight, where would you put the 7 weight so that the beam would balance?

Rosa: At 11.

Frank: At 11, want to put it at 11, please. How did you figure 11, Rosa?

Rosa: (indistinct response)

Frank: Okay, shall we try it? Didn't work. Okay, thank you very much.

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Frank: Let's try another one, Jeff. This time I'm going to give you a 5 weight. Where would you put the 5 weight so that the beam will balance? Alright, can you explain why you put it there?

Jeff: It's half as heavy, so put it out half as much.

Frank: Okay, shall we try it? Didn't work. Want to try it again?

Jeff: Yes.

Frank: Where did you put it now?

Jeff: Twice as far.

Frank: Why did you do that?

Jeff: It's half as heavy.

Frank: Half as heavy, so what does that mean? You have to put it farther away. Do you think it will work now?

Jeff: It works.

Frank: Okay, one more. Jeff, I'm going to take the 10 weight and I'm going to move it at the 9 spot. Okay, now I'm going to give you a 7 weight and I would like you to put it on the side so that the beam will balance. (LONG PAUSE) Okay, can you figure it out?

Jeff: No.

Frank: Do you have any idea where it could go?

Jeff: It should be down towards the end.

Frank: Towards the end. Want to put it someplace. Can you figure why you put it there?

Jeff: The weight's a little bit heavier.

Frank: Okay. Shall we try it? Okay, you can't figure out where it should be.

Jeff: No.

Frank: Okay, Jeff, thanks a lot.

RATIO PUZZLE

Frank: This morning I measured him in my office with some buttons and I found him to be 4 buttons tall. Okay.

Harley: Yes.

Frank: Now, I've got another friend, Mr. Tall, who I didn't bring here this morning but I measured him this morning in my office with the same buttons and he was 6 buttons tall. Okay?

Harley: Yes.

Frank: Now, Harley, what I would like you to do is I would like you to measure the height of Mr. Short with these paper clips.

Harley: Yes.

Frank: Kinda keep this close to you. Okay?

Harley: Okay. What height? From his feet to his head?

Frank: Yes.

Harley: He is 5-1/2.

Frank: He is 5-1/2 paper clips tall. Okay, Harley, could you figure out how tall Mr. Tall is in paper clips?

Harley: It's 5, it's proportional. 4, 5-1/2 is 6 to cross multiplying at 33 divided by 4. Which is 8 and 1/4.

Frank: So Mr. Tall is 8-1/4 paper clips tall. And how did you figure that out now.

Harley: By proportion. Four buttons to 5-1/2 should be 6 to 8-1/4.

Frank: Okay. Let me ask you another question, Harley. Mr. Tall has a car, and the car is 14 buttons wide, 14 paper clips wide. Could you tell me how wide that car is in buttons?

Harley: Okay, it's the same proportion. It's 14 buttons no it's 14 paper clips wide. Okay, it's 14 paper clips, x is to 14 paper clips as 6 is to 8-1/4, 6 buttons to 8-1/4 paper clips which is a proportion as 14 x 6 which is 80-1/4 divided by 8.25. Which is approximately 10. Little bit over 10.

Frank: So you are saying that Mr. Tall's car is 10 buttons wide.

Harley: Yes.

Frank: And you figured it out by what?

Harley: A proportion.

Frank: Thanks, Harley.

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Frank: How did you get that answer, Jackie? 8.25 paper clips.

Jackie: Well, because Mr. Short was about 5'5", I just set up a ratio.

Frank: You set up a ratio. Can you explain how you set up that ratio?

Jackie: Well, Mr. Short is 4 buttons and Mr. Tall is 6 buttons and Mr. Short is 5'5" in paper clips so I put that over x.

Frank: Okay. That's how you got the answer.

Jackie: Yes.

Frank: Okay. Let me ask you another question, Jackie. Mr. Tall has a car and the car is 14 paper clips wide. Can you tell me how wide that car is in buttons? (PAUSE)

Jackie: Is this okay to do? Well, I guess it would be.

Frank: What's that Jackie?

Jackie: Well, if one's tall and the other is width, can you do that?

Frank: Can you do what? Make a proportion?

Jackie: Yes.

Frank: Try it. How did you figure that out, Jackie?

Jackie: Setting up a proportion.

Frank: Could you explain that, how you set up that proportion?

Jackie: Okay. The car is 14 paper clips so you put that over x buttons and Mr. Tall was 8.25 paper clips and 6 buttons so I just figured it.

Frank: And you solved for the answer.

Jackie: Yes.

Frank: Okay, thanks Jackie. That was very good.

****

Tracey: Nine paper clips tall.

Frank: How did you figure that out Tracey? Can you tell me?

Tracey: Proportion.

Frank: Could you explain it please.

Tracey: Alright, it is 4 buttons x 6 paper clips reduces 2 to 3 so you want to set up a proportion between 6 buttons and x paper clips that equals 2 to 3, 18 equals 2 x. x equals 18 over 2 equals 9.

Frank: Okay, Tracey. That's pretty good. Let me ask you another question, Tracey. Mr. Tall has a car and the car is 14 paper clips wide. Could you tell me how wide that car is in buttons.

Tracey: 14 paper clips.

Frank: Wide.

Tracey: Okay, that is 2 over 3 equals 14 over x equals 2, 21.

Frank: Okay, can you explain how you got that answer, Tracey?

Tracey: I did the same thing. 2 buttons for 3 paper clips is 14 buttons for x paper clips cross multiply and solve for x.

Frank: Okay, thank you very much.

****

Eddie: 5-1/2 paper clips tall.

Frank: Okay, good, Eddie. Could you tell me how tall Mr. Tall is in paper clips.

Eddie: I don't think I will be able to figure it. Mr. Short is 4 buttons tall that would be about 2 more buttons taller ---------which comes out inaccurately as 5- 1/2. What did I say? 5-1/2?

Frank: 5-1/2.

Eddie: 5-1/2 paper clips tall. I don't think I will be able to.

Frank: Okay.

****

Martha: 5 paper clips and 1/2.

Frank: 5-1/2 paper clips. Now could you figure out for me how tall Mr. Tall is in paper clips?

Martha: Okay. 2, 4. Mr. Short is 4, right?

Frank: 4 buttons.

Martha: And Mr. Tall is 6 buttons.

Frank: You want some paper or something, Martha? Here's some paper and a pencil if you want.

Martha: 1, 2, 3, 4, 5, that's 5-1/2.

Frank: Paper clips.

Martha: Paper clips equals 4 buttons and 6 buttons so half of 5-1/2 is 2-1/2 also half of half is a fourth which would be 2 and 3/4 so it would be 6-3/4 equals.

Frank: So you're saying Mr. Tall is 6 and.

Martha: No, that's Mr. Short. No, Mr. plus two, -----------

Frank: Explain to me how you got that?

Martha: Okay, let's see. Mr. Short is 5-1/2 paper clips.

Frank: Yes.

Martha: Which is 4 buttons. Okay? And a half and Mr. Tall is 6 buttons.

Frank: Right.

Martha: Which would be 1/3 of 4 buttons. And 1/3 of 6 buttons is 2. Okay, so I added 2 buttons to 4 which would make Mr. Tall and I figured half of 5-1/2 is 2-1/2 plus the 1/4 which is 2 and 3/4.

Frank: Okay. You say he is 8-3/4 buttons tall.

Martha: I think so.

Frank: Okay, 8-3/4 paper clips.

Martha: Paper clips, right.

Frank: Okay, good.

Module 4 Formal Thought [Film]

Introduction

In Piaget's theory, concrete thought is characterized by serial ordering, simple classification, conservation logic and other operations applied to objects that a person is able to observe or manipulate directly. Formal thought includes these operations but goes beyond them to utilize other processes in situations where one does not deal with tangible objects. Formal processes often involve proportional reasoning, separation of variables, elimination of contradictions and class inclusion or exclusion operations.

In the film "Formal Thought" that you are about to see (1), you will observe Rita Peterson and Robert Karplus working with students between the ages of twelve and seventeen as they face four tasks. The young people's approaches are intended to illustrate the characteristics of concrete and formal thought described in Module 2 and to demonstrate that a group of high school students is likely to make use of both types of reasoning patterns. Though self-regulation is not emphasized in the film, you might look for situations in which a student finds that his procedure was not adequate and has the opportunity to use the new data for another attempt. Since this film was made in 1971, research with college students has produced similar results.

Objectives

To assist you in describing and/or identifying responses that indicate concrete thought and formal thought applied to simple Piagetian tasks.

Procedure

The staff will make available a film-showing schedule at the beginning of the workshop. "Formal Thought" lasts about thirty-five minutes but need not be seen in its entirety. Reading the film notes first will acquaint you with the four tasks that are used in the film.

After viewing the film, please go to the discussion center, where the workshop staff will meet with small groups of participants to help you exchange ideas, raise questions, and clarify your thoughts regarding Modules 1-4.

Film Notes

Proportional Reasoning (Ratio Puzzle)

In the first scene, JANET displays her command of proportional reasoning operations by determining a ratio of two measurements and then using this ratio to calculate the dimensions of an object that she cannot observe (2). In the second scene, PETER appears to approach this task in the concrete operational manner, but changes his procedure when asked to explain.

Separation of Variables (Flexible Rods)

In order to study the reasoning processes that one goes through in separating variables, Inhelder and Piaget performed a series of experiments with a simple device similar to that shown in the film (3). From front to back the rods are, in order, thin round steel, thin round brass, medium round brass, medium flattened aluminum, thick round brass, thick round wood. This "flexibility" apparatus permitted the adjustment of five variables (length, thickness, material, cross-section, weight) and required the experimental subjects to vary each factor independently if a complete solution was to be obtained. A complete description of the experiment appears in Chapter 3 of reference 3.

Combinatorial Logic (Chemical Mixtures)

Formal thought is characterized by the development of propositional logic. In turn, this logic depends upon the establishment of a combinatorial system that is manifested in a person's ability to link a set of associations or correspondences with each other in many possible ways. The "coloring liquids" problem shown in the film requires the student to combine a number of solutions (1=dilute sulfuric acid, 2=water, 3=hydrogen peroxide solution, 4=sodium thiosulfate solution, g=sodium iodide solution) to obtain the brown color of free iodine; the differences between the formal method which goes through the full range of possibilities each time and the concrete operational thinker's method of one-by-one combination which leaves many steps untouched are sharply illustrated here. This experiment is described in Chapter 7 of reference 3.

Application of Proportional Reasoning (Equal Arm Balance)

The last task shown in the film, balancing the beam, requires students to apply proportional reasoning and other elements of formal thought to a more difficult problem. The students we watch exhibit a variety of approaches. JOCELYN has an intuitive idea that increased distance compensates for increased weight, but she used the weight difference rather than the ratio to make a prediction. (Using differences rather than ratios in a situation like this is a characteristic of concrete operational thinkers.) ROBERT is able to handle a simple situation requiring the utilization of a 2:1 ratio, but he is unable to generalize the procedure to treat a more complicated application of proportional reasoning. Students who perform in this manner are often considered to be in transition from the concrete operational mode to the formal one. VLADIMIR clearly displays that he uses a formal thought process to arrive at the solution to the problem.

References:

(1) "Formal Thought" (16 mm film) available from Davidson Films, Inc., 3701 Buchanan Street, San Francisco, CA 94123 (Rental fee, $30.00).

(2) Elizabeth F. Karplus, Robert Karplus, and Warren Wollman, "Intellectual Development Beyond Elementary School IV: Ratio, the Influence of Cognitive Style," School Science and Mathematics, October 1974.

(3) Bärbel Inhelder and Jean Piaget, The Growth of Logical Thinking from Childhood to Adolescence, Basic Books, New York, 1958.

Module 5 Analysis of Physics Problems and Test Questions

Introduction

You may be wondering how to apply the concept of developmental stages in your physics teaching. To help you with this, we have prepared modules 5-11 dealing with differing aspects of instruction. Module 5 concentrates on the analysis and writing of physics problems and test questions. As you read the examples we have selected, keep in mind the characteristics of concrete and formal thought described in Module 2. A matter that we find difficult to resolve concerns how to give all students, regardless of the reasoning patterns they use initially, practice in problem solving. Furthermore, evaluation through tests should give all students an opportunity to show what they have learned in physics and with respect to formal reasoning patterns. The article "Physics Problems and the Process of Self-Regulation" by Anton E. Lawson and Warren T. Woolamn in Module 11 describes how problems could be used to advance students' reasoning.

Objectives

To assist you in classifying physics problems (homework or test) as "concrete" or "formal."

To assist you in writing physics problems that are "concrete" or "formal," as needed.

Procedure

This module provides for a sequence of activities in which you will study twelve physics problems. The first four serve as background for our explanation of how a problem's demand for concrete or formal reasoning may be identified. The last eight are examples to which we and you can apply the criteria. Please find a partner with whom you can work and exchange ideas during this module. Then use the activities in the attached instructional materials in the order given.

Module 5 Instructional Materials

1. Exploration

The first problem we are presenting here has been given to students in an engineering physics course, and some of their solutions are reproduced on the next page. The students had not covered lens optics in their course, so that they had to rely on general problem-solving strategies rather than on a memorized equation. Note the extent to which students A and D engaged in self-regulation, beginning to advance to a higher level of reasoning because of discrepancies they perceived in their first results. Students B and C did nothing further, apparently satisfied with their accomplishment and oblivious to the inconsistencies and dimensional errors.

<u>Problem 1</u> (a) The focal length of a convex lens is the distance from the lens where light from a distant source comes to a focus after it passes through the lens. (See Figure 1.) The focal lengths of two identical, thin, convex lenses are the same and measured to be 20 cm each (F_1=20 cm, F_2=20 cm). The two lenses are placed one over the other as shown in Figure 2 and taped together at their edges only. The focal length of this combination, F_c, is 10 cm. Write an equation that gives the focal length of a lens combination that consists of two lenses having identical focal lengths.

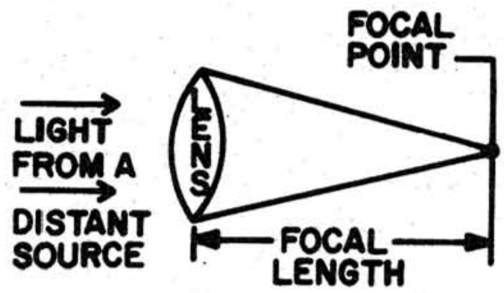


Figure I

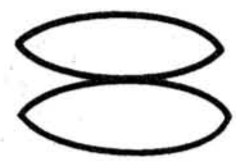


Figure 2

(b) One of the 20 cm focal length lenses is replaced by a lens having a focal length (F_3) of 5 cm. The focal length of the resulting combination is measured to be 4 cm. Write an equation that can be used to calculate the focal length of a lens combination that consists of two lenses of unequal focal lengths.

	Answer (a)	Answer (b)
Student A:	If $F_1 = F_2$, then $F_c = \frac{1}{2} F$	$F_c = \frac{F_1}{F_3} = 4$ cm However, since the same principles are acting when the lenses are identical and when they are different, the equations for the two systems should be the same. So I really don't know how to derive the equation.
Student B:	$F_c = \frac{F_1}{4} + \frac{F_2}{4}$ $F_c = \frac{1}{4}(F_1 + F_2)$	$Fd = F_1 + F_3$ $Fd = 20 + 5$ $4 = 25$ $6.25\, Fd = F_1 + F_3$ $Fd = \frac{F_1 + F_3}{6.25}$
Student C:	$F_c = \frac{F_1}{2} = \frac{F_2}{2}$ iff $F_1 = F_2$	$F_c = \frac{F_2}{f_3}$ iff $F_3 < F_2$

Student D:

Several possibilities

$F_t = \frac{F_1 + F_2}{4}$ or $\frac{\text{average of lens focal length}}{\text{number of lens}}$

$(f_1)(\frac{1}{f_2}) = f_t$ This is too obvious and is not consistent with the other problem

$F_1 = 20 \quad F_2 = 20 \quad F_t = 10$

$F_1 = 20 \quad F_2 = 5 \quad F_t = 4$

Ah ha! $\frac{1}{20} + \frac{1}{20} = .1 \qquad \frac{1}{.1} = 10$

$\frac{1}{5} + \frac{1}{20} = .25 \qquad \frac{1}{.25} = 4$

So $\rightarrow$ $\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f_t}$

Now please write out solutions to Problems 2, 3, and 4, making a sketch and carefully stating the equations (definitions, laws, principles) that are the starting point of your procedure.

Problem 2 At what distance from the earth's center would a standard kilogram weigh 1 newton? At what distance would a body with a mass of 1 gram weigh 1 dyne?

Problem 3 An unbalanced force of 5.0 newtons on an object produces an acceleration of 20 meters/sec2. What is the mass of the object?

Problem 4 An inductor made of copper wire has been wound on a long cylindrical form of cross-sectional area $10^{-3}m^2$. The field at the center of the inductor is 0.1 webers/m^2 when the current is 4.0 amp. The resistance of the winding is 25 ohms and its inductance is 0.2 henry. How long is the winding on the form?

2. Criteria for Classifying Problems as "Concrete" or "Formal"

You have probably assessed the three problems to be of quite differing difficulty, with #3 the easiest and #4 the hardest. How would students react to them? Problem 3 can be solved by direct substitution into Newton's second law which relates force, mass, and acceleration. A student who uses concrete reasoning patterns and has memorized the terminology and the law should be able to do that, even though he does not understand all the ramifications of Newton's law when applied with various kinds of boundary conditions. We, therefore, call Problem 3 a "concrete" problem.

Problem 2 is more difficult even if the student has memorized the form of the law of gravitation in terms of the acceleration of gravity at the earth's surface, which is not mentioned in the problem statement. Coordinating the law of gravitation with the definition of weight and the conditions at the earth's surface requires formal thought; hence we consider Problem 2 to be a "formal" problem.

Problem 4 would strike the concrete thinker as completely impossible, since the length of the winding is usually not stated explicitly in formulas for inductance and magnetic field that he can memorize. To solve the problem, the student has to realize that the magnetic field depends on the density of windings while the inductance depends on the total number of windings. Since both the field and the inductance are given, the length can be found. The resistance, which depends directly on the length of wire, according to a very popular formula, is useless here, because resistivity, wire diameter and form shape are not given.

From these three problems certain patterns are visible, and we shall now expand them to formulate a classification into "concrete" problems (solvable by concrete reasoning patterns through straightforward use of a learned definition or equation) and "formal" problems (solvable only after an overall analysis and some improvisation). Here are some clues for distinguishing between the two types:

<u>Clues for "concrete" problems</u> --affirmative answers to:

C1. Can I use a formula to solve the problem?

C2. Could I observe the variables in the problem directly?

C3. Are the calculations simple, not requiring proportions, graphical interpretations of abstract variables, or choosing among models or theories?

C4. Are the given data necessary and sufficient?

<u>Clues for "formal" problems</u>--affirmative answers to:

F1. Do I need to combine formulas or derive a new one?

F2. Do I need to introduce variables in addition to the ones given or asked about?

F3. Do I need to decide which approximation or theory is appropriate to the conditions of the problem?

F4. Do I need to select relevant data from the extraneous or be concerned that the problem might admit no solution or more than one solution?

F5. Do I need an overall plan before I can start with an equation?

3. Applications of the Criteria

Keep these clues in mind as you examine Problems 5 to 7, which will serve to illustrate the classification scheme further. We suggest that you not spend time now actually working out solutions.

Problem 5 What is the displacement of a car that travels at a steady speed of 40 miles/hour for three hours on a straight road?

Concrete -- all the items above, especially C1.

Problem 6 A space capsule travels along a straight line from the earth to the moon. Considering only the earth-moon system, at what distance from the earth is the gravitational force on the capsule equal to zero? Introduce symbols for astronomical data such as distances.

Formal -- especially F1, F2, and F5.

Problem 7 Find the momentum and energy of a 150-grain 30-06 bullet with a speed of 2500 ft/sec. How fast must a 200 lb. deer move to have the same momentum? (7000 grains = 1 lb.)

Formal -- "yes" on C2 and C4, but also "no" on C3. The grains-pounds conversion leads us to this classification, but we admit that the problem may fall between the two types.

Now classify the following problems using the clues described earlier together with any criteria that you have developed. Please write your answers and reasons and compare them with your partner's.

Problem 8 A man in a sailboat is stranded in a dead calm. He wishes to reach an island whose shore is at a distance D from his location.

(a) Suppose he tries to propel himself by throwing an object of mass m off the boat. In what direction should he throw it? Make a diagram.

(b) Suppose the boat and its content have mass m and the man throws the object at speed v; how long would it take him to reach the island? (Neglect friction.)

(Circle one) Concrete Formal

Reasons (you may refer to the items by number):

Problem 9 A cue strikes a billiard ball, exerting an average force of 50 newtons over a time of 10^{-2} seconds. If the ball has a mass of 0.20 kg, what speed does it have after impact?

(Circle one) Concrete Formal

Reasons (You may refer to the items by number):

Problem 10 At the instant a traffic light turns green, an automobile starts with a constant acceleration a_x of 5 ft/sec.^{2}. At the same instant a truck travelling with a constant speed of 30 ft./sec. overtakes and passes the auto. How far beyond the starting point will the auto overtake the truck?

(Circle one) Concrete Formal

Reasons (you may refer to items by number):

Problem 11 Six joules of work is done when a charge is moved through a potential difference of 3.0 volts. How large is the charge?

(Circle one) Concrete Formal

Reasons (you may refer to items by number):

Module 5 Review Questions

1. The problem that follows has several parts. With your partner, classify each part as "concrete" and/or "formal"; explain how a part might have either classification, depending on which formulas the student has memorized.

Problem 12 A car moving with constant acceleration covers the distance between two points 180 feet apart in 6.0 seconds. Its speed as it passes the second point is 45 feet/second.

a. What was the car's average speed between the two points?

b. What was the car's speed at the first point?

c. What was the car's acceleration?

d. At what distance before the first point was the car at rest?

"Concrete" parts and procedures:

"Formal" parts and procedures:

2. Look back at Problem 4 or 6, which were considered to be "formal," and rewrite one of them jointly with your partner so it is accessible to a concrete thinker. If you determine that this task is impossible, please state your reasons.

"Concrete" rewrite:

Answers: 1 a, b, c: "concrete;" 1 d: "formal"

Module 6 Analysis of Learning Materials

Introduction

Module 6 continues with the application of the concept of developmental stages in your physics teaching. The module concentrates on the analysis of physics texts and film loops, which provide important instructional inputs for students. As you read the excerpts we have selected for your review, keep in mind the characteristics of concrete and formal thought explained in Module 2. Also, remember that all students, regardless of their developmental stage, will find the text easier and will understand a new topic in a more broadly-based way if they can progress gradually from a concrete view of the subject. Of course, some students will progress further than others in grasping all the implications and subtleties contained in their reading.

Objectives

To assist you in classifying text passages and film loops as to their requirements for concrete and formal reasoning patterns.

Procedure

This module includes four text passages, two film loops and a review item to be analyzed for their demand on a student's reasoning patterns. We have highlighted certain features of these excerpts to indicate what makes a passage more or less accessible to the use of concrete reasoning only. In conclusion, we have listed criteria that you may use to evaluate physics texts or to help you prepare instructional materials of your own. Please choose a partner with whom you can work and exchange ideas during the module. Then use the activities in the attached instructional materials in the order given.

Module 6 Instructional Materials

1. Excerpts A and B: Coulomb's Law

The first two excerpts we have chosen deal with Coulomb's law. Since the mathematical formulation of Coulomb's law makes use of direct and inverse proportions, formal reasoning is undoubtedly required for full comprehension. Nevertheless, a careful explanation that takes into account concrete thinking patterns can help the concrete or transitional students, present in substantial numbers in high school and college classes, grasp some of the underlying relationships among force, distance, and magnitude of charge, at least qualitatively. The formal thinker is also going to be helped to a richer understanding, achieved more easily, by such an explanation.

An important matter not identifiable from the excerpts is the student's concept of force. If force was defined in terms of actions and examples (deformation of a spring or rubber band, bending of a beam, weight), the student at the stage of concrete thought will have a chance to enlarge his understanding through the electrostatic application. If force was defined in terms of other concepts (mass and acceleration), no presentation of Coulomb's law will be understandable in terms of concrete reasoning patterns.

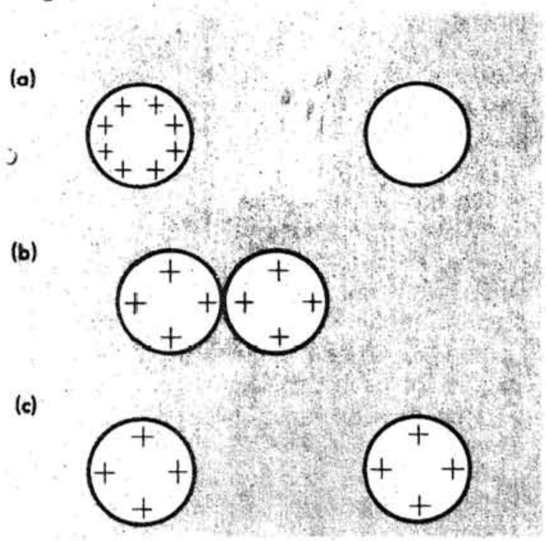
In the margins next to the text passages we have identified items that require identifiable patterns of reasoning on the part of the reader. In our opinion, Excerpt A makes an effort to communicate by means of concrete patterns of reasoning, but Excerpt B does not. Please read the two excerpts now, discuss their content and the marginal notes with your partner, and then continue on to the next excerpts.

Excerpt A

Electric Charge and Electric Force

In an electrically neutral body the effects of positive and negative electric particles cancel. A positively charged body contains uncanceled positive particles, and a negatively charged body contains uncanceled negative particles. Thus the charge of a body depends on the uncanceled excess of positive or of negative particles, measured from neutral.

The force between two charged bodies depends on their separation and increases with the excess of positive or of negative electric particles on each body. Just how does the force depend upon the excess of electric particles? To answer this question we need a scheme to divide the excess of particles in a known way—in half, in thirds, etc. Suppose we touch a charged metal sphere with an identical uncharged sphere (Fig. 27–2). Then the electric particles will move around until they are shared equally by both spheres. Each sphere will have half the original charge.



The sharing of electric charge. When a charged sphere is touched to an identical uncharged one, the excess of electric particles divides equally. The final distribution of charge must be symmetrical, as shown in (c).

What happens to the electric forces when charges are shared? We measure the force of repulsion between two charged spheres A and C at a certain separation. Then we halve the charge on A by sharing it with an identical sphere B. The force of repulsion between A and C (still at the same separation) is *also* cut in half. Furthermore, we get the same force when A is replaced by B, the identical sphere with which it shared its charge. Apparently, charge and force are proportional, as we might have guessed.

Dependence on charge separation:

Explained in detail in the preceding section, which also illustrates the design and action of the torsion balance.

Charge sharing:

Reference to a sphere with eight positive charges by means of a diagram. Note, however, the unphysical conception suggested by the arrangement of charges in fig. 27-2b.

Comparison of Forces:

Identifies importance of keeping the same separation.

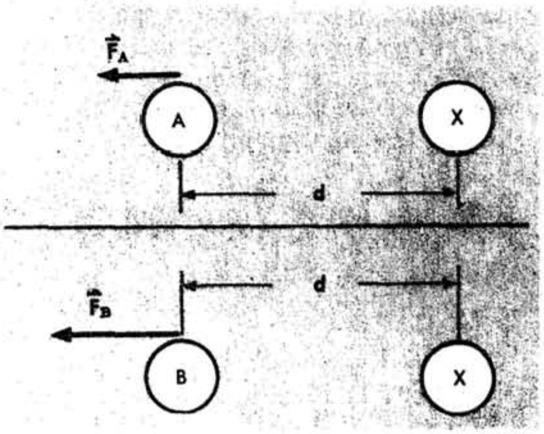
Such experiments give us a way of comparing charges quantitatively. Two charges are equal if they experience equal forces at a given distance from any third charge. One charge is twice another when it experiences twice the force. When a charge is halved by charge sharing, the force exerted on it by a third charge is also halved. In general, charges are compared by the ratio of the forces exerted on them by any other charge at a given distance. This *ratio* does not depend on the magnitude of the "other" charge nor on the distance apart (Fig. 27-3). Equivalently, we can compare the ratio of the forces exerted *on* the "other" charge by each of the two charges being compared.

Now let us summarize our knowledge in algebraic language. The electric force on a charge q is proportional to the charge: $F \propto q$. When this force is the force of interaction on the charge q by another small body of charge Q, the force is also proportional to the other charge. We can write this proportionality to both the charges as $F \propto qQ$.

We now have a definite meaning for charge, and we know how the electric force depends on the charges. We can combine this knowledge with Coulomb's experiments. They tell us that the force is inversely proportional to the square of the separation r between the charges. So we arrive at the complete expression for the force of interaction between two charges. The magnitude of the force on either charged body is

$$F = \frac{kQq}{r^2},$$

where the proportionality factor k depends only on the units in which we measure forces, separations, and charges.



To compare two charges, *A* and *B*, we place them in turn at the same distance from any other charge *X*, and measure the forces. The ratio of the charges equals the ratio of the forces: $q_A/q_B = F_A/F_B$. What do you think is the ratio of the forces exerted on *X*?

Comparison of charges:

Interrupts the explanation and is therefore not correctly placed for a concrete-thinking reader who is concentrating on how the electric force depends on the magnitude of the charge.

Applicability to point charges:

Bodies described as "small."

Excerpt B

COULOMB'S LAW

The first quantitative investigation of the law of force between charged bodies was carried out by Charles Augustin de Coulomb (1736–1806) in 1784, utilizing for the measurement of forces a torsion balance of the type employed 13 years later by Cavendish in measuring gravitational forces. Coulomb found that the force of attraction or repulsion between two "point charges," that is, charged bodies whose dimensions are small compared with the distance r between them, is inversely proportional to the square of this distance.

The force also depends on the quantity of charge on each body. The net charge of a body might be described by a statement of the excess number of electrons or protons in the body. In practice, however, the charge of a body is expressed in terms of a unit much larger than the charge of an individual electron or proton. We shall use the letter q or Q to represent the charge of a body, postponing for the present the definition of the unit of charge.

In Coulomb's time, no unit of charge had been defined, nor had any method been developed for comparing a given charge with a unit. Despite this, Coulomb devised an ingenious method of showing how the force exerted on or by a charged body depended on its charge. He reasoned that if a charged spherical conductor were brought in contact with a second identical conductor, originally *uncharged*, the charge on the first would, by symmetry, be shared equally between the conductors. He thus had a method for obtaining one-half, one-quarter, and so on, of any given charge. The results of his experiments were consistent with the conclusion that the force between two point charges q and q' is proportional to the product of these charges. The complete expression for the force between two point charges is therefore

$$\boxed{F = k\frac{qq'}{r^2},} \qquad (24\text{–}1)$$

where k is a proportionality constant whose magnitude depends on the units in which F, q, q', and r are expressed. Equation (24–1) is the mathematical statement of what is known today as *Coulomb's law:*

The force of attraction or repulsion between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

Applicability to point charges:

"Point charges"identified as having small dimensions compared to the distance of separation, another ratio to be taken into account.

Dependence on charge separation:

Summarizes the result in one sentence, without reference to specific examples at this point.

Charge sharing:

Abstract reasoning and general conclusion only.

Comparison of forces:

Summarizes proportionality without reference to separation.

2. Excerpts C and D: Kinetic Energy

Study Excerpts C and D with your partner, taking note of the marginal comments as you did before. Evaluate their demand for concrete or formal patterns of reasoning, then compare with our evaluation on the next page.

Excerpt C

14·6 Kinetic energy

You probably have learned that the distance required to stop a car increases fourfold when its speed doubles. Have you ever wondered why? When a bicycle rider approaches a hill, he usually pedals as fast as he can so that he will get to the top of the hill more easily. Just how far up will his speed carry him? In both these examples, there is a transfer of energy from kinetic energy to another type: thermal energy of the brakes, or gravitational field energy of the bicycle, rider, and earth system.

As we have said in Chapter 4, kinetic energy is the energy stored in moving objects. Thus, the kinetic energy of the car determines how far it will advance as the brakes bring it to a stop. The bicyclist maximizes his kinetic energy as he approaches the hill.

When a force acts on a particle, its velocity or momentum changes, and usually its energy changes also. In this section we will derive a mathematical model for the relation of kinetic energy to speed. We will show how this relation can be used in conjunction with the law of conservation of energy to predict the motion of objects under many circumstances, such as the car coming to a stop and the bicycle moving uphill.

Derivation. Instead of constructing the model in the light of experimental results, we will derive it from Newton's theory. Imagine a particle at rest (zero speed, zero kinetic energy) that is acted upon by a constant net force until it is moving with the velocity **v**. The kinetic energy of the particle is, according to the law of conservation of energy, equal to the work done by the net force (Eq. 14·16). To find the work, we have to calculate the distance through which the particle moved while it was being accelerated by the action of the force. This problem is very similar to the problem of free fall solved in Section 14·4. There, too, a constant force speeded up a particle that was initially at rest. The principal differences between that and the present tasks are that now the force can be any force (not only the force of gravity), and the motion can occur in any direction (not only vertically). Still, the motion and the force are in the same direction, because the particle starts from rest (Fig. 14·20).

The relative position of the particle is equal to one half of the velocity times the time (Eq. 14·17 from Eq. 14·10). The net force also can be related to the actual velocity (equal to the change of velocity) and to the elapsed time (Eq. 14·18 from Eq. 14·5). Since the force, the velocity, and the relative position are all in the same direction, the component of the displacement along the force direction is equal to the magnitude of the relative position (Eq. 14·19). When the formulas are combined to calculate the work and therefore the kinetic energy, we obtain a mathematical model (Eq. 14·20).

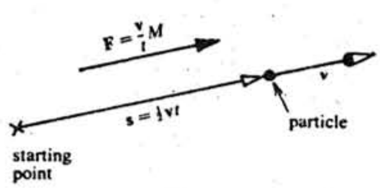


Figure 14·20 The kinetic energy of a particle is equal to the work done by a constant force that accelerates the particle from zero velocity to its actual velocity. The force required and the position relative to the starting point reached by the particle are related to the velocity by Eqs. 14·17 and 14·18.

Equation 14·16

kinetic energy	KE
work	W
net force	**F**
displacement component along the force direction	Δs_F

$$KE = W = |\mathbf{F}|\Delta s_F$$

Equation 14·17

position relative to starting point	**s**
velocity	**v**
elapsed time	t

$$\mathbf{s} = \tfrac{1}{2}\mathbf{v}t$$

Equation 14·18

mass M

$$\mathbf{F} = M\frac{\mathbf{v}}{t}$$

Equation 14·19

speed v

$$\Delta s_F = |\mathbf{s}| = \tfrac{1}{2}vt$$

Equation 14·20

$$KE = |\mathbf{F}|\Delta s_F = M\frac{|\mathbf{v}|}{t} \times \frac{1}{2}vt$$

$$= \frac{1}{2}Mv^2$$

Introduction:

Two familiar examples, described with reference to kinetic energy

Feedback:

The concept of kinetic energy has evidently been introduced earlier, though not related to the mass and speed of the moving .

Introduction of a net force:

Anticipation of relation between kinetic energy and speed

Selection of a constant force:

Reference to Newton's theory, but no rationale for constant force.

Introduction of work:

Reference to energy conservation

Algebraic derivation:

Leans on earlier treatment of free fall from rest which was illustrated with stroboscopic photographs and tables of time-distance data; the discussion paraphrases the equations very concisely and without new examples; keeps direction of motion general

Final Conclusion:

The specific form of the speed-kinetic energy relation is exhibited

Excerpt D

7–5 Kinetic Energy and the Work-Energy Theorem

In our previous examples of work done by forces, we dealt with *unaccelerated* objects. In such cases the *resultant force* acting on the object is zero. Let us suppose now that the *resultant force* acting on an object is *not zero*, so that the object is *accelerated*. The conditions are the same in all respects to those that exist when a single unbalanced force acts on the object.

The simplest situation to consider is that of a *constant resultant force* **F**. Such a force, acting on a particle of mass m, will produce a constant acceleration **a**. Let us choose the x-axis to be in the common direction of **F** and **a**. What is the work done by this force on the particle in causing a displacement x? We have (for constant acceleration) the relations

$$a = \frac{v - v_0}{t}$$

and

$$x = \frac{v + v_0}{2} \cdot t,$$

which are Eqs. 3–12 and 3–14 respectively (in which we have dropped the subscript x, for convenience, and chosen $x_0 = 0$ in the last equation). Here v_0 is the particle's speed at $t = 0$ and v its speed at the time t. Then the work done is

$$\begin{aligned} W &= Fx = max \\ &= m\left(\frac{v - v_0}{t}\right)\left(\frac{v + v_0}{2}\right)t = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2. \qquad (7\text{–}11) \end{aligned}$$

We call one-half the product of the mass of a body and the square of its speed the kinetic energy of the body. If we represent kinetic energy by the symbol K, then

$$K = \tfrac{1}{2}mv^2. \qquad (7\text{–}12)$$

We may then state Eq. 7–11 in this way: *The work done by the* resultant *force acting on a particle is equal to the change in the kinetic energy of the particle.*

Introduction:

Completely abstract statement about forces, objects, and accelerations.

Feedback:

Reminder of behavior of unaccelerated objects to set the stage for now doing something else.

Introduction of a net force:

Reminder of relation between force and acceleration

Selection of a constant force:

Implied reference to Newton's law, but no rationale for constant force.

Introduction of work concept:

No rationale for suddenly asking about work

Algebraic derivation:

Quotes results from motion in one dimension with constant acceleration which was illustrated with time-distance and time-speed graphs; specializes to x-axis, but drops subscript.

Final conclusion:

The formula resulting from the algebraic operations is used to define the kinetic energy.

In our opinion, the first half of Excerpt C can be understood by the use of concrete patterns of reasoning and will therefore give all readers a better understanding (gut-feeling) of energy relationships. In spite of being intended for very different readers than Excerpt D, the remainder of Excerpt C is discouragingly similar to D. Still, the reader of C can omit the section entitled "Derivation" and come to grips with kinetic energy in a qualitative way; the reader of D gains at best a very formula-based notion of kinetic energy, with no idea how this "energy" is related to the energy he has met in his every-day life, chemistry courses, etc.

3. Communicating by means of Concrete Reasoning Patterns

By referring to the characteristics of concrete reasoning patterns described in Module 2, you can construct a list of items that will help you communicate at the concrete level. You can also review the features of formal reasoning patterns and then take special care to avoid these, or to call attention to those elements of formal thought that are used in the discussion because they appear unavoidable.

You can make a presentation more concrete by:

1. Beginning with concrete situations.

2. Illustrating the arguments with specific examples.

3. Providing "action models" or procedures that enable the student to work out an answer or verify a conclusion through concrete actions (in which he may often imagine himself) rather than through deductive or algebraic reasoning.

4. Providing a clear overview of a complicated explanation in advance, indicating the purpose and the principal steps.

5. Making clear references to formal operations when these are used:

    a. Identify variables that are held fixed while others change;

    b. State assumptions that are made;

    c. Paraphrase equations in words, and don't use equations as principal parts of a sentence, "$E = mc^2$ is a consequence of Einstein's Relativity Theory;"

    d. Use diagrams to illustrate steps of the reasoning;

    e. Enumerate some specific instances when new classes or categories are defined.

6. Proceeding directly from known or previously explained ideas to new ones; don't start with "Let us assume that . . ." or "It is convenient to . . .".

7. Providing pictures of apparatus that is referred to.

4. Film Loops

After you and your partner have completed your work on the text passages, go to one of the film loop projection stations in the module area. You will find two film loops, (1) "Superposition of Pulses on a Spring" and (2) "Conservation of Energy." Please view them in the order 1-2, and read the film notes for each one so you can evaluate a student's reaction to them. Determine the loops' suitability in terms of their demand for concrete and formal reasoning patterns on the part of the viewer; use your experience with the text passages as basis for your analysis. Then read our comments on the next page. If you wish, view the loops a second time to examine their scenes more closely.

5. "Superposition of Pulses on a Spring"

This film loop would ordinarily be used by a student who had been introduced to the superposition principle in class discussion, or who had been asked to read about it in the text. The film does not attempt to provide a discussion of the principle, but only exhibits the phenomenon of superposition.

It will be apparent to you that this example can be quickly understood by a concrete operational thinker. The phenomenon of superposition is clearly shown at normal speed and in slow motion. Various aspects of the process that might be overlooked if only a single spring were used are highlighted by the ingenious technique of sending pulses along three identical springs, supported side by side. Note the way in which the demonstration is presented, proceeding from the simple to the more complex aspects of the phenomenon being illustrated. (For example, the longitudinal case <u>followed</u> the transverse illustration.)

Observe that even if a student overlooked the film notes, he would still derive a considerable amount of information from the film because of the direct way in which the phenomenon is presented. Since this film requires no formal reasoning operations to be performed by the viewer, it may be classified as suitable for concrete thinkers.

6. "Conservation of Energy"

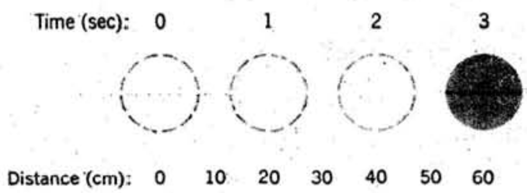
This film came to our attention when a graduate student who was using it with a group of freshmen in a physics course for non-science majors complained that the film did not contain enough information for the viewer to obtain the results quoted. The film shows a glider being accelerated along an air track. The captions assert that one can show from the data provided that the work done on the glider is equal to the change in the glider's kinetic energy. Can you identify the problem that the graduate student was having? How might you modify the film to make it at least partly understandable by the use of concrete reasoning patterns? (Hint: re-read our comments about text excerpts C and D.)

Module 6 Review Questions

Excerpt E

Excerpt E is presented for your reading and analysis. At its conclusion on the next page we have posed four questions related to the sections numbered in the margin.

Several times we have used the phrase "uniform motion." Precisely what does this mean? Consider the motion of an air puck on a horizontal surface. Figure 6–2 illustrates such 1 a puck moving to the right. The circles represent positions that the puck occupied at different times as it moved. These positions might have been determined by examining suc-



Successive position of a puck floating on a film of air moving to the right.

cessive frames of film taken by a motion-picture camera. As we see from Figure 6–2, the distance traveled by the puck in each 1.0-sec interval is the same, namely 20 cm. Assume, now, that the speed of the camera is doubled. The time between successive frames would be reduced to 0.5 sec. If the puck is engaged in uniform motion, then the 2 distance between any two successive positions of the puck would be 10 cm. If, for *any* equal time intervals we choose the distance intervals are also equal, then the motion is uniform.

The *speed* of an object in uniform motion is defined as the ratio of a distance interval to the corresponding time interval. This can be written as an equation:

$$\text{speed} = \frac{\text{distance interval}}{\text{time interval}} \qquad (6\text{–}1)$$

Usually scientists prefer to write such equations in symbols. The symbol commonly used for speed is v. The v really stands for velocity. To specify the velocity completely, you must know not only the speed but also the direction of the 3 motion. Until the direction of motion assumes more importance in our discussion, we will use the two words interchangeably. The distance interval can be thought of as the difference between two readings of position, x, read from a meter stick at rest parallel to the path of the object, and the time interval can be thought of as the difference between two readings of time, t, read from a clock. The symbol Δ before a quantity means a change in that quantity, so Equation 6–1 symbolically becomes

$$v = \frac{\Delta x}{\Delta t}$$

If the motion is not uniform, it is still possible to define an *average velocity*. Take any distance interval and divide by the corresponding time interval:

$$v_{av} = \frac{\Delta x}{\Delta t} \qquad (6\text{–}3)$$

4

Equations 6–2 and 6–3 are very similar. The difference is that for uniform motion, the speed calculated from Equation 6–2 is independent of the interval selected, whereas for nonuniform motion, the average velocity calculated from Equation 6–3 may come out to be a large number for one particular interval and a small number for a different interval.

Please discuss your answers to these questions with your partner and/or other workshop participants. You might compare with the items on page 6-8 and the reasoning patterns described in Module 2. Our ideas are briefly described at the bottom of the page.

1. What reasoning pattern is required by the opening of the excerpt, Item 1? How might the opening have been made more concrete? How might it have been made more formal?

2. What level reasoning pattern is required to follow the generalization from the original example introduced in Item 2? Is this necessary to define uniform motion? Does it go far enough to define uniform motion? Could it have been done more clearly?

3. How does Item 3 help the reader? Should the text have given more emphasis to the directional requirements on uniform motion, possible in connection with Item 2? Should this explanation of the symbol v have been omitted?

4. In Item 4, the average velocity is defined by an arithmetic procedure. What level of reasoning pattern is involved in this definition? What level of reasoning pattern is involved in the explanation that relates this definition to the case of uniform motion? How well is the reader prepared for the transition from uniform to non-uniform motion?

5. Can you spot any sections that require formal reasoning patterns outside the numbered items? Explain your reasons and suggest other ways of handling the material. Do you have any comments on the overall organization of Excerpt E?

Brief answers:

1. Concrete in view of the example, but could have been improved by recalling one of the "several times."
2. Formal, especially when going to "any" time interval.
3. Seems unnecessary and confusing; the term "speed" could serve until direction is brought in.
4. Concrete for formula, formal for explanation; another puck example with unequal intervals would help.

List of Sources

1. An Approach to Physical Science: Physical Science for Nonscience Students, (Brief Edition.) PSNS Project Staff and A. A. Strassenburg, John Wiley & Sons, Inc., N.Y. 1974 (Excerpt E)

2. Introductory Physics: A Model Approach, Robert Karplus, W. A. Benjamin, Inc., N.Y. 1969 (Excerpt C)

3. Physics, Part I, David Halliday and Robert Resnick, John Wiley & Sons, Inc., N.Y., 1966 (Excerpt D)

4. Physics, Second Edition. Physical Science Study Committee, D.C. Heath and Co., (Excerpt A).

5. College Physics, 4th Edition, Francis W. Sears, Mark W. Zemansky, Hugh D. Young, Addison-Wesley Publishing Co., Reading, Mass., 1974 (Excerpt B).

6. "Superposition of Pulses on a Spring;" Loop No. 81293, Encyclopedia Britannica Educational Corporation (Super 8-Color).

7. "Conservation of Energy;" Loop No. 80-276, Ealing Corp., Cambridge, Mass. 02140.

Module 7 Self-Regulation

Introduction

How can students be made more aware of their own reasoning? This question identifies one aspect of formal thought. It must be answered if students are to proceed to formal thought by self-regulation, the process whereby an individual advances from one stage of reasoning to the next. We have alluded to self-regulation in several of the earlier modules, but concentrated on the characteristic reasoning patterns associated with each stage. In this module we shall describe self-regulation in detail.

Objectives

To assist you in describing self-regulation.

Procedure

Join with a group of four to eight other participants for this entire module. The introductory group activity will be followed by individual reading of an essay on self-regulation. In conclusion, we suggest a discussion with your group of the concept of self-regulation and some of its implications.

Module 7 Instructional Materials

1. Exploration

In an attempt to simulate the experience of a student using concrete reasoning patterns in a physics class that requires formal thought, we have constructed a puzzle that requires you to make drawings while looking into a mirror. In our experience, only a few persons can make drawings under these conditions with some facility, most have initial difficulty but can teach themselves, and a few have serious blocks that appear to prevent them from mastering the skill.

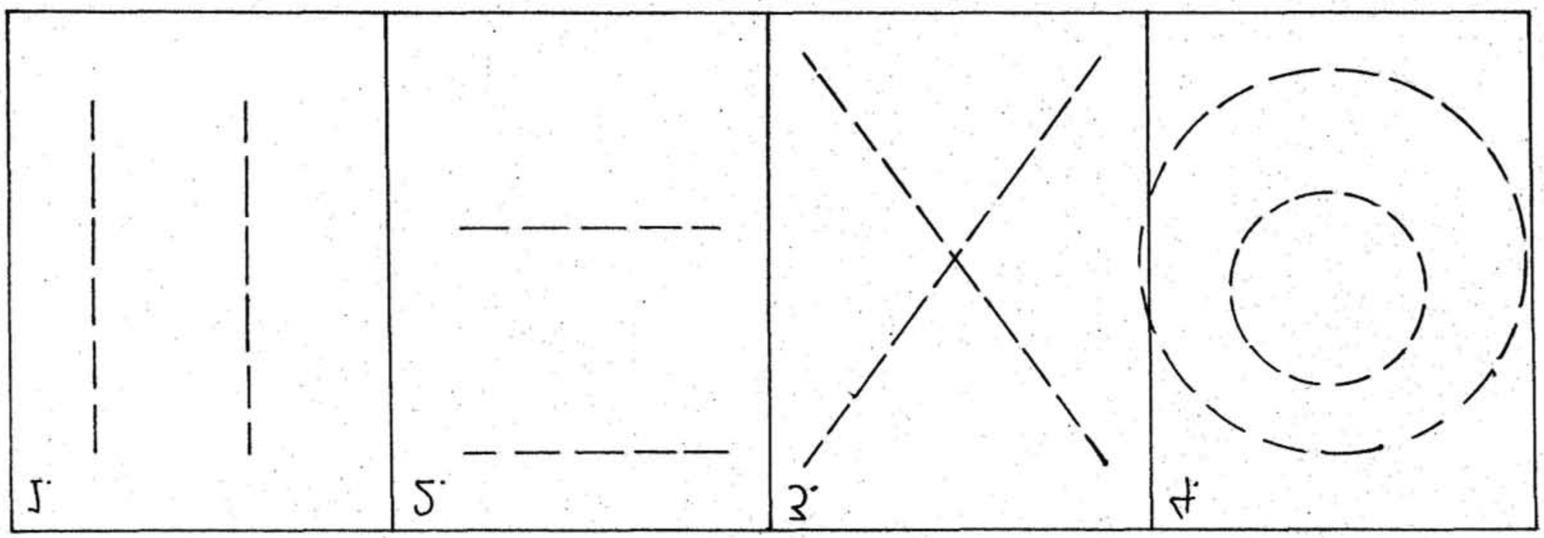
Join with your group of participants to use the mirror puzzle with pages 7-3 and 7-4 (either furnished separately by the workshop or torn from these instructional materials). Take turns with your colleagues to draw the patterns suggested in items A, B, and C. Try to become conscious of your own technique while you are drawing and observe carefully while your colleagues are working to identify their learning strategies. After each person's turn is concluded, the "observers" might tell him what they thought he was trying to do, and he could then describe how he perceived his own efforts.

After everyone from your group has used the mirror puzzle, exchange ideas regarding your efforts and difficulties. Did your thinking patterns change while you were using the puzzle? What feedback from your actions was especially helpful? What new procedures did you adopt? What errors persisted in spite of your best efforts? What direction and/or shape of line was easiest to draw while looking into the mirror? Which was the most difficult? Did right vs. left-handedness seem to affect the result? Could you do better with your eyes closed than open?

Please stay with your group as you turn to page 7-5 for the essay on self-regulation. The concluding discussion will involve you and your colleagues.

A. Please write your name while looking only into the mirror.

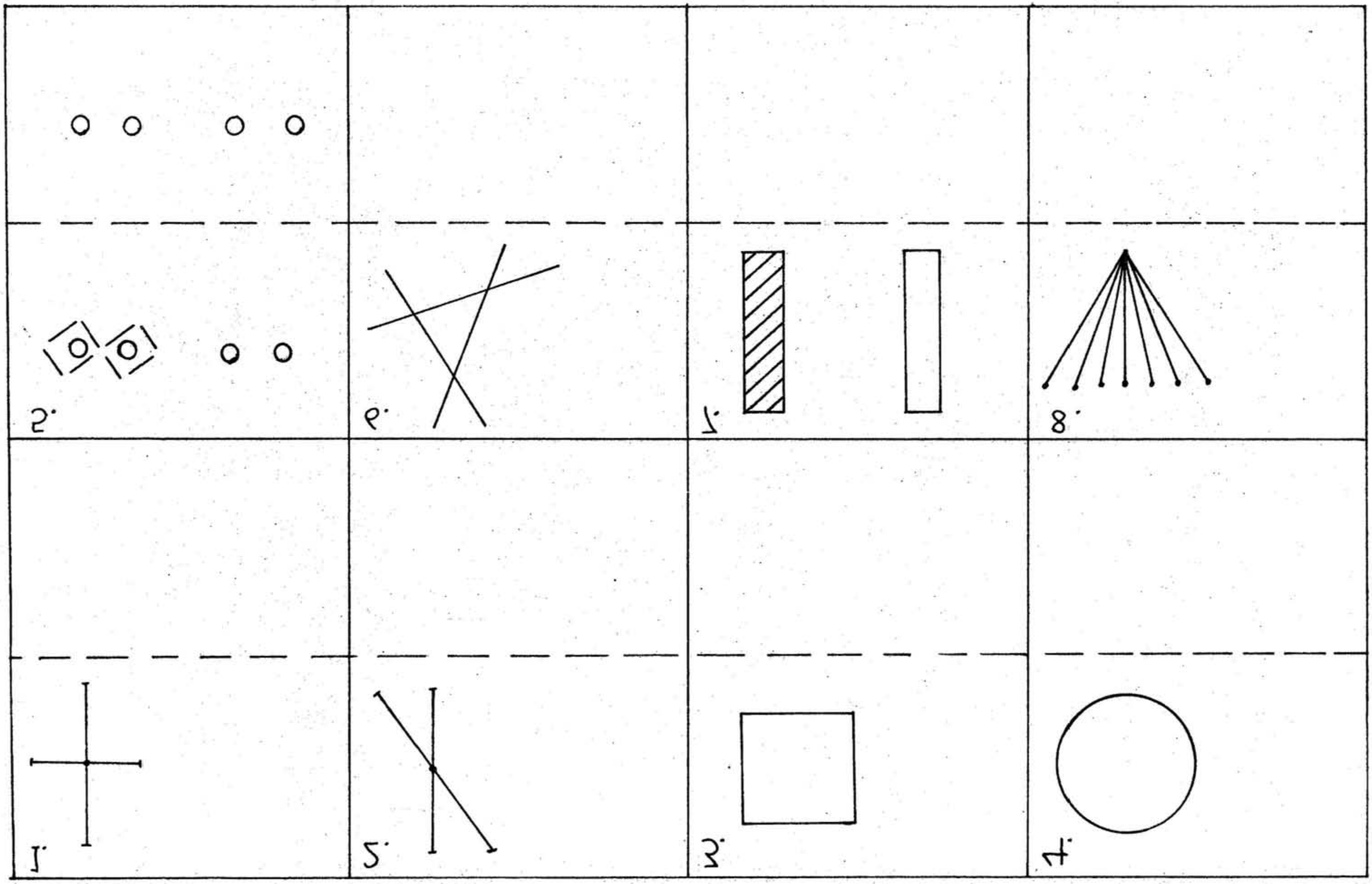
Write it so it appears normal when read in the mirror.



B. Please trace each of these figures while looking only into the mirror.

Keep your pencil in contact with the paper as much as possible.

C. Please copy each of these figures one or more times in the rectangle beside the pattern.



2. Essay. Self-Regulation

Self-regulation is the second key concept in Piaget's theory, supplementing the concept of stages of intellectual development introduced in Module 2. Self-regulation is the process whereby an individual advances from one stage to the next, at least within one realm of ideas.

According to the theory, each stage represents a relatively stable state of mind in which feedback derived from a person's thoughts and actions supports the reasoning patterns characteristic of that stage. These patterns form an interrelated system of understandings and operations called mental structures. When you approached the mirror puzzle, you were using structures based on direct visual feedback from your experience without mirror. Your responses to the reflected images may have been at first inappropriate, making you draw lines in the wrong directions. In other words, you interpreted the new experience in terms of your old structures, an action for which Piaget uses the term assimilation. Usually such assimilation results in success -- you don't often use a mirror to write -- but sometimes it does not.

As another and very different example, consider the relationship of pizza price to pizza size. A child using concrete reasoning patterns will expect to pay more for a large pizza than for a small one, but he will not connect the size to the price quantitatively. When asked about the price of a sixteen-inch pizza compared to an eight-inch one, he will predict that it costs twice as much, "Because it's twice as big." Imagine his dismay when he finds that the large pizza costs four times as much! How can that be explained other than in terms of extortion by the pizza parlor proprietor?

Just as in your encounter with the mirror, extended interactions with the environment are likely to lead to contradictions, i.e. situations in which the individual's patterns of reasoning lead to expectations that are not confirmed by what actually happens. Then the stable state of mind is upset and a change in the mental structures must be brought about, a change Piaget called accomodation. The process leading from assimilation to accommodation is called self-regulation.

Self-regulation is an active process whereby a person searches for new reasoning patterns and new relationships that will resolve the contradictions he has encountered. A very important aspect of self-regulation leading to formal thought is awareness of one's own reasoning. You were engaged in "self-regulation" with respect to the mirror puzzle after you recognized your difficulties and were attempting to control your hand movements, perhaps by thinking of the line's appearance on the piece of paper as it would appear without mirror intervention. Or, you might have distinguished between the need to draw toward your hand rather than away from it. Whatever the specific method, when the changes required are not too great, then the individual's further investigations and experiences are likely to lead him to reorganize his patterns of reasoning into appropriate new structures. Confirmation of these new structures through further experiences -- occasions for assimilation -- will maintain the new stable state until additional contradictions are encountered.

If the required changes in mental structures are great, however, a person may be especially susceptible to the influence of peers, teachers, or parents who can suggest useful avenues for investigation or may even describe a more appropriate pattern of reasoning. "It's the area of the pizza that determines the price, and the area varies as the square of the diameter." Such direct teaching, however, is usually not effective unless the learner has had previous experiences with ideas such as area and square, and can subsequently test them against his own observations. He must get encouraging feedback from the environment to make sure that the interplay of thought and action, an essential part of self-regulation, continues until the new mental structures are firmly established.

Unfortunately, Piaget's theory becomes murky as concerns more details about the process of self-regulation and the nature of effective teaching programs that allow self-regulation to be initiated and completed. An individual who uses formal reasoning patterns in some areas of experience is more likely to engage in self-regulation in a new domain because he is aware of his own reasoning, can recognize his shortcomings, and can search more effectively for new structures. To be helpful, a teaching program must strengthen these tendencies and discourage unquestioning acceptance of poorly-understood principles and procedures. In the next three modules we shall present more specific suggestions about how physics instruction can be adapted to facilitate self-regulation.

3. Discussion

Join again with your group to discuss the concept of self-regulation in the light of your teaching experience and what you gained from the earlier modules. You might assume that only a few of your students have formal mental structures for dealing with physics content, that more of them have such structures for dealing with algebra or geometry, and that some have no formal mental structures at all.

Suggested questions:

1. How will the various groups of students respond to text passages like those analysed in Module 6?
2. How will the various groups of students respond to problems like those described in Module 5?
3. How may the various groups of students be helped to initiate self-regulation in regard to Newtonian mechanics?
4. How may the various groups of students be helped to initiate self-regulation with respect to electricity?
5. What role might the physics laboratory have in contributing to self-regulation of the various student groups?

Module 8 Learning Activities for Self-Regulation

Introduction

It is quite clear from the earlier modules in this workshop that a teacher's awareness of students' patterns of reasoning will influence his choice of subject matter, level of presentation, selection of text, and assignment of homework problems. We shall now describe some ways in which the learning activities can be planned so as to enhance the opportunities for self-regulation after a student is introduced to a new idea.

On the basis of Piaget's developmental theory, concrete learning activities play a central role in the improvement of a student's reasoning. The physics laboratory, therefore, is an especially important part of instruction. Does it make any difference what kind of laboratory exercise we ask a student to perform? We believe that the answer is yes, and we shall describe what we have learned from Piaget's work that is applicable to labs and other aspects of teaching. We have called the resulting pattern of instruction a "learning cycle," since it may be used repeatedly for each successive topic or lab session in a course.

Objectives

To enable you to describe the "learning cycle" approach to teaching.

To assist you in designing laboratory activities that encourage self-regulation.

Procedure

This module provides for a laboratory investigation of physical pendula and two essays: on the learning cycle and on the physics laboratory. Please carry out the activities in the order described in the attached instructional materials. We recommend that you find a partner with whom you can compare notes and exchange ideas during this module.

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Module 8 Instructional Materials

1. Exploration

To help you and your partner approach this module in an inventive frame of mind, we ask you to begin with the laboratory investigation introduced on this page. In the module area you will find the following equipment: support stands, timers, meter sticks, string, spring scales, and various objects that may be suspended. Suspend one of the objects, set it swinging, and observe its motion. Then think of some properties of the system that you can vary, look for some other properties that might be affected by the variations, and make measurements to determine quantitative relationships that seem to interest you. You may use objects in your possession in addition to the ones provided.

Please record your observations and data here. State any conclusions you reach.

After about ten to twenty minutes, join with a group of other workshop participants to discuss some ways in which the above "exploration" might be followed up in a student laboratory exercise.

Please turn to the next page for the first essay.

2. Essay. The Learning Cycle

Suppose you are planning to begin your course's section on geometrical optics. Would you begin it by:

(a) Listing the assumption of the ray model for light, from which the results of geometrical optics can be derived?

(b) Arranging for a laboratory period in which your students could assemble light sources, lenses, mirrors, plastic blocks, and glasses of water into optical systems to observe image formation under various conditions?

(c) Reminding your students of their everyday experiences with light and invite them to describe some of the properties of light that are revealed by their observations?

(d) Describing the transfer of energy by means of electromagnetic radiation of various frequencies, and then specializing to the visible part of the spectrum?

(e) Providing a laboratory as in (b), but making certain that your students could work with "pencils" of light, as emitted by a laser or a source with a good collimator?

(f) Providing a laboratory where your students are assigned to measure accurately the focal lengths of convergent and divergent mirrors and thin lenses on a carefully aligned optical bench?

Certainly, the resources available to you and the level of students will influence your choice. Compare your reactions with our comments on the alternatives:

(a) This procedure is frequently used because of its conciseness but it is likely to be difficult for your students, especially those using concrete reasoning patterns, to assimilate. They do not know the basis of the assumptions and therefore cannot evaluate when and how these are to be used.

(b) We would recommend an approach of this kind, where the student has a great deal of freedom to use his own judgment and try out his own ideas as he gains practical experience with the objects he will study theoretically later. See also (e).

(c) In the absence of laboratory materials, we would recommend this approach to connect the new ideas about light propagation with the student's previous experience; demonstrations with student participation would help

(d) This rather theoretical approach would be inappropriate at the beginning of the topic, because it highlights the wave nature of light which is disregarded in geometrical optics except insofar as it limits the applications.

(e) Since light "rays" play an important part in geometrical optics, we would consider this a very helpful addition to the lab. An ordinary comb with coarse teeth can be used very effectively to make a bundle of light "rays" whose behavior can be followed.

(f) This type of laboratory prevents the student from asking his own questions and satisfying his own curiosity. The concept of focal length needs to be defined and understood before this lab can be worthwhile. At a later time in the course it might be quite appropriate, though we favor a more open approach.

The preferred approach in (b) or (e) is an example of the "exploration" phase in the learning cycle which we recommend for the planning of teaching activities. The entire learning cycle consists of three phases that we call exploration, invention, and discovery. During exploration the students learn through their own more or less spontaneous reactions to a new situation. In this phase, they explore new materials or ideas with minimal guidance or expectation of specific achievements. Their patterns of reasoning may be inadequate to cope with the new data, and they may begin self-regulation. The laboratory exercise opening this module gave you an "exploration" experience.

During the "invention" phase, you define a new concept, introduce a new principle, or explain a new kind of application to expand the students' knowledge, skills, or reasoning. This step should always follow exploration and relate to the exploration activities. It will thereby assist in your students' self-regulation. In the example of geometrical optics above, for instance, alternative (a) represents a possible "invention" phase, perhaps introduced via (c) as an intermediate step to relate exploration and invention. Do encourage individual students to "invent" part or all of a new idea for themselves, before you present it to the class.

During the last phase of the learning cycle, "discovery," a student finds new applications for the concepts or skills he has learned earlier. The measurement of focal lengths of a variety of optical systems (single and multiple lenses, glasses of water) would be an appropriate discovery activity to follow the introduction of geometrical optics. Other discovery activities could involve the theoretical analysis of various optical elements and systems for object-image relationships. The discovery phase provides additional time and experiences for self-regulation to take place. It also gives you the opportunity to introduce the new concept repeatedly to help students whose conceptual re-organization proceeds more slowly than average, or who did not adequately relate your original explanation to their experiences. Individual conferences with these students to identify their difficulties are especially helpful.

As another example of the learning cycle, we direct your attention to this essay. We did not begin it with a definition of the learning cycle, but rather tried to place you in a situation of considering alternative teaching strategies according to your own experience and preferences, to be compared with our thoughts. That served as "exploration," the best we could think of in the context of this module. Next we described the three-phase learning cycle, the "invention" in this essay, with references to your exploratory experience with the optics example. Finally, we should like you to examine, after the conclusion of this workshop, our entire workshop plan, which is also formulated according to a learning cycle. That examination will form a "discovery" activity for you, we hope!

After concluding the essay, please discuss the following items with your partner and/or other workshop participants and staff.

1. Suppose you are teaching an introductory course in Newtonian Mechanics. What "exploration" activity might be suitable at the very beginning of the course? What "exploration" activity might be suitable to introduce the topic of rigid body rotation? Use this space to write down some good ideas that emerge from the discussion.

2. Suppose you are teaching an introductory course on electricity and magnetism. What might be the focus of some "invention" activities?

3. What might be some "discovery" activities to follow the items you listed for #2? What might be some "exploration" activities to precede the items in #2? Make notes of the ideas that are expressed.

4. Most advanced physics courses are strictly "blackboard and chalk." Pick a particular course with which you have worked recently and suggest "exploration" activities that might be introduced. Keep in mind the fact that many of the students may not have assimilated all the material that was covered by the prerequisites. Make notes about ideas that are brought up.

5. Do you see a relation between the learning cycle and self-regulation? How do you and your partner view the relationship?

3. Essay. The Laboratory and Self-Regulation

Suppose you are asked to develop a laboratory exercise on the pendulum for beginning general physics students. Rank the following procedures in terms of how you perceive their usefulness in encouraging self-regulation for the students; use 1 for the most useful and 4 for the least useful.

Rank

A. Provide the students with a mass on a string. Indicate the relevant variables of the system and suggest that they verify the square root relationship between the length of the string and the period of oscillation. ____

B. Provide the students with a mass on a string. Supply a list of possible variables of the system, i.e., angle of swing, mass, length of string, acceleration of gravity, the period of oscillation, etc. Supply a list of possible relationships between variables, e.g., the period oscillation is directly proportional to the mass, the length of string is directly proportional to the period, etc. Ask the students to identify the relevant variables and the most appropriate relationships between them. ____

C. Provide the students with a variety of periodic systems, e.g., a cork floating on water, a baseball bat swinging by a hole in its handle, a clock pendulum, a mass on a string, a uniform metal rod with pivot holes in it. Ask the students to identify common variables of these systems and to search for quantitative relationships between the variables. ____

D. Provide the students with a mass on a string. Indicate that for small angles of oscillation there is a relationship between the length of the string and the period of oscillation. Challenge them to discover it based upon their data and then compute the length of string required for a 10 second period. ____

According to our learning cycle model to induce self-regulation, an introductory period of exploration or openness in a laboratory exercise is to be recommended. Hence, procedures B and C are superior to A and D. Furthermore, C is a more open and exploratory procedure than B and may encourage the student to examine a number of aspects of a swinging object that you may not think are important or interesting, but that appear important to him. Procedure C enables the students to begin where they are in their understanding of periodic motion and enlarge their concrete experiences with such systems without having the instructor impose his own reasoning on their activities. Hence, we believe that C is the preferred procedure to use.

Procedure B also provides a good deal of openness while directing the students toward variables determined by the instructor. A variant of this procedure would be a good discovery activity. It tends to focus the activities of the students and make their efforts more efficient if content

goals are important. Predictions and expectations in advance of the experiments can be exploited to produce some contradictions in the thinking of the students and start them on self-regulation. Extreme cases not tested directly or concretely can also encourage self-regulation. Hence, we favor procedure D over procedure A. In fact, procedure A has little to recommend it as far as we are concerned.

The social interactions that occur in the laboratory setting are important for starting self-regulation. Testing one's ideas against the ideas of one's peers is a profitable way to spend some time during the laboratory period. Individual contact between the instructor and the student is possible in the laboratory. Such instructor-student dialogues can be very valuable when the instructor asks the student to justify his results. Helping students to become aware of their own thinking is a major function of the instructor if he wishes to encourage his students along the path of self-regulation. Such common thinking tools of physicists as checking the dimensions or units of an answer, making an order of magnitude estimate, and seeing if the answer makes sense at the extreme values of the variables are all aspects of the self-regulation process that can be learned as a part of laboratory activities.

4. Laboratory on Objects that Swing

In this exercise, we present a laboratory activity arranged according to the learning cycle into exploration, invention, and discovery. The students were given four pages, one with the title and instruction for exploration (see below), a second page organized as data sheet, an "invention" page, and an "application" (i.e., discovery) page. Since the last three pages required student recording of data or answering of questions, we are presenting them in reduced format with the data, answers, and work of one pair of students. Please examine these pages and look for evidence of concrete and formal reasoning patterns, self-regulation, and failures to respond to inconsistencies. Then look at our comments on page 8-12.

SWINGING OBJECTS

Purpose: Examine the properties of objects that swing to and fro when suspended on a string.

Equipment: Objects, string, timer, meter stick, and supports.

EXPLORATION

Explore the properties of a swing that consists of an object suspended on a string. What are the properties of that system that you can vary? Measure quantitatively these properties and the period of time required for the object to make ten complete swings to and fro.

When you are satisfied that you have examined all aspects of your system, ask the instructor for the invention page.

Please record all the activities you pursue, even ones that may lead to a dead end. You will be evaluated on the completeness of your records as well as the reasonableness of your conclusions.

#1) For the first test, we are varying the weights, while keeping the pull-back and string length constant. The length of time between ten full swings is recorded. The string length is constant to the extent that only the size of the weight causes an increase in the distance to the center of mass.

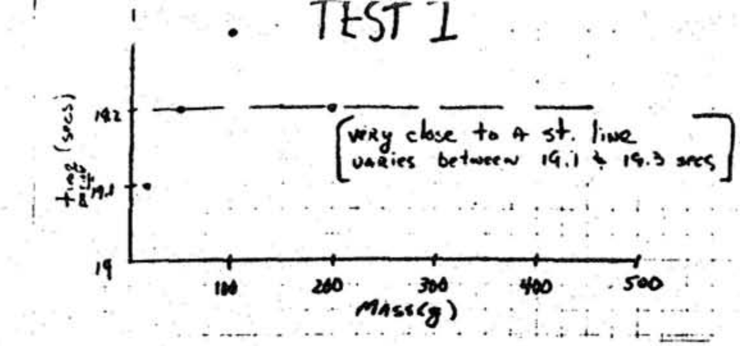
#2) For the second test, the mass of the object and the string length are held constant, while the pullback is varied. Pullback is made in 10 cm intervals and time is recorded for 10 swing intervals.

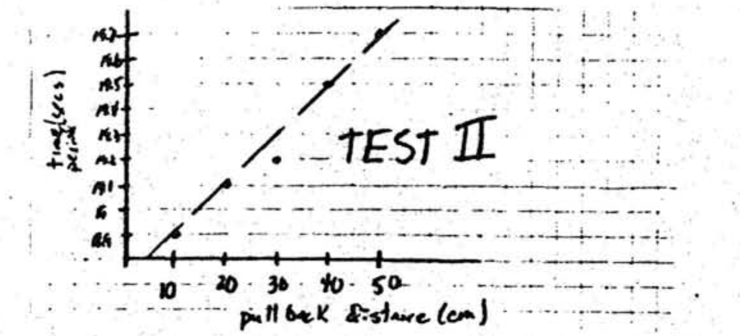
#3) Test number three involves changing the length of the supporting string, while holding the mass and pullback constant. The pendulum is allowed to swing for 10 swings and the time recorded.

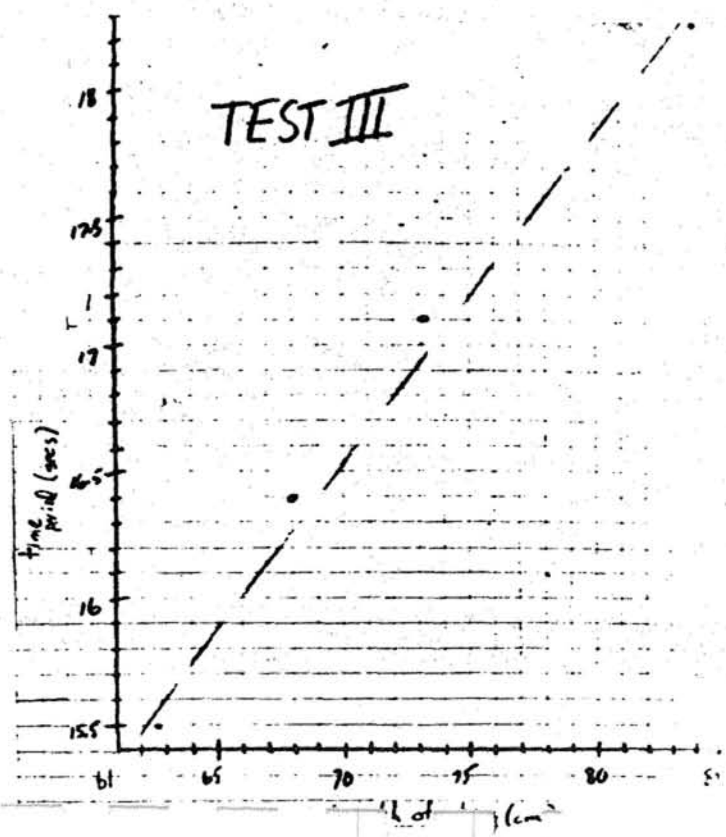
Data Sheet

Quantitative Values of the Variables of the System (label the columns)					Time required for ten to and fro swings of the obj
TEST 1 mass	LENGTH	CONSTANT PULL BACK			
200g	91cm	"			19.2
100g	90.6	"			19.3
50g	89.55	"			19.2
20g	89.5	"			19.1
500g	94cm	"			19.3
TEST 2					
CONSTANT mass 200g	CONSTANT LENGTH 91cm	PULL BACK			
200g	91cm	10cm			18.9
"	"	20cm			19.1
"	"	30cm			19.2
"	"	40cm			19.5
"	"	50cm			19.7
CONSTANT mass 200g	LENGTH OF STRING	PULL BACK CONSTANT (30)			
"	68 cm	"			16.4
"	61.6 cm	"			15.5
"	52.5 cm	"	OMIT		OMIT
"	73.1 cm	"			17.1
"	84 cm	"			18.3

If you need additional data space, record your data on the back. If you have finished with your data collection activities, please ask for the invertic page.







INVENTION PAGE

The time required for a swinging object to make one complete trip, to and fro, is called a period.

Which of the properties of your system seemed to have the least effect upon the period of oscillation? It would appear that test I is closest to having the least affect upon the period of oscillation. By varying the mass and keeping a constant pull-back distance of 36.6cm our periods of time varied only from 19.1 to 19.3 secs. So it appears that having a constant pull back distance and changing mass has little affect on period of oscillation.

Which of the properties of your system seemed to have the most effect upon the period of oscillation? Test II has a slight affect upon the period of oscillation, in it we varied the pull-back distance keeping the mass constant. In our test III though we had varied the length of the string keeping the pull back rate constant. In this test our periods of time varied much more, so it would seem that varying the length of the string had the most affect upon the period of time.

What relationships between the period of oscillation and the quantitative values of a property of the system can you identify? In order to express these relationships in a quantitative way you may wish to draw a graph of your data.

Test 1 on graph paper appears to be a constant while tests 2,3 appear to be more like proportions.

What conclusions can you reach? State them here:

For test one it appears that little affect upon the period of oscillation is achieved by varying mass with constant pull back [It appears constant], while in the other two parts a proportion seems to result between (period & pullback in 2) & (period and length of string in 3). Quantitative analysis could be calculated in tests 2 & 3

Discuss what you have written on this page with another group of experimenters. List the similarities and differences between your results and theirs here:

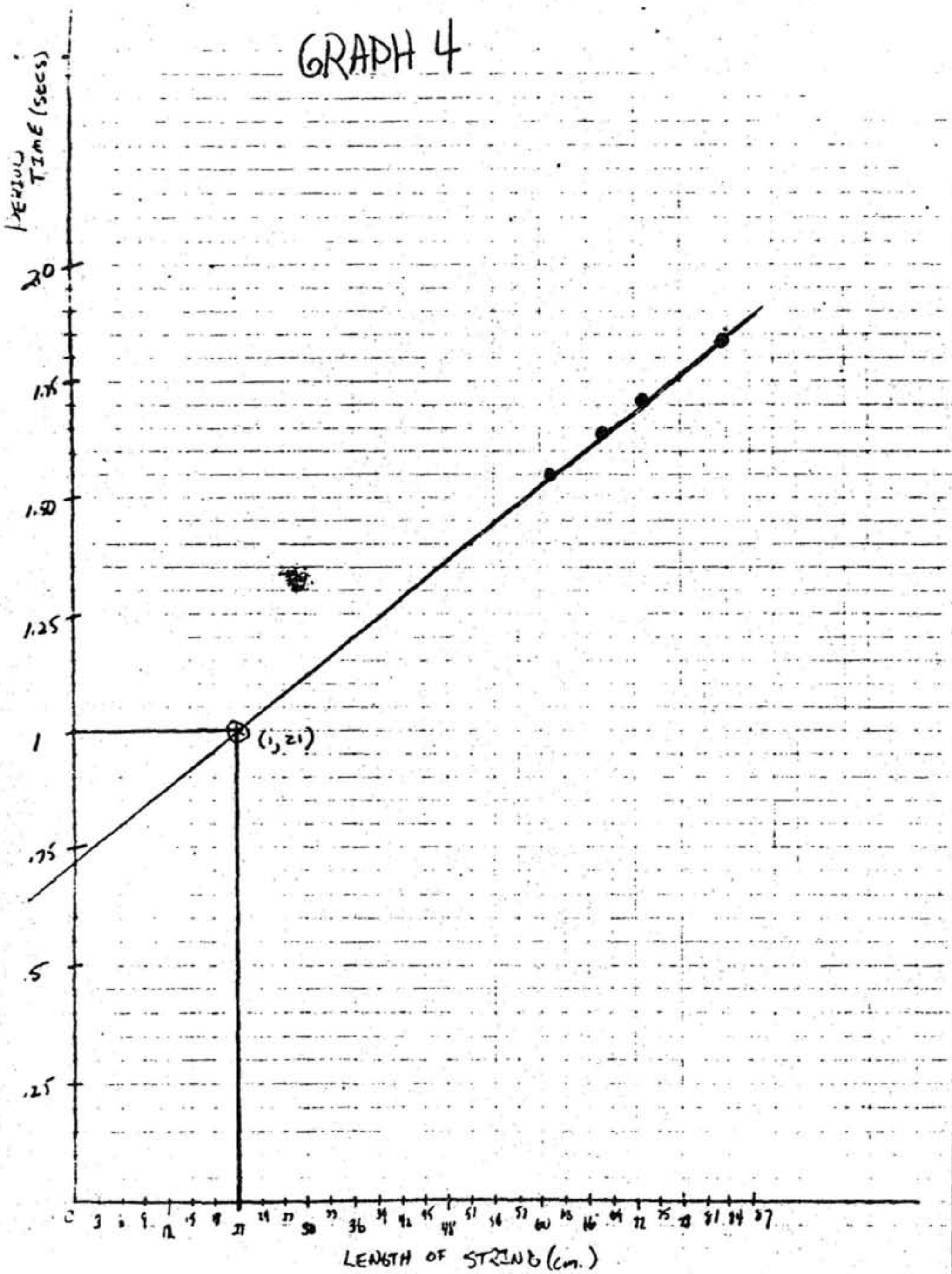
Comparing with another group seemed to lead to the same conclusions.

(1.) Keeping constant pull-back with varying mass leads to a constant equation.

(2.) Keeping constant mass and varying the pullback or angle causes a slight difference, but the difference was minimal and would be due to mechanical error.

(3.) Keeping constant mass and ∠ and varying length of string, causes an increase in oscillations per sec

Ask the instructor for the application page.



APPLICATION PAGE

Now based upon your previous results either construct or explain in detail how you can construct a system of a swinging object on the string which:

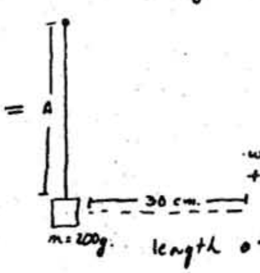
(a) has a period of oscillation of 1.0 second.

A = length of string

200 g = constant

pullback (or angle) = 30cm

21cm. =

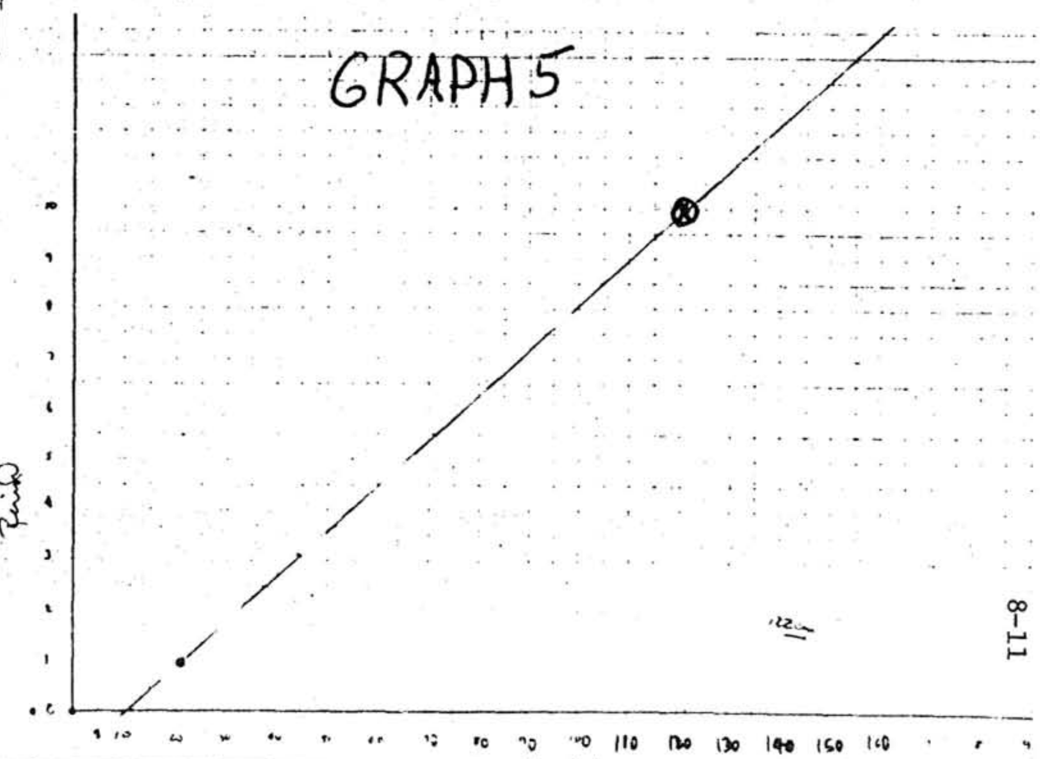


Using test three data and graph we extended the graph (see graph 4) to include the one sec. oscillation interval. This gave us a 21cm. length of string

(b) has a period of oscillation of 10.0 seconds.

Re-draw graph in opposite direction and go thru same steps as above ANSWER comes out 122cm

Common sense indicates this isn't right, but that is our value.



Comments on Student Work:

You will have noticed that these students set out systematically to examine the various properties of the system of a swinging object. They carefully isolated variables (mass, amplitude, length), a formal reasoning pattern.

The students' graphical analysis at the end of the experiment, however, does not show the self-regulation one might have expected. Graph 4 shows that a string of zero length would have a 0.70 second period, while Graph 5 leads to the prediction of a 0.0 second period for a string of 10.0 cm length. The inconsistency of these two inferences does not concern the students, though they do assert that their result for the 10 second period contradicts common sense. The use of an analytical tool (plotting points) and applying it to data (drawing a straight line) without self-regulation are characteristic of the step-by-step following of instructions characteristic of the concrete stage. Apparently this laboratory activity was not successful in leading these students to self-regulation with respect to data analysis.

Discuss the following items with your partner if they did not come up during your earlier conversation:

1. What other instances of concrete or formal reasoning can you identify in the students' work?

2. What aspects of their work could you use to guide them into self-regulation? Point out the discrepancies or other starting points you noticed.

3. Think about some laboratory activities of <u>your</u> students. Do they reflect the learning cycle approach? Do they require the students to follow a "recipe?" How might they help to initiate self-regulation?

Module 8 Review Questions

Please discuss these questions with your partner after marking your answers.

1. Suppose you are asked to design a laboratory exercise on the topic of Ohm's Law for beginning physics students. Rank the following procedures in terms of how you perceive their usefulness for encouraging self-regulation on the part of the students. Use 1 for **the most** useful, 2 for the next, etc. (For our answers, see the bottom of the page.)

	Rank: self-reg.	Rank: content
A. Provide the students with a 1.5 volt battery, some known resistors, and an ammeter. Ask them to verify the V=IR relationship.	____	____
B. Provide the students with some 1.5 volt batteries, some known resistors, and an ammeter. Supply them with a list of of the possible variables of the system: the number of batteries, the number of resistors, the current, the length of connecting lines, etc. Supply a list of possible relationships between the variables, e.g., the voltage is directly proportional to the number of resistors in the circuit, the current is proportional to the square root of the resistance, etc. Ask the students to identify the relevant variables and the most appropriate quantitative relationships between them.	____	____
C. Provide the students with a variety of batteries, a variety of resistors including some slide wire type, and a multimeter. Ask them to identify the variables of a circuit and find quantitative relationships between the variables.	____	____
D. Supply the students with a battery, two known resistors, a galvanometer of unknown calibration and several unknown resistors. Ask the students to compute the resistances of the unknown resistors.	____	____
E. Supply the students some lengths of unlabeled metal wires and a resistivity table. Ask them to identify the metals by using Ohm's Law and the definition of resistivity. Provide the necessary apparatus.	____	____

2. Now reread the list and rank the items according to their usefulness in transmitting content about Ohm's Law. Does this ranking agree with your self-regulation ranking? If not, why do you think that there is a difference?

Answers: We would rank the alternatives B, C, D, E, A; the last three, about whose relative ranking we do not feel very strongly, would be suitable as discovery activities when self-regulation is in an advanced stage. A would be improved by including a circuit element that did not follow Ohm's Law.

Module 9 Analysis of Physics Concepts

Introduction

Most physics teachers think about their courses in terms of topics covered, concepts explained, and principles applied. Our effort in this workshop has been to call your attention to another important dimension of physics teaching, your students' patterns of reasoning. By this time, you have probably concluded that most physics courses are addressed primarily to students who can use formal reasoning patterns with ease, and we would agree with that. Yet there are also the students who use formal reasoning patterns only with difficulty and in limited areas. To help you analyze course content and present it in a way that will be understandable to more of your students, we suggest that you classify physics concepts according to the reasoning patterns necessary to understand the meaning you wish to communicate. Concepts may then be called "concrete " or "formal," in analogy to the stages of reasoning. This module presents examples and explanations of "concrete" and "formal" concepts.

Objectives

To assist you in classifying physics concepts on the basis of the patterns of reasoning needed to understand them.

Procedure

We have arranged this module in the form of a learning cycle built around the distinction between concrete and formal concepts. Please find a partner with whom you can join in the activities. Then undertake the designated exploration, invention, and discovery activities described in the attached instructional materials. An audiotape to supplement the invention phase is available; we suggest you listen to it at a certain time as indicated in the text, but you may wish instead to proceed to some of the discovery activities before listening.

Module 9 Instructional Materials

1. Exploration

Four concepts commonly introduced in an introductory physics course are listed below. Determine from your teaching experience whether a student could develop an initial understanding by the use of concrete reasoning patterns together with actual experience using suitable materials. Begin by discussing each of the topics listed below with your partner and briefly outlining to one another the instructional experiences you would provide for students at your institution. Then identify in writing the reasoning patterns necessary and laboratory experiences that could be used. If you believe that a concept could be introduced at various levels, use the simplest one here.

Interaction:

Electrical Conductor:

Ideal Gas:

Light Wave:

Please come to an agreement with your partner on each item before continuing to read.

2. Invention

In our opinion, "interaction" and "electrical conductor" can readily be understood in terms of familiar actions, observations, and examples. In other words, these concepts can be derived from using concrete reasoning patterns. Such concepts are called concrete concepts. The concepts of "ideal gas" and "light wave" must be understood in terms of other concepts (pressure, volume, electric field, etc.), functional relationships (ideal gas law, wave function), inferences, and/or idealizations. Those understandings are not the direct result of concrete experiences but are theoretical elaborations that require application of formal reasoning patterns. Such concepts are called formal concepts. Many concepts, of course, have more than one meaning and may therefore be concrete or formal, depending on their treatment. Thus, temperature as read on a thermometer is a concrete concept; temperature as a measure of the average molecular kinetic energy is a formal concept.

It may be good to mention at this time that the concrete vs. formal distinction is not equivalent to the familiar concrete vs. abstract distinction. All concepts are abstract, abstracted from many specific instances and concrete examples. Interaction is abstract in that it is very general, applicable to all objects that influence one another, regardless of whether they exchange energy or momentum, modify the chemical composition, or (if living) infect with a disease. The abstraction process involved in the interaction concept, however, depends on reasoning patterns appropriate to the concrete stage, and the concept has been taught successfully to second and third grade children in the framework of everyday objects and their interactions.

The light wave concept is also abstract, though more restricted in applicability than interaction. Yet the meaning of light wave depends essentially on Maxwell's electromagnetic theory, which can be understood only through the use of propositional reasoning, functional relationships, abstract variables, idealized models, and other formal reasoning patterns. We might add that the concept of electrical conductivity is a formal concept, even though we considered electrical conductor concrete because it could be identified by direct empirical criteria.

Please listen to the audiotape on Self-Regulation and Physics Concepts now. For your convenience, the script is included at the end of these instructional materials.

3. Discovery

To allow you to apply your present understanding of the distinction between concrete and formal concepts, we have constructed a list of items we should like you to classify. Discuss each item with your partner to help you clarify your ideas, but record your own views if the two of you disagree. To help you justify your classifications, we have included here a slightly edited version of the concrete and formal reasoning patterns originally given in Module 2.

The formal reasoning patterns most frequently required for the understanding of physics concepts are:

F1. understands concepts defined in terms of other concepts or through abstract relationships such as mathematical limits.

F2. imagines all possible combinations of conditions even though not all may be realized in nature.

F3. separates the effects of several variables by holding all but one constant.

F4. uses theories or idealized models.

F5. recognizes and applies functional relationships, such as direct and inverse proportion.

The concrete reasoning patterns most frequently required for the understanding of physics concepts are:

C1. understands concepts defined in terms of familiar actions and examples.

C2. applies conservation reasoning.

C3. establishes one-to-one correspondences and arranges data in increasing or decreasing sequence.

C4. makes simple classifications and successfully relates systems to subsystems, classes to subclasses.

The differences between these reasoning patterns might be summarized as follows: the concrete patterns employ simple operations applied to real objects and experiences, but not to relationships, hypothesized objects, or postulated properties. A concept can usually be considered concrete, therefore, if one can grasp its meaning through direct experience. If a concept derives its meaning principally from its position within a theoretical system, it has to be classified as formal.

Here are the concepts for your exercise. We have included answers for the first two items to illustrate how you might refer to the above lists of reasoning patterns when you give your reasons.

	Concept	C or F	Reasons
1.	Pressure	C	Defined operationally through a barometer reading, with pressure differences defined by a manometer (C1). Pressures can be compared (C3) but not used to calculate gas volumes or forces exerted on container surfaces.
	Pressure	F	The usual definition, force per unit area, depends on the force concept (F1) and on proportions (F5).

	Pressure	F+	Pressure is the time-average effect of molecular bombardment of the containing surface (F1, F4, F5). This concept derives its meaning from the kinetic-molecular theory, a theoretical system in modern physics.
2.	Shadow	C	Can be observed easily and is familiar (C1). Correspondence of obstacle shape and shadow shape can be established (C3), as can qualitative size relationships.
	Shadow	F	Ratio and proportions are used to describe size relations of obstacle and shadow in terms of light source, obstacle, and shadow positions (F3, F5).
	Shadow	F+	The concept of shadow is qualified by the diffraction of light according to the wave theory (F4). This concept's meaning is affected by the theoretical system of the electromagnetic theory of light. (Note: introduction of the quantum theory would escalate the conceptual level another step.)
3.	Temperature		
4.	Vertical		
5.	Latent heat		
6.	Wave interference		

If you have reached the conclusion that many physics concepts, though not necessarily all, can be interpreted on either the concrete or formal level, then you will be able to relate this activity to teaching through self-regulation. As was explained on the audiotape, learning that begins with a concrete version of a concept is likely to make a more secure connection with the student's previous understandings and preconceptions. After he encounters some limitations of this concept -- for instance, the difficulty of making quantitative predictions from pressure defined concretely in terms of a barometer reading -- he can extend its significance to that of a formal concept through self-regulation.

Please look back at the above concept list now, and do the following together with your partner: for each item that you classified on two or more levels, think of an activity that would bring out the shortcomings of the concrete version and thereby initiate self-regulation.

Module 9 Review Questions

Please work on these items together with your partner.

1. Name two physics concepts that can only be understood by use of formal reasoning patterns (i.e., they have no "concrete" version).

2. Name two physics concepts for which you can identify three or more levels of meaning. Briefly define each level.

3. Select one of the concepts you have named in #1 or 2, or a concept mentioned earlier in this module, and briefly work out a learning cycle of exploration, invention, and discovery that might be built around it.

4. Compare the learning activities that might be used for a formal concept with those that might be appropriate for a concrete concept (or the concrete version of the same physical quantity).

Module 9 Audiotape "Self-Regulation and Physics Concepts"

A Conversation between Robert G. Fuller and John W. Renner

Robert Karplus:	This is the audio tape accompanying Module 9 in the Workshop on Physics Teaching and the Development of Reasoning. The workshop was prepared under the auspices of the American Association of Physics Teachers with partial support from the National Science Foundation. The speakers are Bob Fuller, who is a little confused, and Jack Renner, who helps to explain.
Jack Renner:	How are you doing?
Bob Fuller:	Well, I'm a bit confused. These last two modules had something to do with the concept of self-regulation and I'm not sure I understand it. Think you could help me a little bit?
Jack Renner:	Well, that is a confusing concept, and you know, it is so important for any teaching activities that are based on the intellectual development theory of Piaget that maybe I should take a few minutes to run over its meaning with you. Think of it like this. Whenever a student encounters an unfamiliar object, unfamiliar situation, or new event - in short, has a new experience - he interprets that new experience in terms of his existing patterns of reasoning, which form a system of understandings and operations called mental structures. Assimilation is Piaget's term. If the new experience is sufficiently complex and unfamiliar to the student, he will only understand it in terms of what he already knows and will not develop an appreciation of the entire meaning the teacher had intended. Development of a greater depth of understanding requires a change in the student's mental structures, a change Piaget calls accommodation. To change the structures, the student must have extensive exploratory experiences as was explained in Module 8. After an appropriate mental reorganization or accommodation, the intended impact of the new experience can be more fully felt. The process leading from assimilation to accommodation is self-regulation. After accommodation the student is in the position of re-interpreting his other knowledge in terms of the new mental structures.
Bob Fuller:	Oh, I see. You start by assimilating into your present structures, then through self-regulation, you can accommodate to the new experiences. Sounds like some kind of new jargon to me. I wonder if you could give me some more specific example, maybe taken from physics.
Jack Renner:	All right. The first physics course I ever had was in college. I remember the instructor very well, Dr. Tom Bedwell, who was a superior instructor, and he really drove home the concept of velocity. Velocity is the change of distance with respect to time. Thought I, "Big deal! That's speed. Just exactly

Renner (cont'd): what you read from a speedometer. Vectors are not important to the speedometer of my Model A." (That kind of dates me, doesn't it?) I promptly forgot all about the direction aspect of velocity.

Next, we encountered acceleration through an experience in the laboratory with a spark-gap device. That apparatus was, as I remember it, a free-fall apparatus and it delivered to me a nice tape that I could use to see that the carriage fell farther each successive unit of time. Therefore the carriage had to be traveling faster and the velocity had to increase during each interval of time. I could then appreciate the concept of acceleration, that is, a change of velocity with respect to time. I know my reasoning was, at best, early formal operational and that ratio of a ratio gave me some trouble; but in a short time I was saying centimeters per second per second just like everyone else. The holes in the tape made by the spark provided the concrete experience that led me to change my mental structures. Notice, Bob, that once again I did not pay any attention to the vector aspect of acceleration. Nor did the experience require this to be done! I had achieved self-regulation without it, I thought, and to a degree, I had.

Then the roof fell in. Uniform circular motion! Speed is constant and the object is accelerating. Impossible, said I. When the speedometer on my Model A reads constant, I am not accelerating. The patient instructor then reinforced the idea of velocity to a thoroughly confused physics student. I discovered that velocity and acceleration were completely different than I had thought them to be. My entire mental structure regarding velocity and acceleration had to be changed, I had to undergo a completely new self-regulation.

Now, when the instructor drew arrows over the V and A symbols, those arrows really meant something to me and led me to an entirely new set of understandings about Newtonian mechanics. I had finally changed my mental structures, the ultimate outcome of self-regulation (it was a lengthy and uncomfortable process, yet essential for my understanding).

Bob Fuller: Oh, yes, I think I've had similar experiences with self-regulation as a physics student myself. Now let me ask you a question that's really got me confused. I picked up this module that says something about analyzing physics concepts for formal and concrete concepts and now I find at the beginning all of this introduction to the idea of self-regulation. What has that got to do with it?

Jack Renner: That's a very good question. The basic answer to that question is that, in order to initiate self-regulation, you, the physics teacher, must do something with the physics subject matter. Think back to what I said earlier about how self-regulation starts. The student assimilates the outcome of a new experience to his present mental structures. If these mental structures are based on concrete reasoning

Renner (cont'd): patterns, and the student is presented with content that requires formal thought, he is in trouble. Without the aid of concrete experience and the opportunity for self-regulation, he will resort to rote memorization and learn a recipe. So you must begin with concrete concepts. Learners with concrete mental structures need exploration experiences that will lead them to comprehend concrete concepts. Data from such exploration plus the introduction of new concepts may then initiate self-regulation that will ultimately make the student think about the world in a formal way.

Bob Fuller: Oh, I see; so ability to be able to analyze physics concepts into concrete and formal categories might be very helpful for me as a physics teacher. What then is a concrete concept or a formal concept in physics?

Jack Renner: Well, Bob, a concrete concept is one about which the student can develop understanding through exploring concrete objects, concrete events, and/or concrete situations. Those explorations must produce concrete information that can be used to introduce the concept. In other words, for a concept to be concrete, the learner has to be able to develop understanding of it through actual experience. Consider the series circuit. A student can actually observe the fact that the elements in the series circuit are connected each one to the next, and that if you follow from one element to the next, you will come back to where you started. An aspect of the series circuit is that anything moving in the circuit, moves through or over every element. Furthermore, if you define an ammeter as a black box that measures what is moving in the circuit the student can insert the ammeter in the circuit at any one of several places and observe the same reading throughout. Thus a series circuit can actually be experienced. Many concrete discoveries can be made with the series circuit concept.

Temperature, Bob, is another concrete concept if it is related to hot and cold, which can be experienced, and can be measured with a thermometer. So, a concrete concept is one of which the student can develop an understanding through direct experience.

Bob Fuller: Oh, I get it, Jack, that seems fairly easy. Then just about anything I cover in the introductory physics course is probably a concrete concept.

Jack Renner: I wish that were true, but it isn't. Consider the idea of pressure. Now that's a common concept that we always have in physics courses. Pressure is normally defined as a ratio, force per unit area. To understand pressure, the student must understand force and area. While a single force can be experienced, generalizing the idea so force can be thought of as acting on one unit of area requires the student to use a formal reasoning pattern. Hence pressure viewed in

Renner (cont'd): this way is a formal concept. Pressure viewed as the reading of a barometer, however, is a concrete concept, just as temperature defined as a thermometer reading was a concrete concept.

Bob, the nuclear atom is another formal concept. For it to have meaning, the student must grasp the theoretical constructs of plus charge, minus charge, electron, proton, and neutron. None of those can be experienced; none is based upon experience.

Bob Fuller: Oh, I see, Jack; so that really means that a lot of the concepts we use in the basic models we use in physics are formal concepts.

Jack Renner: That's right. A formal concept is one that has meaning because of its position within a hypothetical deductive system. The concept of light polarization, for example, has meaning only in terms of the wave theory. Temperature viewed as mean molecular kinetic energy is a formal concept deriving its meaning from the kinetic molecular theory. Often teachers try to make formal concepts concrete by introducing a tangible model, such as styrofoam balls for atoms, ball bearings for molecules, water waves for light waves. Yet many students only learn about the model from such an experience. They do not construct the related system of postulates and deductions, and do not recognize the relationship of the theory to the concrete materials used to represent the idealized entities of the theory. Examples and careful explanations do help to clarify concepts, but models and examples do not of themselves turn formal concepts into concrete concepts.

Bob Fuller: Now you've got me scared, Jack. What am I going to do with a course in which I have students who are still using concrete operational mental processes?

Jack Renner: Well, students with concrete mental structures cannot properly assimilate formal concepts. Therefore, and this we believe to be the primary message of this module, these students can initiate self-regulation only if they have concrete experiences and the opportunity to begin with an understanding of concrete concepts in the topic to be mastered. After they reflect on the meaning of their experiences, self-regulation will lead them to build the formal mental structures with which they can then assimilate the necessary formal concepts.

Bob Fuller: Oh, I see. Well, thank you very much, Jack. I am eager to go home and try these ideas out in my physics classroom.

Jack Renner: Glad to help.

Robert Karplus: This is the end of the Module 9 audio tape. Thank you very much for listening. Please rewind the tape back to the beginning so another workshop participant can use it. Goodbye.

Module 10 Teaching Goals and Strategies

Introduction

Most physics classes include students who use concrete reasoning patterns on some occasions, formal reasoning patterns on others. Most likely their approach to a new kind of problem will include a mixture of techniques derived from their previous learning, their awareness of their own reasoning, and their ability to engage in self-regulation. Obstacles to their success may stem from misconceptions they formed as a result of poorly assimilated prior learning experiences. So what? What does that tell me about the goals and strategies I might choose for my teaching? In this module we shall pursue the implications of the students' needs to begin learning by using their existing mental structures, but to form new ones through self-regulation as part of their progress. Since this module outlines the last workshop activities, we invite you to bring up during the discussion any related matters about which you have questions.

Objectives

To assist you in selecting teaching strategies that will encourage self-regulation on the part of your students.

To assist you in balancing course goals aimed at content with those aimed at improved reasoning.

Procedure

Please find a partner with whom you can discuss some of the points raised while you read the two essays in the attached instructional materials. After you complete the reading, join a discussion group to compare your ideas concerning course goals and teaching strategies with those of other participants and workshop staff. For your reference, we have included a brief recapitulation of the major ideas proposed in the workshop.

Module 10 Instructional Materials

1. Essay. Teaching Strategies for Self-Regulation

How can you emphasize learning and progress in reasoning through self-regulation for your students? Though we cannot offer a widely-tested prescription, we can describe some steps we have found useful.

1. Plan your teaching to start with more concrete (operational) definitions of the important concepts and gradually introduce more formal meanings. Introduce new concepts and definitions with the help of concrete examples, demonstrations, and experiences for your students. Forces, for instance, can be illustrated with springs, bow-and-arrow, magnets, friction, and plumb lines. Waves can be illustrated by a ripple tank, a slinky, and a long elastic rope. A Cartesian coordinate system can be represented by three dowels tied together and marked X, Y, Z. A balloon can be used to represent a Gaussian surface, a pencil the normal vector, and a pen the electric field.

2. Regardless of the text you use, become aware of its strengths and weaknesses by reading it carefully to identify the demands for reasoning it places on its readers. We have often been amazed when we did that!

3. Use the learning cycle to organize laboratory activities and discussion sessions by always beginning with a task the students can define and organize partially for themselves. (Asking, "Do you have any questions?" is not such a task, but describing a simple physical situation and challenging students to pose a problem derived from it is one.)

4. Supplement the text by remarks in the lectures or in study guides that will especially help students with concrete mental structures.

5. Propose unlikely observations, unsatisfactory hypotheses, or incorrect conclusions "tongue-in-cheek" and challenge your students to evaluate these. A good example is the "capillary sprinkler": after students learn to compute the capillary rise of water in a tube, describe a tube that is too short for the rise derived from its diameter -- what will happen to the water at the top?

6. Encourage students to interact with one another during discussions, laboratories, or problem-solving sessions. Students can learn a great deal from one another during group efforts at school or at home, supervised or unsupervised. Students using formal reasoning patterns serve as role models for the more concrete thinkers, while the latter will challenge, through their questions and difficulties, the explanations and ideas provided by the text or their more advanced colleagues' short-cuts in reasoning.

7. Allow students who have made a mistake to present their complete incorrect procedure for analysis by their classmates. Change the emphasis of your teaching from the "right answer" to an understanding of the method.

8. In conversation during discussions, office hours, or tutorial sessions call your students' attention to their own reasoning. You might ask them to explain or justify their conclusions, predictions, and inferences regardless of whether these are correct or incorrect. "Are you sure of that?" "What is the evidence?" "Could you explain that to me?" "Is there another way of thinking about that problem?" are questions that might be asked of a group or of an individual student.

9. When you select problems for an assignment or test, keep in mind that a problem makes demands on physics knowledge and on mental structures. Use "I.Q.Test" type of problems, in which complicated and ingenious reasoning overshadows the physics, only as supplementary material for the more advanced students.

10. Assign specially constructed problems that encourage students to evaluate their own reasoning as described in Module 8. Encourage students to come to office hours or tutorial sessions for a review of their work on these problems so they may receive individual assistance that can help initiate self-regulation. If necessary, reduce the staff assigned to discussion sections, which rarely meet this need.

11. Use your students' performance on their physics activities to assess their reasoning patterns with respect to physics. While the tasks presented in the first few modules of this workshop have been designed for standardized interpretation of the results, we do not recommend their use to you unless you are interested in conducting research in this field and wish to compare your observations with those made at other institutions. If that is the case, please consult some of the references in Module 11 for a description of research studies. If that is not the case, you will get sufficient insight into your students' mental structures by listening carefully as they respond to their physics problems or ask questions in your lectures. Please keep in mind that you are concerned less with whether their answers are right or wrong, and more with their procedures for finding it.

In addition to these specific approaches we urge you to become more aware of your own interaction with your students. Do you tell them all the "answers" and expect them to give these back to you on a test? (Not recommended) Do you reveal that you are sometimes unsure of how to proceed but use certain techniques for identifying and evaluating alternatives? (Recommended) Do you try to recognize the misconceptions that may block their understanding (e.g. treating energy as vector, not distinguishing the integrals over electric field in Gauss's law and the definition of potential)? (Recommended)

Discuss a few items on the above list with your partner and then list below some teaching techniques that you have used to further the reasoning patterns of your students.

2. Essay. Course Goals: Content or Reasoning

It would be much easier to teach students who already apply formal reasoning patterns in their physics studies than to teach students who need to experience self-regulation first. And yet, the instructor who intends to cover new material must expect to allow for self-regulation if he wishes the students to come to a good working understanding of the new ideas. How much time will be needed depends on the level of the course and preparation of the students. Less time will be needed in an advanced course whose students have formed some of the formal mental structures previously. More time will be needed in an introductory course whose students are less experienced and may include a small number with no formal mental structures at all.

In view of these considerations, we should like to rephrase the question in the title of this essay to "Course Goals: Content With or Without Reasoning?" The reasoning patterns are closely related to the subject matter you select. Usually physics teachers have defined course goals exclusively according to the major topics covered, with a great deal of freedom for the individual instructor as regards emphasis and elaboration of details. Now you have to consider including goals related to your students' reasoning. Are these compatible with all the content goals? Are the topics in your course sequenced in order of increasing use of formal mental structures? Is there sufficient opportunity for concrete experience in the laboratory? Are there provisions for making students aware of their own reasoning so that they can initiate self-regulation?

3. Discussion

Please join with a group of participants and workshop staff to discuss some of the following questions. On the next page we have a recapitulation of the major points presented in this workshop for your quick reference.

1. Have you any indications of concrete reasoning patterns used by students in your courses? Describe some of your observations.

2. Do you feel a need to make the development of reasoning, as described in this workshop, an important course goal to which you will subordinate some other goals? If so, what kinds of changes will you make? How could you tell your students about this goal?

3. What possibilities are there within your courses for helping your students build formal mental structures?

4. What contributions can the traditional physics lectures make to self-regulation and the building of formal mental structures?

5. What contributions can the physics laboratory in your course make to self-regulation and the building of formal mental structures?

6. What contributions can discussion sections or office hours in your course make to self-regulation and the building of formal mental structures?

7. How might new course formats, such as Keller plan or Audio-Tutorial be particularly appropriate for stimulating self-regulation and building formal mental structures?

4. Recapitulation of Major Ideas

1. Piaget's theory describes two stages of logical reasoning in human intellectual development, the stage of concrete thought and the stage of formal thought. Earlier stages identifiable in the behavior of very young children may be called pre-logical.

2. Each of the two stages is characterized by certain reasoning patterns that reflect the mental structures used by the individual to classify observations, interpret data, draw conclusions, and make predictions.

3. The two stages are idealizations, in that most persons after age twelve use formal reasoning patterns under some conditions and concrete reasoning patterns under others. The latter is likely to occur whenever the subject matter is unfamiliar, as is the case for a student beginning work in a new academic discipline. The former is likely to be the case for an experienced worker in the academic discipline.

4. The process whereby an individual advances from the use of concrete reasoning patterns in an area of knowledge to the use of formal reasoning patterns is called self-regulation. Self-regulation begins with one's awareness that the concrete reasoning patterns are inadequate and proceeds through direct experience with the phenomena supplemented by the introduction of the related organizing principles and major concepts.

5. A person who has only concrete mental structures is likely to proceed through self-regulation in a new subject much more slowly than a person who has developed some formal mental structures in connection with other studies. The latter person benefits from the possibility of transferring the formal mental structures to the new area, especially if the new and old are closely related as is the case with mathematics and certain topics in physics.

6. Some students who are required to learn formal-level material in a subject in which they have so far developed only concrete mental structures -- or possibly no mental structures at all -- may draw on their own experience in related areas and their awareness of their own learning problems to go through self-regulation spontaneously. Other students, with less experience or self-awareness, are not likely to experience self-regulation; instead, they will memorize certain prominent formulas and procedures, but will apply these unreliably.

Module 11 Suggested Reading

Introduction

This module contains reprints of several articles related to the ideas of stages of development and self-regulation and a bibliography of books and articles that you may wish to study after you complete the workshop.

Objectives

To provide you with examples of applications of the instructional techniques that you were introduced to in the workshop, and to make available a bibliography that you can use for further study.

Procedure

If you would like further background information on Piaget's theory as related to physics instruction, read one or more of the three reprints selected from AJP and TPT that are included in the instructional materials for this module. If you would like additional information on Piaget's theory in general, read the article by Piaget reprinted here or consult the books and articles listed in the bibliography -- most are available in paperback and many can be obtained in any college or university bookstore.

INSTRUCTIONAL MATERIALS

This module contains the following materials:

1. Reading list of suggested books and articles.

2. Joe W. McKinnon and John W. Renner, "Are Colleges Concerned with Intellectual Development?" American Journal of Physics 39, 1047 (1971).

3. John W. Renner and Anton E. Lawson, "Piagetian Theory and Instruction in Physics," Physics Teacher 11, 165 (1973).

4. John W. Renner and Anton E. Lawson, "Promoting Intellectual Development Through Science Teaching," Physics Teacher 11, 273 (1973).

5. Jean Piaget, Journal of Research in Science Teaching Vol. 2, pp. 176-186 (1964).

6. Anton E. Lawson and Warren T. Wolman, "Physics Problems and the Process of Self-Regulation" The Physics Teacher 13, 465 (1975).

Module 11 Instructional Materials

Books

1. Anderson, DeVito, Dyrli, Kellog, Kochendorfer and Weigand, Developing Children's Thinking Through Science, Prentice-Hall, N.J. 1970.

2. Ruth M. Beard, An Outline of Piaget's Developmental Psychology for Students and Teachers, Basic Books, Inc., N.Y. 1969.

3. David Elkind, Children and Adolescence, Interpretive Essays on Jean Piaget, Oxford Univ. Press.

4. Richard I. Evans, Jean Piaget: The Man and His Ideas, E.P. Dutton, Co. N.Y. 1973.

5. Hans G. Furth, Piaget for Teachers, Prentice-Hall, Inc., Englewood Cliffs, N.J. 1970.

6. Herbert Ginsburg and Sylvia Opper, Piaget's Theory of Intellectual Development, Prentice-Hall, Inc., Englewood Cliffs, N.J. 1969.

7. Richard M. Gorman, Discovering Piaget, Charles E. Merrill Publishing Co., Columbus, Ohio, 1972.

8. Bärbel Inhelder and Jean Piaget, The Growth of Logical Thinking from Childhood to Adolescence, Basic Books, N.Y. 1961 (There is a paperback classroom edition of this book)

9. John L. Phillips, Jr., The Origins of Intellect: Piaget's Theory, W. H. Freeman and Co., San Francisco, 1969.

10. Jean Piaget, Genetic Epistemology, W.W. Norton & Co., New York, 1970.

11. Jean Piaget, The Psychology of Intelligence, Littlefield, Adams, & Co., Paterson, N.J. 1968.

12. Jean Piaget, Six Psychological Studies, Vintage Books, Random House, N.Y. 1967.

13. Jean Piaget, To Understand is To Invent, Grossman Publishers, N.Y. 1973.

14. John W. Renner, Robert F. Bibens, and Gene G. Sheperd, Teaching Science in the Secondary School, Harper and Row, N.Y. 1974, Chapter 4.

15. M. F. Rosskopf, L. P. Steffe, and S. Taback, Eds., Piagetian Cognitive-Development Research and Mathematical Education. Reston, Va.: National Council of Teachers of Mathematics, 1971.

Selected Articles

1. Entire issue, Journal of Research in Science Teaching, Vol. 2, 1964, (Articles by Piaget, Karplus, Ausubel and Duckworth).

2. Arnold B. Arons, "Anatomy of an Introductory Course in Physical Science," Journal of College Science Teaching, April 1972.

3. Arnold B. Arons, "Toward Wider Public Understanding of Science," American Journal of Physics, 41, 769 (1973).

4. Arnold B. Arons and John Smith, "Definition of Intellectual Objectives in a Physical Science Course for Preservice Elementary Teachers," Science Education, 58, 3, pp. 391-400, 1974.

5. B. S. Craig, "The Philosophy of Piaget and its Usefulness to Teachers of Chemistry," J. Chem Ed., Dec. 1972, 807-809.

6. David Elkind, "Piaget and Science Education." Science and Children, Nov. 1972.

7. Elizabeth F. Karplus and Robert Karplus, "Intellectual Development Beyond Elementary School I: Deductive Logic," School Science and Mathematics, LXX, 5 (May, 1970) pp. 398-406.

8. Robert Karplus and Rita Peterson, "Intellectual Development Beyond Elementary School II: Ratio, a Survey," School Science and Mathematics, 70, 9 (December, 1970), pp. 813-820.

9. Edward G. Palmer, "Accelerating the Child's Cognitive Attainments Through the Inducement of Cognitive Conflict: An interpretation of the Piagetian Position." Journal of Research in Science Teaching, 3, 318-325 (1965).

10. Jean Piaget, "Intellectual Evolution from Adolescence to Adulthood," Human Development, 15, 1 (1972).

Tenn.) cost approx. $2000. An alternative choice might be to purchase a PDP/8E with high-speed paper tape reader (total cost approx. $11 000), or equivalent from another manufacturer.

[8] *Teaching Computing in Universities* (Her Majesty's Stationery Office, London, 1970); Phys. Bull. **21**, 482 (1970).

[9] A survey of four computer dictionaries gave no definition for minicomputers. From *The New York Times*, 5 April 1970, Sec. 3, p. 1:

> *Maxi Computers Face Mini Conflict*, by William D. Smith.
>
> Mini vs Maxi, the reigning issue in the glamorous world of fashion, is strangely enough also a major point of contention in the definitely unsexy realm of computers.
>
> The definition of a minicomputer depends on to whom you are speaking. Descriptions range from electronic calculators to the IBM System 3 that sells for $42 000.
>
> A consensus opinion would probably include as minicomputers machines that cost less than $25 000 and that include some type of input–output device such as a teleprinter, a memory of about 4000 words, and circuitry capable of performing calculations under the control of stored programs written in some form of higher-level computer language such as FORTRAN or BASIC.
>
> The major manufacturer of minicomputers is the Digital Equipment Corporation. Other major makers include the Hewlett–Packard Corporation, the Data General Corporation, Varian Associates, Honeywell, Computer Automation, Inc., Motorola, the Raytheon Corporation and Mini-Computer Systems, Inc.

Are Colleges Concerned with Intellectual Development?

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The assumption is often made by college professors that incoming freshman students think logically. Using tests designed by the Swiss psychologist Jean Piaget to evaluate logical thought processes, the authors found that 66 of 131 freshmen exhibited characteristics of the concrete operational thinker, while another 32 did not meet the criteria for formal operations. Professors further compound the problem by failing to recognize the kinds of experiences incoming freshmen students must have to move toward more logical thought. McKinnon, using a newly developed inquiry-oriented science course based upon Piagetian criteria, found a highly significant difference between those students who were exposed to the course and like students who were not. The authors concluded that secondary and elementary teachers do not take advantage of inquiry-oriented techniques so necessary to the development of logical thought because college professors do not provide examples of inquiry-oriented teaching.

INTRODUCTION

Are colleges and universities making inadequate evaluations of student ability to think logically? Is the unrest today in many universities caused by student evaluation of problems based upon emotion rather than logic? Do student claims that curriculums are irrelevant, trivial, and inadequate in terms of the magnitude of the problems facing mankind today have substance, or are these students unable to evaluate logically the structure and necessity of those curricula? These questions, together with suspicions voiced by various professors of science about the inability of their freshman students to think logically about the simplest kind of problems, led the authors to question whether or not most college freshmen do think logically. This doubt about the ability of the entering freshman to think logically led to the following hypothesis: The majority of entering college freshmen do not come to college with adequate skills to argue logically about the importance of a given principle when the context in which it is used is slightly altered.

Since these students have been accepted by boards of admission that based their decisions upon high school transcripts and various established entrance examinations such as the American

TABLE I. A comparison of operational level of 131 students on Piagetian data.

	Male	Female	Total number	Per cent
Formal	25	8	33	25
Post-concrete	12	20	32	25
Concrete	16	50	66	50
Mean Piagetian score	12.82	9.45	Average 10.74	

College Test (ACT) and the Scholastic Aptitude Test (SAT), a different means of evaluation was sought. The evaluative system used is one based upon the ability of the student to think critically about problems, the answers to which would be found in his experiential background and could not be derived from memorized data.

WHEN DO STUDENTS BEGIN TO THINK LOGICALLY?

The scheme of evaluation of the ability to think logically which was used has been developed and verified by a Swiss psychologist, Jean Piaget, during many years' research with children. There is, however, no indication that his work has been extended to include entering college students, particularly American students. In addition, no work can be found with American children which verifies his conclusions that children begin to think logically between ages 11–15.

Piaget[1] found that children progress through various stages of mental manipulation and that these steps cannot be circumvented. Prior to thinking about abstract ideas, a student must undergo a period of physical manipulation of objects using the basic principles upon which the abstraction to be developed depends. This stage Piaget identifies as the *concrete stage* of thought. A student may handle concepts quite adequately, but until he has had many manipulative experiences he cannot recognize those concepts in the context of a broader generalization, of which the manipulative experiences and the concepts are simply a subset. Inhelder and Piaget[2] found that from 11–15 years of age most Swiss children should become *formal operational*, i.e., capable of abstract logical thought. The concern of this research was whether or not this was true for American college freshmen, i.e., had those students become formal operational?

A STUDY OF THE ABILITY OF COLLEGE FRESHMEN TO THINK LOGICALLY

McKinnon[3] studied responses to tasks given 131 members of the freshman class at an Oklahoma university in which students had to think logically about problems of volume conservation, reciprocal implication of two factors, the elimination of a contradiction, the separation of several variables, and the exclusion of irrelevant variables from those relevant to problem solutions. These tasks had initially been developed by Inhelder and Piaget[2] for determining the patterns of thought of children and the ages at which changes in those thought patterns occur.

Table I presents the test results for these 131 students using the foregoing tasks and the criteria specified by Inhelder and Piaget for demonstrating formal operational thought. Each student was graded from 0 through 4 on each of the tasks. Should a student score a total of 14 or more points on the five tasks, he was judged as definitely being at the formal operational stage. To achieve 14 points, he had to score at least 3 points on the

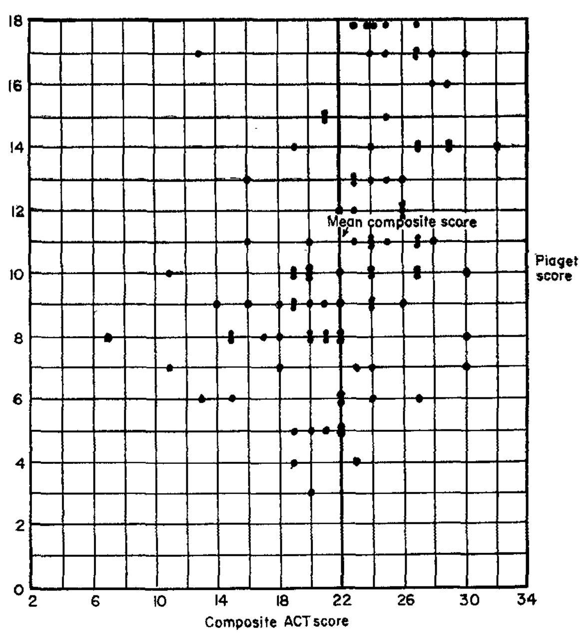

FIG. 1. A comparison of ACT score versus Piagetian score for 94 freshman students.

tasks for which 4 points were possible. If a student scored an average of 2 points or less on each of the five tasks, he was judged to be at the concrete stage of operations. Those students who scored more than 10 but less than 14 points were judged to be moving from the concrete stage to the formal stage of thought.

The findings, as shown in Table I, are that 50% of the entering college students tested were operating completely at Piaget's concrete level of thought and another 25% had not fully attained the established criteria for formal thought. The average score for all students was 10.74, with the males scoring significantly higher than females. An examination of the performance of the students on the various tasks used follows:

1. Of the college freshmen tested, 17% of them did not conserve quantity (the result of a change of form), while another 10% failed to recognize equivalence of volume. Thus, 27% of those students tested were at the lowest concrete operational state or less.
2. Reciprocal implication involved the student in the problem of reflecting a ball and the necessity to relate incident and reflected angles. This task was second only to the problem of density in the number of failures recorded—64% scored 2 or less.
3. The elimination of a contradiction involved the student in relating weight and volume of floating and sinking objects in a meaningful way. More than $\frac{1}{3}$ of those tested did not relate weight and volume. Typically, they recognized weight only. Seldom was there a proportionality expressed; 67% of the students tested on this task were concrete operational.
4. The separation of variables task gave evidence that 50% of entering college freshmen could not recognize the action of a potential variable and find a way to prove the action of that variable.
5. The task of excluding irrelevant variables showed that 33% of the students tested could not eliminate variables of no consequence in a swinging pendulum, while another 18% could do no more than order the effects of weight.

In the research, a comparison was made of the score obtained by each student on the various Piagetian tasks given him and this score was correlated with his ACT composite score. (See Fig. 1.) A graph of these two scores shows that Pearson product–moment correlations were high for those students scoring at the average ACT composite of 22 or better, but correlations of -0.05 were found for students scoring less than that average. The university where this study was made ranks high in terms of the average ACT scores when compared with all other colleges and universities in Oklahoma[4] and is well above average for all regions of the United States.[5] Almost 75% of that university's entering freshmen, however, were either partially or completely concrete operational. What evidence exists, therefore, to demonstrate that logical thought can be promoted among all levels of students?

CAN INQUIRY-ORIENTED COURSES PROMOTE LOGICAL THOUGHT?

The University of Oklahoma Science Education Center has, for some time, been investigating the effects of inquiry-oriented teaching upon both teachers and pupils. Various new courses in science which utilize the inquiry approach have been evaluated. Porterfield[6] compared teachers of reading who had inquiry educational experiences in science with those who had not. He found that the former tended to use more questions requiring analysis and synthesis and other high-level cognitive thought patterns than did the latter group. Wilson[7] found much the same in a study of 30 classes of elementary children when fifteen of the teachers had been exposed to inquiry experiences in science and fifteen had not. Schmidt[8] found similar results by investigating the teaching in social studies done by teachers who had and had not been involved with inquiry in science. Friot[9] found in a study of seventh, eighth, and ninth grade science that courses placing emphasis upon the inquiry approach allowed students to be able to function at a much higher level of logical thought than those courses in which students did not have that inquiry experience.

Stafford used the development of conservation reasoning in children as an evaluative tool to determine whether or not inquiry-oriented science experiences move first graders toward the acquisition of concrete operational thought. The specific unit he used was *Material Objects*.[10] Stafford found: "... those first grade children who have experiences with the unit achieved the ability to

TABLE II. A comparison of the growth in logical thought processes of the experimental and control groups.

Group	Stage	Pre-test Females	Pre-test Males	Post-test Females	Post-test Males	Net gain Females	Net gain Males	Total
Experimental	Formal	4	11	14	16	10	5	15
	Post-concrete	14	6	17	8	3	2	5
	Concrete	24	10	11	3	−13	−7	−20
Control	Formal	4	14	7	17	3	3	6
	Post-concrete	6	6	11	7	5	1	6
	Concrete	26	6	18	2	−8	−4	−12

conserve much more rapidly than did those children who did not have these experiences."[11] *Material Objects* is an inquiry-centered unit and Stafford concluded: "... children so taught do show more rapid intellectual development than do those children not having such experiences."[11]

Finally, McKinnon,[12] in a study of the effect of an inquiry-centered science course on entry into the formal operational stage of concrete operational freshman college students, found a highly significant difference between those students enrolled in the course and a like group who had not been exposed to the course.

The data of Table I gave evidence of the ability of students to think logically. The data of Table II show the effect of the inquiry-centered course upon freshman students' ability to think logically. A net gain in favor of the experimental group resulted in 15 students moving into the formal stage of thought—compared with six for the control group. The post-concrete gain was, respectively, five and six, with the experimental group showing a net movement of 20 out of this category compared with 12 for the control group, a net gain of more than 50% for the group exposed to the influence of the new science course. The material of the science course did not include references to the tasks which were part of the test instruments; therefore, changes in ability to think logically were caused by added opportunities for inquiry. Another comparison in terms of the mean Piagetian scores for the two groups is shown in Table III.

After obtaining individual pre-test–post-test differences and summing them up for each group, an F ratio of 6.24 was obtained. This value is significant in favor of the test group at the 0.001 level of confidence; therefore, the hypothesis must be accepted that a properly designed course in science for freshman college students does enhance their logical thought patterns by increasing their ability to hypothesize, verify, restructure, synthesize, and predict.

The preceding research gives evidence that students do not think logically. However, research carried out on newly developed courses does give evidence that the logical thought processes can be enhanced. Therefore, who is at fault and what steps must be taken to alleviate the situation?

AN EVALUATION OF EDUCATIONAL RESPONSIBILITY USING THE INQUIRY APPROACH

If students do not think logically when they enter college, who has not discharged his responsibility? The immediate answer to the foregoing question is, the high school. That answer, however, needs to be examined.

Piaget states formal operations begin to emerge around 11 years of age But Friot[9] found that 82% of eighth and ninth grade children (ages 13 and 14 years) were still concrete operational. Thus, children probably enter senior high school two to three years behind the age set by Piaget for

TABLE III. Pre-test and post-test Piagetian mean scores for both experimental and control groups.

Group	Experimental n	Experimental Piaget score	Control n	Control Piaget score
Pre-test	69	10.77	62	10.81
Post-test	69	12.32	62	11.14

entering into formal operation. While some of this age difference might be attributed to differences in the samples of Piaget and Friot, the entire 82% cannot be. The answer to the question of who is responsible for the lag in intellectual development seems to be the elementary school. But that answer, too, needs to be examined.

Begin that examination with another question. Who is teaching in the elementary and secondary schools? Teachers who have been educated in the existing colleges and universities. Those teachers have been subjected to four years of mainly *listening* experience. They have been lectured to, told to verify, given answers, and told how to teach. Lest you think the foregoing happens entirely in the colleges and/or departments of education, remind yourself that *all the content taken by a teacher* (which represents a substantially greater number of credit hours than do courses in education) *is taken in other colleges and/or departments.* Teachers are, in other words, not having the kinds of experiences with inquiry which Piaget says they must have in order to allow logical thought processes to develop. Future teachers are not having learning experiences in college which will permit them to learn the value of inquiry in educating a child. The foregoing rather dogmatic statement was substantiated by Gruber[13] when he found that only 25% of those attending NSF Institutes showed interest in inquiry-oriented science teaching, while Torrance[14] found that only 1.4% of elementary and 8.4% of secondary social studies teachers listed independent and critical thinking as important educational objectives. These statistics suggest that pre-college teachers place little value upon logical thought as an outcome of 12 years of schooling. Considering the paucity of research on implementation of logical thought as an educational objective, these educators' values will not change. The responsibility, then, for the small percentage of high school students attaining formal operations rests in part at the door of the institutions of higher education. They have assumed that their role is to tell. Future teachers, therefore, assume that telling is teaching and when they get their first class, they tell, tell, tell! All the while, very little, if any, intellectual development is going on. If, then, a college student develops logical thought, such development is more by accident than design. One of the criteria Piaget cites for intellectual development is that of social transmission. Just possibly more intellectual development goes on in dorms, fraternities, sororities, and student hangouts than in the classroom because social transmission occurs in these places and little occurs in classes. To test our assertions, walk down the hall of any building on any campus and stop outside any classroom door and listen to who is talking. In most instances only information is being transmitted by the instructor.

Stafford and Renner[11] hypothesized that "... specialized educational experiences in inquiry-centered science teaching encourage a teacher to become sensitive to children, functionally aware of the purposes of education, and equipped to lead children to learn how to learn in all subject areas." The importance of this hypothesis is in the phrase "... all subject areas.", for inquiry methodology is not only the province of science, but all the other disciplines as well. Unfortunately, few other teaching areas have recognized the importance of the inquiry approach.

With the exception of a few new courses in the social science areas, most educators have chosen to ignore the lead taken by science and mathematics in devising new courses from kindergarten through the 12th grade. In many cases, the colleges have failed to use inquiry even when teaching the new curricula. This point was well illustrated by Gruber. Therefore, the blame must, in the last analysis, be placed, at least partially, upon the shoulders of those who teach at the college level and who insist upon ignoring the rapidly accumulating evidence in favor of the inquiry approach.

Renner and Stafford also pointed to the necessity of the teacher becoming "... functionally aware of the purposes of education ..." which in far too many cases they are not now. Unless teachers are aware of the primary purpose of education being the development of the learner's intellectual ability, they will not pursue teaching by giving the student opportunities for exploration using all his senses. Rather, they will continue to teach students what the teacher wants them to know and not what the students want to learn.

Finally, the total accumulation of research to date leads to the following hypotheses: (1) The secondary educational experience does not now

promote logical thinking in most students. (2) An abundance of inquiry-oriented courses taught by teachers who are products of college and university professors who practice and profess inquiry must come into being in the secondary schools before an alternative to the first hypothesis can be accepted. Those experiences will have to be developed by many colleges.

Those hypotheses have profound educational implications since a serious problem has been shown to exist and the means for its alleviation have also been shown to be available to the profession. If colleges and universities do not try to solve the problem by assuming the responsibility for the intellectual development of their students, but continue to look at their primary purpose as the transmission of information about the several disciplines, the elementary and secondary schools will continue to fail in their mission of truly educating students. The needed changes, however, can come only through acceptance of inquiry by *all* of those who teach the teachers.

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Radiation Field of a Charge Moving on a Straight Line

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A derivation of the radiation field of a charge accelerating on a straight line is presented that makes use of Gauss' law in a direct manner and does not make use of the concept of lines of force.

We derive the radiation field of an accelerating point charge from the following assumptions: (1) electric effects are transmitted with the velocity c; (2) Gauss' law holds good in all inertial frames of reference; (3) the electric field of a charge moving uniformly is known.

These are the assumptions made by J. R. Tessman and J. T. Finnel[1] to derive the radiation field of a point charge moving on a straight line. However, we shall not make use of the concept of lines of force, and Gauss' law shall be used in a most direct way.

Consider the following kinematic sequence on a straight line of a particle with the charge q:

(a) The charge moves with constant velocity v_1 until $t=t_0$. At $t=t_0$ we designate its position by O.

(b) The charge moves with constant velocity v_2 thereafter. We only suppose that v_1, v_2 are less than c.

Piagetian Theory and Instruction in Physics

John W. Renner and Anton E. Lawson

Jean Piaget and his associates have been gathering data and formulating important theoretical observations about the intellectual development of children since 1927. Although it has taken American psychologists and educators a relatively long time to become acquainted with his work, it is becoming apparent that we can gain much by a careful evaluation of his efforts and their educational implications.

Numerous texts[1] have become available in recent years attempting to explain Piaget's theory and its educational significance. The primary purpose of this paper is similarly to explain his ideas, and further to expand a scheme of instruction and classroom procedures that arise as a consequence of that theory.[2] When possible these ideas will be put forth using examples in physics context in an effort to elucidate difficult ideas.

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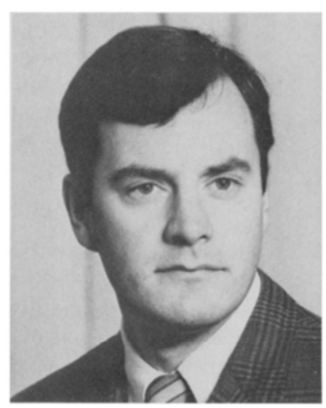

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Mental Structures

A central idea in Piaget's work and fundamental in understanding his theory is the concept of mental structure. It would be satisfying to be able to indicate the physiological and chemical nature of these structures, but at this point in the study of human mental functioning that is not possible.[3] Instead their existence in the brain is hypothesized from observable behavior; determination of their exact nature awaits further research. These hypothesized mental structures function to organize the environment so that the organism can function effectively. In this sense the construction of these structures carries adaptive value for the individual. An analogous situation is found in the genetic adaptation of evolving species. Basically, then, mental structures represent a more or less tightly organized mental system to guide behavior.

During development of the human infant to adulthood, these structures must be built within the brain. A complete developmental sequence of the structures is not genetically given to the child; they must be learned. According to Piaget, the building and rebuilding of these mental structures is what underlies the process of intellectual development. These structures control how and what we think and guide behavior. In other words, structures actually represent our knowledge.

Since science educators are deeply concerned with intellectual development and the building of mental structures about everything from the metric system to the theory of relativity, two questions need to be asked: (1) How are structures built? (2) Once the structure is built is it static or can it be altered?

These two questions are not mutually exclusive, and we will answer the second one first. Structures can be altered, and that may be a more than

adequate definition of education—the building and rebuild- of structures. The answer to the first question should then give us good insights into how learning takes place and how instruction should be planned.

The Building of Mental Structures – A Problem

An important point must be made before examining the process by which mental structures are formed according to Piaget. Structures do not come from simply making a mental record of the world by keeping eyes and ears open. Unfortunately, it would appear that many teachers subscribe to this view. Work done by Van Senden with congenitally blind persons provides an interesting example of this point.[4] These persons, who had gained sight after surgery, could not identify objects without handling them. They were unable to distinguish a key from a book, when both lay on a table. Also they were unable to report seeing any difference between a square and a circle. The important idea to note is this: Whether the task is to simply distinguish objects in the environment or complex relationships such as $F=ma$, acceleration, or velocity, the ability to develop the understandings requires much more than a simple photographing of the environment.

According to Piaget a person is unable to perceive things until his mind has a structure which enables its perception. Without the development of a mental structure things which seem obvious to an adult, such as the difference between a key and a book, a square and a circle, are simply not perceived by beginners. But this leads us to a fundamental problem. If learning is the building or rebuilding of mental structures, and if structures are needed in order to perceive and learn and are not derived from simply copying the external world, then where do they come from?

Plato's answer to this question was simple. The structures were innate and developed through the passage of time and the growth of the brain. Of course at the other end of the spectrum is the belief that these structures derived directly from the environment. This is the classical empiricist's view; but we have already seen that this view is untenable.

Piaget rejects the Platonic view, except to admit that certain very primary structures must be present at birth. Piaget's view is that the development of structures derives from a dynamic interaction of the organism and the environment which he calls equilibration.

The Building of Mental Structures – Equilibration

From birth, basic structures enable the child to begin interacting with his surroundings. As long as that interaction is successful the basic structures continue to guide behavior. However, owing to the child's inborn drive to interact with his environment he meets contradictions, i.e., things which do not fit his present mental structures. These contradictions produce a state of disequilibrium. In other words, his present mental structures are disrupted and must be replaced. Through continued investigation and guidance from others, the child alters or accommodates his disrupted mental structure. Once this is accomplished he is then able to assimilate the new situation. The new structure that is developed is then tried. If the structure guides behavior so that the child's efforts are rewarded (reinforced) the structure is also reinforced. In this manner the child builds new mental structures and adapts to new situations.

The above-described process underlies all development according to theory. The entire process of development of mental structures is viewed as a process of *equilibration* or self-regulation. This process results in the development of progressively more complex and useful mental structures.

The Building of Mental Structures – Contributing Factors

The role of three main factors, *experience, social transmission,* and *maturation* can be isolated in the process of equilibration. It is apparent that experience is a necessary part of learning. With no contact with the environment, no contradictions of present structures arise and no possibility for further exploration into the situation that produced the contradiction is possible.

There are basically two kinds of experience – physical, and logical–mathematical. This distinction is important because the different experiences lead to different kinds of mental structures.

Physical experience is exactly what the phrase connotes – actual physical action on the objects in the world. This physical experience leads to the development of structures about objects. At some point, however, the learner begins to see more in his interaction with the world than just objects. He sees that his actions with objects produce some kind of order themselves. An example of this is when a learner discovers that ten objects, when counted left to right provide the same result as when counted right to left. In other words, the action itself has properties. The learner now can make the generalization that the sum of any set of objects is independent of their order. Now the student has a mental structure that he can utilize in many situations and that is a logical–mathematical structure. The structures then enable the learner to operate logically within his environment. The basic behavioral patterns directed by the mental structure are called operations. In the early structure-building stages the opportunity for the learner to interact with concrete material is mandatory.

Piaget has not projected to what academic level the necessity for interaction with material exists; he says, "...coordination of actions before the stage of operations needs to be supported by concrete material."[5] A literal interpretation of that statement would be that, regardless of age, the student must have materials to perform actions with until he can begin to utilize logical–mathematical operations. Our research with kindergarten and elementary school children,[6] junior high school students,[7] and college freshmen,[8] all studying science, supports our interpretation of the foregoing quotation.

The factor of experience, then, helps students to build operational-structures which can ultimately lead them to think abstractly about the world around them. In other words, it is experience with the materials of the discipline that produces the person who can understand abstract content and *not* studying abstract content which produces students who can interact with the materials and invent abstract generalizations. This says to science teachers that the laboratory *must precede* the introduction of an abstract generalization.

Fig. 1. *Jean Piaget. Photograph by the Science Curriculum Improvement Study.*

Piaget's second factor, *social transmission,* also provides a basis for structure building. The very young child – and some not so young – operate from a very egocentric frame of reference. He cannot see things objectively because he always looks at them as related to himself. Such a thinker cannot objectively view and/or evaluate anything. In order to shake the learner from an egocentric view of anything, he must experience the viewpoints and thoughts of others. He must, in other words, interact with other people. If he does not, he has no reason to alter the mental structures which he gained from an egocentric frame of reference. Social interaction can lead to conflict, debate, shared data, and the clear delineation and expression of ideas. All of these require that the student carefully examine his present beliefs which will, according to the Piagetian model, develop and change structures. In order to have all of this happen, however, students must be encouraged to talk with each other and their teachers. Data from an experiment must be shared, discussed, retaken, and rediscussed. Students, "...should converse, share experience, and argue."[9] The factor of social interaction is valuable in building and rebuilding structures, but it is insufficient because the learner can receive valuable information via language or via education directed by an adult only if he is in a state where he can understand this information. That is, to receive this information he must have a set of experiences that enables him to assimilate this information.

Maturation, the third factor, must also be considered. Evidence indicates that these structures require time to develop. Old structures cannot be accommodated to new experiences all at once. The process of development is slow, as any teacher can attest.

Perhaps this personal example will help clarify how these three factors interact in the process of equilibration to change structures. Our first contact with $V=IR$ was a rather traumatic experience. We vaguely understood that it involved the conservation of energy, but concentrated upon memorizing what the symbols meant and how to juggle the formula. In short, an advanced state of disequilibrium was our lot! When meter readings were substituted for the very abstract terms of potential difference and current, the symbols began to have meaning, and after a good deal of thinking equilibrium was achieved. Then a series circuit with one source and more than one resistor and parallel circuit was introduced. The notion that in a series circuit the total potential difference, V_t, of the source equaled the sum of all voltage drops, V_i, $i=1,2,3,...n$, around the circuit brought on another disequilibration. Once again meter reading (objects) were salvation; we began to really understand that

$$V_t = \sum_{i=1}^{n} V_i,\ i=1,2,3,\ldots,n,$$

really was a conservation of energy statement. Now $V=IR$ was a concept which was available for use and once again equilibrium was achieved. Parallel circuits presented no problem and Kirchhoff's laws were nearly obvious.

This example demonstrates that the science laboratory clearly has a place in promoting equilibration and disequilibration. Data from an experiment can be very threatening, because they too often produce disequilibrium. But to the sensitive, concerned science teacher, disequilibrium is an opportunity; he can now introduce the student to the major conceptualizations of the discipline which will produce equilibrium. This sequence of events suggests that perhaps the principal role of the teacher is to promote disequilibrium and equilibrium, because through the process of equilibration structures are built and rebuilt. Equilibration proceeds through experience with the materials worked with and the social interaction of those around us.

The Learning Cycle

An instructional technique incorporating much of Piagetian theory has been developed and refined by the Science Curriculum Improvement Study, University of California,

Berkeley. Their procedure is basically a three-phase process: (1) exploration, (2) invention, and (3) discovery.

Exploration involves the students in concrete experience with materials. As a consequence of these initial explorations, which sometimes may be highly structured by the teacher or on other occasions relatively free, the learner encounters new information which does not fit his existing structures. This produces disequilibrium. At the appropriate time, determined by the teacher, he suggests a way of ordering the experiences. In essence, the teacher invents a new structure which often involves a new concept. This phase, termed *invention,* is analogous to Piaget's structure building and promotes a new state of understanding or equilibrium. The question now is: Can the new situation be applied in other situations? During phase three, *discovery,* further application of the inventions are discovered by the students. Discovery experiences serve to reinforce, refine, and enlarge the content of the invention.[10]

Again an example from physics may help to clarify these points. Experience in the laboratory with voltage and resistance, seeing the effect these have on current, and recording all these data is exploration. These exploratory experiences, if provided at the appropriate time, will promote disequilibrium and lead students to question relationships. Since it would take a brilliant student to invent the notion that $V=IR$, the formal statement of that relationship is left up to the teacher. The teacher, having explained the relationship, has in effect provided a way of ordering the student's experience. This is invention. Now the student is in a position to make discovery with this new concept. He can apply it to various types of circuits, magnitudes of voltage, current, and resistance, practically any type of situation he can design. That is the true notion of discovery. Exploration, invention, and discovery are the three phases of the learning cycle and represent a process which will lead the learner to move from physical action to abstract mental operations. Science in general – and in our opinion physics in particular – has a unique opportunity to lead students to build structures. Are we utilizing it? There is much evidence to suggest we are not.[11]

Levels of Thinking

Piaget's theory has gone further than describing how mental structures are formed. He has outlined the basic structures that dictate behavior from birth to adulthood. The structures fall roughly into four categories. Each category or stage incorporates and adds to the structure of the previous stages. If Piaget is correct, it becomes imperative for educators to understand these stages of development. They provide a possible key for adapting instruction to the learner's capabilities. They further suggest types of activities which could promote intellectual development.

The child at birth is in a state Piaget calls *sensory–motor.* During this period, which lasts until about 18 months, the child acquires such practical knowledge as the fact that objects are permanent. The name of the second stage describes the characteristics of the child – *preoperational,* the stage of intellectual development before mental operations appear. In this stage, which persists until around seven years of age, the child does not, for example, reverse his thinking; he exhibits extreme egocentricism, centers his attention upon a particular aspect of a given object, event, or situation, reasons transductively, and does not demonstrate conservation[12] reasoning. In other words, the child's thinking is very rigid.

At about seven years of age the thinking stages of children begin to "thaw out" – they show less rigidity. The stage the child has entered is called *concrete operational.* Those structures which permit the reversal of thinking *et al.*, which are denied a pre-operational thinker, begin to show themselves as the child moves more and more deeply into the concrete operational stage. The child can now perform what Piaget calls mental experiments – he can assimilate data from a concrete experience and arrange and rearrange them in his head. In other words, the concrete operational child has a much greater mobility of thought than when he was younger.

The name of this stage of development – concrete operational – is representative of the type of thinking of this type of learner. As Piaget explains this stage: "The operations involved...are called 'concrete' because they related directly to objects and not yet to verbally stated hypotheses."[13] In other words, the mental operations performed at this stage are "object bound" – operations are tied to objects. This point must be firmly entrenched in the minds of teachers, because when working with students who are moving through this stage they must focus their teaching on the object – the actuality – and not on the abstract. Density, for example, is an abstraction – lenses are concrete.

As the child begins to emerge from the concrete operational stage of thought, according to the Piagetian model, he enters the last stage called *formal operational.* According to Piaget, this occurs between 11 and 15 years of age. A person who has entered that stage of formal thought "...is an individual who thinks beyond the present and forms theories about everything, delighting especially in considerations of that which is not."[14] Formal operational thought is capable of reasoning with propositions only and has no need for objects. It should be pointed out, however, that for this type of thought to occur it must be developed through the use of objects. For that reason this type of thought can be described as propositional logic. An analysis of formal operations reveals that they "...consist, essentially of 'implication'... and 'contradiction' established between propositions which themselves express classifications, seriatations, etc."[15] The formal thinker can form hypotheses and test them. To do this, he must isolate and control variables and exclude irrelevant ones. This type of thought can truly be described as abstract.

The maximum educational gain that comes from the study of science is derived from the isolation and investigation of a problem. Quite obviously this involves the formulation and stating of hypotheses and using a form of thinking which can be described as, if..., then,..., therefore. That is, of course, propositional logic. In other words, science teaching should promote formal thought. But it cannot do so if concrete operational thinkers are asked to interact with science on a formal operational level and their teacher teaches them as though they think formally. Concrete operational learners must interact with science at that level; they *cannot* do otherwise. Only then will they build the struc-

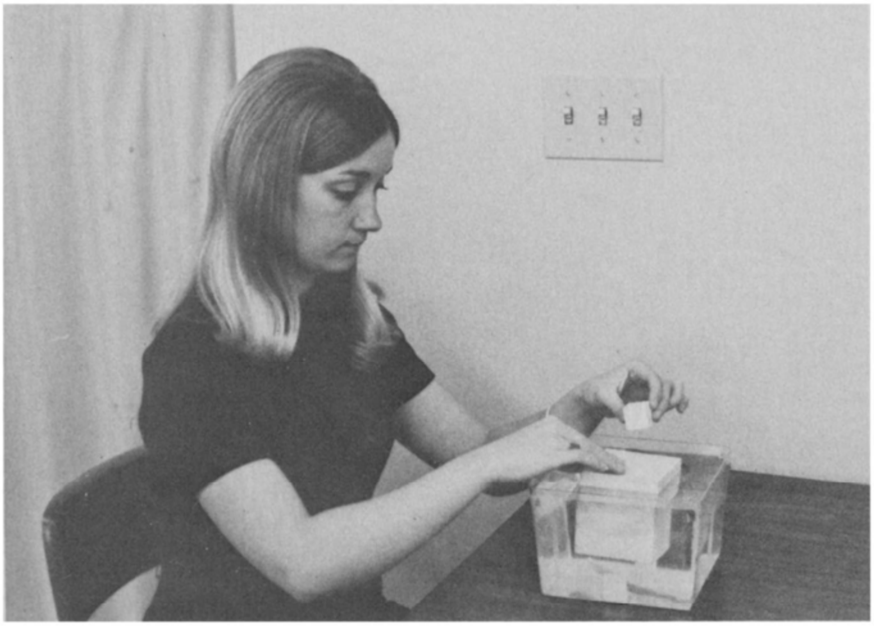

Fig. 2. *Eliminating the contradiction in the sinking-floating problem is useful to identify formal thinking.*

tures that promote their intellectual development toward formal thought.

Where are today's science students in the development of formal thought? If the programs of study available for high school physics are examined, for example, the fact that they require the use of abstract thinking is immediately apparent. The same can be said for most of the new curriculum developments in science. As Kohlberg and Gilligan recently said: "Clearly the new curricula assumed formal operational thought rather than attempting to develop it."[16] Is such a statement justified? Can science taught at the pre-collegiate and college levels promote formal thought? What can teachers do, if anything, as they select and arrange curricula and interact with students to promote formal thought? A later article in this journal will address itself to those questions.

[The second part of this article will appear in the May issue of *The Physics Teacher.*]

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Promoting Intellectual Development Through Science Teaching

John W. Renner and Anton E. Lawson

The previous article in this series, ["Piagetian Theory and Instruction in Physics," Phys. Teach. **11**, 165 (1973)] discussed the process of intellectual development and the intellectual level concepts of Jean Piaget and briefly commented upon the relation of those ideas to teaching and learning physics. The purpose of this article is to comment upon the thought patterns of secondary school and first-year college students and to suggest types of experiences students need to have to enable them to move toward acquiring formal thought.

We start with the assumption that all students deserve the opportunity to develop the capacity to think with the "If..., then..., therefore..." form – in other words, to develop formal thought. Three questions immediately arise:

(1) What type(s) of thought do secondary school and first year college students use?

(2) How can the student's level-of-thought be assessed?

(3) What can educational institutions do to change the type(s) of thinking students do?

Levels of Thought, Students, and Content

If you reflect back to the first article we prepared on the topic of learning, you will recall that we pointed out that learners begin to leave the pre-operational stage at around seven years of age. At this point, they enter the concrete operational stage of thought and, according to Piaget, move more and more deeply into that stage until somewhere between 11 and 15 years of age. That is the time when they begin to move into the last stage of intellectual development – formal operational thought.

Now the transition from concrete to formal thought is of the utmost importance to teachers who work with students in grades 10-12 in the secondary schools and in their first years of college. *If* students have achieved the ability to think formally, the teacher can proceed to lead them to deal in the great abstractions of science because they can think with form, "if..., then..., therefore...," or propositional logic. These teachers need not be as concerned with providing students direct experience with the materials of the discipline as those teaching concrete operational thinkers. But if students are concrete operational, they cannot think with propositional logic and *all* they learn will come from interacting with the materials of the discipline. These statements carry with them serious implications for science teaching, indeed for all types of teaching which deal with abstractions. Therefore, the validity of these statements must be carefully evaluated. At this particular time such an evaluation has not been carried out to any satisfactory extent. However, to any teacher who has had the experience of having his students simply not comprehend what to him seemed eminently clear, Piaget's hypothesis becomes extremely compelling.

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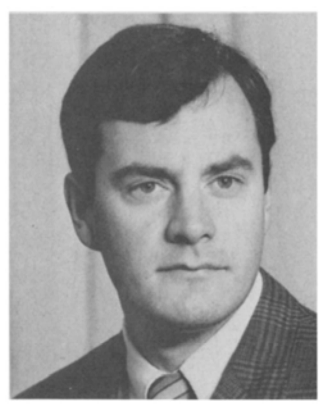

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Basically one can grasp why Piaget asserts that "if..., then..., therefore..." thinking is required to understand abstract concepts if you understand the nature of the abstract concepts themselves. The abstractions in physics, as well as in biology and chemistry, are in actuality models created by scientists to explain observable data. These models do not arise directly from the observations; rather, they simply represent attempts to construct an explanation or model which implies what is observed. The scientist creates the model (we do not know how) and reasons *if* his model is true, *then* consequences should be found. If the predicted consequences are indeed found, he has *therefore* supported his model. The process is hypothetico-deductive or in the if..., then..., therefore... form. For a student to fully grasp the meaning of the abstract models he, too, must be able to think in the if..., then..., therefore... form. The inertia principle, for example, has to be deduced and verified from its implied consequences. Strictly speaking, it does not give rise to observable empirical evidence.

Consider Newton's second law, $F = ma$. That law is always stated (and properly so) in terms of the mass of a body. Now mass is not a concrete concept – it is an abstraction. All matter that students have *experienced* exists in a gravitational field. Therefore what students have experienced is not mass but weight. This point is of little consequence to a formal operational thinker; mass is an abstract concept he can comprehend and do mental experiments with. To succeed in understanding $F = ma$ (particularly when identifying its units) however, the learner must be able to do mental experiments with abstract concepts. Now look at acceleration – a rate of change of a rate of change. A rate of change is a concrete concept; miles/hour, cents/pound, and pounds/foot are all situations with which a learner can have concrete experiences. But when you change that rate of change so that you are referring to miles/hour/second, providing experience which will lead a student to that is nearly impossible. (To make acceleration even more abstract, it is usually written, for example, as ft/sec^2.) About the best that can be done is to let the student experience the fact that as an object slows down, the time intervals required to travel equal distances get progressively longer. Now consider the experience students have had with forces. Those experiences have no doubt been pushes and pulls and have probably been measured in pounds. Now a student takes an abstract quantity (mass) which he has not experienced and multiplies it by a second very abstract quantity (acceleration) and produces a third quantity called force. But here the force is not measured in pounds but in kilogram-meters/second^2 and is called a newton. *There is nothing concrete about that entire process. It is a complete abstraction.* Now if a student is a formal thinker, he can probably handle that abstraction – *he can't if he is concrete operational.* Do not misread *can't* to mean "doesn't want to"; it means exactly what it says, *can't.*

Couple Newton's second law with the calorie, transverse waves, the particle theory of light, the gauss and maxwell, and the second law of thermodynamics and you have a pretty good sampling of a first-year physics course. You also have a fair list of abstractions. Those are abstract topics for which formal operations are a necessity. How does a teacher determine whether or not his class can handle such abstract topics?

Assessing Student Level of Thought

What we have done in the area of determining student success with tasks which reflect formal operational thought has been greatly influenced by four sources:

1. Bärbel Inhelder and Jean Piaget, *The Growth of Logical Thinking From Childhood to Adolescence* (Basic Books, New York, 1958), Chaps. 1-7.

2. *The Developmental Theory of Piaget: Conservation* (John Davidson Film Producers, San Francisco, 1969).

3. Elizabeth F. Karplus and Robert Karplus, "Intellectual Development Beyond the Elementary School: I. Deductive Logic," [School Sci. Math. **LXX**, 398 (May 1970)].

4. Robert Karplus and Rita W. Peterson, "Intellectual Development Beyond Elementary School II: Ratio A Survey," [School Sci. Math. **LXX**, 813 (Dec. 1970)].

The foregoing sources contain many more tasks than will be described here, and you are urged to try them. Here are two tasks which we have used quite extensively.

(1) *The Conservation of Volume* (Source 2, above). This task requires two cylinders of exactly the same size but having different weight (we have used one made of brass and the other of aluminum); those properties of the cylinders are pointed out to the student. He is next presented with two identical tubes partially filled with water and allowed to adjust the water levels until he is convinced that each tube contains exactly the same amount. The student is then asked if when the cylinders are put in the tubes, the heavy cylinder will push the water up more, if the lighter cylinder will push the level up more, or if the cylinders will push the levels up the same. The examiner requires the student to explain his answer, and often it is the explanations and not the initial responses that are most revealing of thought patterns. If the student completes the task successfully, he has provided evidence of beginning formal operational thought.

(2) *The Exclusion of Irrelevant variables*[2] (Source 1, above). The student is presented with a pendulum whose length can be easily changed and three different sized weights which can be used for the pendulum bob. He is told to do as many experiments as he needs to, using many different lengths of string and all the various-sized weights until he can explain what he needs to do to make the pendulum go fast or slow. Again, note that the examiner bases his evaluation on the student's explanations. The variables of string length, angle, and push are also pointed out to the student. If the examinee recognizes that length is the only relevant variable, he is about to enter into the formal operational thought period. If he not only excludes the irrelevant variables but hypothesizes a solution to the problem and demonstrates his solution, he has entered the formal period. If the student can state a general rule about pendula in such a way that it can be tested, he is probably capable of working with propositional logic. Although the concept of an oscillating pendulum and its period is not an abstract concept itself (its discovery and construction related directly to a concrete physical experiment), solution of the pendulum problem does indicate the use of propositional

logic and that is a prerequisite to the understanding of abstractions.[3]

Student Performance on the Tasks

Physics is normally taught in the high schools to students in grades eleven and twelve. We administered these tasks, therefore, to 99 eleventh graders and 97 twelfth graders from Oklahoma public schools. The schools were randomly selected, and students in each selected school were also randomly selected. Table I shows what we found.

Table I. *Performance of formal operational tasks by a random sample of high school students.*

Population	Conservation of volume	Exclusion
11th Grade (*N*=99)		
Females (*N*=54)	19	14
Males (*N*=45)	26	23
12th Grade (*N*=97)		
Females (*N*=47)	18	16
Males (*N*=50)	34	20

The data in Table I suggest that out of the population from which physics students are drawn, not many are formal operational. You are urged to administer these tasks to your students. If you are interested in doing some group evaluations of your students, study sources three and four listed earlier. Source three deals with determining student ability to reason abstractly by presenting a problem and then providing one clue at a time. The clues and the original statement of the problem must then be analyzed and used to draw conclusions. Source four assesses student ability to apply the concept of ratio. When using ratios, the student is utilizing proportional thinking which is an essential component of formal thought. Please do not make the assumption that by the time students get to physics in high school only those who think formally enroll. Our high school data from those enrolling in high school physics, though not extensive enough to make a definite statement, suggest that such is not the case. Data will be presented later which show that many concrete operational thinkers are found at the first year college level.

Kohlberg and Gilligan [4] report that in a study of the ability of 265 persons to perform successfully on the pendulum task (exclusion), these results were obtained:

age 10-15 – 45%; age 21-30 – 65%;
age 16-20 – 53%; age 45-50 – 57%.

If you assume that performance on the pendulum task is an indication that formal operational thought is present, the foregoing data suggest what our data do – a large percentage of the adolescent population is not formal operational. Unfortunately, our age ranges and those of Kohlberg and Gilligan do not coincide exactly, and so no more definite statement can be made from those two groups of data.

The conservation of volume and the pendulum tasks were taken by college freshmen. The results shown in Table II were obtained.

Table II. *Performance of college freshmen for formal operational tasks.*

Number of college freshmen	Conservation of volume	Exclusion
185	133	77

The data shown in Table II clearly reflect that the majority of college freshmen have not moved deeply into the formal operational stage of thought – 77 of 185 experiencing success on the exclusion task is not too impressive. We do not mean to infer that performance on the pendulum task is an absolute measure of the achievement of formal operational thought. We *do* mean to infer that performance on these tasks is a strong indication of student ability to use propositional logic. We tested our inference that these two tasks do help isolate formal thinkers – those that use thought patterns which are "the stock in trade of the logician, the scientist, or the abstract thinker."[5] In searching for a test population we ruled out all quantitative fields because the tasks are quantitative in nature. We were reminded that the "if..., then..., therefore" construct is also the stock in trade of the lawyer. In order to survive in the study of law, students have to think mainly on the abstract level. We asked several groups of second and third year law students to react to the two tasks we just described. Table III reflects our results. A total of 66 students reacted to the tasks and 50 of them demonstrated formal operational thought. We feel, therefore, that these two tasks have a good probability of identifying formal thought.

Table III. *Performance of second and third year law students on two formal operational tasks.*

	Concrete Operational	Formal Operational
Conservation of volume (*N*=22)	3	19
Exclusion of irrelevant variables (*N*=44)	13	31

What Educational Institutions Can Do to Foster Formal Thought

Our research has shown us that the level of thought of junior high school students[6] and college freshmen[7] can be changed by providing them inquiry-centered experiences in science. We believe that the principal reason our research has shown an increase in the thought levels of students is because *we accepted that most of them participating in the experiments were concrete operational.* That put squarely

upon us the responsibility for providing concrete experiences with the objects and ideas of the discipline. These students were involved in actually creating some knowledge of their very own. We know that this was the first time some of them had been given that opportunity. We believe that actual involvement with the materials and ideas of science and being allowed to find out something for themselves accounts for the movement toward and into formal thought which we found.

Science teachers in general and physics teachers in particular have a vehicle at their command that makes active student involvement convenient. That vehicle is the laboratory. Both of our research studies had the laboratory at its nerve center. In the case of the college study that laboratory did not too frequently involve hardware and chemicals, but it was a place where data were gathered, ideas were honed, hypotheses were made and tested, and verifications were carried out. That is the true laboratory.

In teaching the majority of physics courses (both college and high school) the laboratory can be used to lead students, through inquiry,[8] to develop understandings of the concepts to be learned. The teacher, then, has three responsibilities to discharge before ever meeting a class:

(1) Isolate those concepts which, when learned, will provide students with an accurate and adequate understanding of the discipline. The teacher must use his understanding of the structure of the discipline in order to select the concepts, and his goal is to provide the learner with *his own* understanding of the discipline's structure. Textbooks are of little help here.

(2) Find those laboratory investigations which when cast in an inquiry framework will, upon completion, allow the student to develop an understanding of the concept being considered. Textbooks are of no help here.

(3) Make sure the investigations are cast into an inquiry framework and be sure the necessary materials are available.

Now classes start.[9] The teacher becomes an asker of questions, a provider of materials, a laboratory participant, and a class chairman and secretary. Perhaps most importantly, he is a discussion leader. He gathers the class together (chairman) and solicits the data they have gathered (secretary). He then leads a discussion on what the data mean (discussion leader). He also makes the necessary conceptual inventions at the proper time, decides when discovery can take place, and when the present concept needs to be related to the next one by exploration. He must also decide when exploration of a completely new concept must begin. This teacher is not a teller, he is a director of learning. Traditional teaching methods embrace the notions that (a) teaching is telling, (b) memorization is learning, and (c) being able to repeat something on an examination is evidence of understanding – those points are the antithesis of inquiry.

The development of formal thought must become the focus of attention of every teacher in the country. The Educational Policies Commission said, in 1961, that the *central* purpose of the school must be to teach students to think and they operationally defined thinking.[10] Such good advice! We would add that the central role of the school must be to teach children to think with form not objects – in other words, to move students into the stage of formal operational thought. Science has the structure to enhance greatly the achievement of this objective. We must not blow our chances to make a maximum contribution to education in general and education in science in particular! Let's establish an environment in our classrooms that encourages and promotes formal thought!

References

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2. Renner and Stafford, Ref. 1, p. 294.
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PART I

Cognitive Development in Children: Piaget

Development and Learning

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My dear colleagues, I am very concerned about what to say to you, because I do not know if I shall accomplish the end that has been assigned to me. But I have been told that the important thing is not what you say, but the discussion which follows and the answers to questions you are asked. So this morning I shall simply give a general introduction of a few ideas which seem to me to be important for the subject of this conference.

First I would like to make clear the difference between two problems: the problem of *development* in general and the problem of *learning*. I think these problems are very different, although some people do not make this distinction.

The development of knowledge is a spontaneous process, tied to the whole process of embryogenesis. Embryogenesis concerns the development of the body, but it concerns as well the development of the nervous system and the development of mental functions. In the case of the development of knowledge in children, embryogenesis ends only in adulthood. It is a total developmental process which we must re-situate in its general biological and psychological context. In other words, development is a process which concerns the totality of the structures of knowledge.

Learning presents the opposite case. In general, learning is provoked by situations—provoked by a psychological experimenter; or by a teacher, with respect to some didactic point; or by an external situation. It is provoked, in general, as opposed to spontaneous. In addition, it is a limited process—limited to a single problem, or to a single structure.

So I think that development explains learning, and this opinion is contrary to the widely held opinion that development is a sum of discrete learning experiences. For some psychologists development is reduced to a series of specific learned items, and development is thus the sum, the cumulation of this series of specific items. I think this is an atomistic view which deforms the real state of things. In reality, development is the essential process and each element of learning occurs as a function of total development, rather than being an element which explains development. I shall begin, then, with a first part dealing with development, and I shall talk about learning in the second part.

To understand the development of knowledge, we must start with an idea which seems central to me—the idea of an *operation*. Knowledge is not a copy of reality. To know an object, to know an event, is not simply to look at it and make a mental copy or image of it. To know an object is to act on it. To know is to modify, to transform the object, and to understand the process of this transformation, and as a consequence to understand the way the object is constructed. An operation is thus the essence of knowledge; it is an interiorized action which modifies the object of knowledge. For instance an operation would consist of joining objects in a class to construct a

classification. Or an operation would consist of ordering, or putting things in a series. Or an operation would consist of counting, or of measuring. In other words, it is a set of actions modifying the object, and enabling the knower to get at the structures of the transformation.

An operation is an interiorized action. But, in addition, it is a reversible action; that is, it can take place in both directions, for instance, adding or subtracting, joining or separating. So it is a particular type of action which makes up logical structures.

Above all, an operation is never isolated. It is always linked to other operations, and as a result it is always a part of a total structure. For instance, a logical class does not exist in isolation; what exists is the total structure of classification. An asymmetrical relation does not exist in isolation. Seriation is the natural, basic operational structure. A number does not exist in isolation. What exists is the series of numbers which constitute a structure, an exceedingly rich structure whose various properties have been revealed by mathematicians.

These operational structures are what seem to me to constitute the basis of knowledge, the natural psychological reality, in terms of which we must understand the development of knowledge. And the central problem of development is to understand the formation, elaboration, organization, and functioning of these structures.

I should like to review the stages of development of these structures, not in any detail, but simply as a reminder. I shall distinguish four main stages. The first is a sensory-motor, pre-verbal stage, lasting approximately the first 18 months of life. During this stage is developed the practical knowledge which constitutes the substructure of later representational knowledge. An example is the construction of the schema of the permanent object. For an infant, during the first months, an object has no permanence. When it disappears from the perceptual field it no longer exists. No attempt is made to find it again. Later, the infant will try to find it, and he will find it by localizing it spatially. Consequently, along with the construction of the permanent object there comes the construction of practical or sensory-motor space. There is similarly the construction of temporal succession, and of elementary sensory-motor causality. In other words, there is a series of structures which are indispensable for the structures of later representational thought.

In a second stage, we have pre-operational representation—the beginnings of language, of the symbolic function, and therefore of thought, or representation. But at the level of representational thought, there must now be a reconstruction of all that was developed on the sensory-motor level. That is, the sensory-motor actions are not immediately translated into operations. In fact, during all this second period of pre-operational representations, there are as yet no operations as I defined this term a moment ago. Specifically, there is as yet no conservation which is the psychological criterion of the presence of reversible operations. For example, if we pour liquid from one glass to another of a different shape, the pre-operational child will think there is more in one than in the other. In the absence of operational reversibility, there is no conservation of quantity.

In a third stage the first operations appear, but I call these concrete operations because they operate on objects, and not yet on verbally expressed hypotheses. For example, there are the operations of classification, ordering, the construction of the idea of number, spatial and temporal operations, and all the fundamental operations of elementary logic of classes and relations, of elementary mathematics, of elementary geometry, and even of elementary physics.

Finally, in the fourth stage, these operations are surpassed as the child reaches the level of what I call formal or hypothetic-deductive operations; that is, he can now reason on hypotheses, and not only on objects. He constructs new operations, operations of propositional logic, and not

simply the operations of classes, relations, and numbers. He attains new structures which are on the one hand combinatorial, corresponding to what mathematicians call lattices; on the other hand, more complicated group structures. At the level of concrete operations, the operations apply within an immediate neighborhood: for instance, classification by successive inclusions. At the level of the combinatorial, however, the groups are much more mobile.

These, then, are the four stages which we identify, whose formation we shall now attempt to explain.

What factors can be called upon to explain the development from one set of structures to another? It seems to me that there are four main factors: first of all, *maturation*, in the sense of Gesell, since this development is a continuation of the embryogenesis; second, the role of *experience* of the effects of the physical environment on the structures of intelligence; third, *social transmission* in the broad sense (linguistic transmission, education, etc.); and fourth, a factor which is too often neglected but one which seems to me fundamental and even the principal factor. I shall call this the factor of *equilibration* or if you prefer it, of self-regulation.

Let us start with the first factor, maturation. One might think that these stages are simply a reflection of an interior maturation of the nervous system, following the hypotheses of Gesell, for example. Well, maturation certainly does play an indispensable role and must not be ignored. It certainly takes part in every transformation that takes place during a child's development. However, this first factor is insufficient in itself. First of all, we know practically nothing about the maturation of the nervous system beyond the first months of the child's existence. We know a little bit about it during the first two years but we know very little following this time. But above all, maturation doesn't explain everything, because the average ages at which these stages appear (the average chronological ages) vary a great deal from one society to another. The ordering of these stages is constant and has been found in all the societies studied. It has been found in various countries where psychologists in universities have redone the experiments but it has also been found in African peoples for example, in the children of the Bushmen, and in Iran, both in the villages and in the cities. However, although the order of succession is constant, the chronological ages of these stages varies a great deal. For instance, the ages which we have found in Geneva are not necessarily the ages which you would find in the United States. In Iran, furthermore, in the city of Teheran, they found approximately the same ages as we found in Geneva, but there is a systematic delay of two years in the children in the country. Canadian psychologists who redid our experiments, Monique Laurendeau and Father Adrien Pinard, found once again about the same ages in Montreal. But when they redid the experiments in Martinique, they found a delay of four years in all the experiments and this in spite of the fact that the children in Martinique go to a school set up according to the French system and the French curriculum and attain at the end of this elementary school a certificate of higher primary education. There is then a delay of four years, that is, there are the same stages, but systematically delayed. So you see that these age variations show that maturation does not explain everything.

I shall go on now to the role played by experience. Experience of objects, of physical reality, is obviously a basic factor in the development of cognitive structures. But once again this factor does not explain everything. I can give two reasons for this. The first reason is that some of the concepts which appear at the beginning of the stage of concrete operations are such that I cannot see how they could be drawn from experience. As an example, let us take the conservation of the substance in the case of changing the shape of a ball of plasticene. We give this ball of plasticene to a child who changes its shape into a sausage form and we ask him if there is the

same amount of matter, that is, the same amount of substance as there was before. We also ask him if it now has the same weight and thirdly if it now has the same volume. The volume is measured by the displacement of water when we put the ball or the sausage into a glass of water. The findings, which have been the same every time this experiment has been done, show us that first of all there is conservation of the amount of substance. At about eight years old a child will say, "There is the same amount of plasticene." Only later does the child assert that the weight is conserved and still later that the volume is conserved. So I would ask you where the idea of the conservation of substance can come from. What is a constant and invariant substance when it doesn't yet have a constant weight or a constant volume? Through perception you can get at the weight of the ball or the volume of the ball but perception cannot give you an idea of the amount of substance. No experiment, no experience can show the child that there is the same amount of substance. He can weigh the ball and that would lead to the conservation of weight. He can immerse it in water and that would lead to the conservation of volume. But the notion of substance is attained before either weight or volume. This conservation of substance is simply a logical necessity. The child now understands that when there is a transformation something must be conserved because by reversing the transformation you can come back to the point of departure and once again have the ball. He knows that something is conserved but he doesn't know what. It is not yet the weight, it is not yet the volume; it is simply a logical form—a logical necessity. There, it seems to me, is an example of a progress in knowledge, a logical necessity for something to be conserved even though no experience can have lead to this notion.

My second objection to the sufficiency of experience as an explanatory factor is that this notion of experience is a very equivocal one. There are, in fact, two kinds of experience which are psychologically very different and this difference is very important from the pedagogical point of view. It is because of the pedagogical importance that I emphasize this distinction. First of all, there is what I shall call physical experience, and, secondly, what I shall call logical–mathematical experience.

Physical experience consists of acting upon objects and drawing some knowledge about the objects by abstraction from the objects. For example, to discover that this pipe is heavier than this watch, the child will weigh them both and find the difference in the objects themselves. This is experience in the usual sense of the term—in the sense used by empiricists. But there is a second type of experience which I shall call logical mathematical experience where the knowledge is not drawn from the objects, but it is drawn by the actions effected upon the objects. This is not the same thing. When one acts upon objects, the objects are indeed there, but there is also the set of actions which modify the objects.

I shall give you an example of this type of experience. It is a nice example because we have verified it many times in small children under seven years of age, but it is also an example which one of my mathematician friends has related to me about his own childhood, and he dates his mathematical career from this experience. When he was four or five years old—I don't know exactly how old, but a small child—he was seated on the ground in his garden and he was counting pebbles. Now to count these pebbles he put them in a row and he counted them one, two, three, up to ten. Then he finished counting them and started to count them in the other direction. He began by the end and once again he found ten. He found this marvelous that there were ten in one direction and ten in the other direction. So he put them in a circle and counted them that way and found ten once again. Then he counted them in the other direction and found ten once

more. So he put them in some other arrangement and kept counting them and kept finding ten. There was the discovery that he made.

Now what indeed did he discover? He did not discover a property of pebbles; he discovered a property of the action of ordering. The pebbles had no order. It was his action which introduced a linear order or a cyclical order, or any kind of an order. He discovered that the sum was independent of the order. The order was the action which he introduced among the pebbles. For the sum the same principle applied. The pebbles had no sum; they were simply in a pile. To make a sum, action was necessary—the operation of putting together and counting. He found that the sum was independent of the order, in other words, that the action of putting together is independent of the action of ordering. He discovered a property of actions and not a property of pebbles. You may say that it is in the nature of pebbles to let this be done to them and this is true. But it could have been drops of water, and drops of water would not have let this be done to them because two drops of water and two drops of water do not make four drops of water as you know very well. Drops of water then would not let this be done to them, we agree to that.

So it is not the physical property of pebbles which the experience uncovered. It is the properties of the actions carried out on the pebbles, and this is quite another form of experience. It is the point of departure of mathematical deduction. The subsequent deduction will consist of interiorizing these actions and then of combining them without needing any pebbles. The mathematician no longer needs his pebbles. He can combine his operations simply with symbols, and the point of departure of this mathematical deduction is logical–mathematical experience, and this is not at all experience in the sense of the empiricists. It is the beginning of the coordination of actions, but this coordination of actions before the stage of operations needs to be supported by concrete material. Later, this coordination of actions leads to the logical–mathematical structures. I believe that logic is not a derivative of language. The source of logic is much more profound. It is the total coordination of actions, actions of joining things together, or ordering things, etc. This is what logical–mathematical experience is. It is an experience of the actions of the subject, and not an experience of objects themselves. It is an experience which is necessary before there can be operations. Once the operations have been attained this experience is no longer needed and the coordinations of actions can take place by themselves in the form of deduction and construction for abstract structures.

The third factor is social transmission–linguistic transmission or educational transmission. This factor, once again, is fundamental. I do not deny the role of any one of these factors; they all play a part. But this factor is insufficient because the child can receive valuable information via language or via education directed by an adult only if he is in a state where he can understand this information. That is, to receive the information he must have a structure which enables him to assimilate this information. This is why you cannot teach higher mathematics to a five-year-old. He does not yet have structures which enable him to understand.

I shall take a much simpler example, an example of linguistic transmission. As my very first work in the realm of child psychology, I spent a long time studying the relation between a part and a whole in concrete experience and in language. For example, I used Burt's test employing the sentence, "Some of my flowers are buttercups." The child knows that all buttercups are yellow, so there are three possible conclusions: the whole bouquet is yellow, or part of the bouquet is yellow, or none of the flowers in the bouquet are yellow. I found that up until nine years of age (and this was in Paris, so the children certainly did understand the French language) they

replied, "The whole bouquet is yellow or some of my flowers are yellow." Both of those mean the same thing. They did not understand the expression, "some *of* my flowers." They did not understand this *of* as a partitive genitive, as the inclusion of some flowers in my flowers. They understood some of my flowers to be my several flowers as if the several flowers and the flowers were confused as one and the same class. So there you have children who until nine years of age heard every day a linguistic structure which implied the inclusion of a subclass in a class and yet did not understand this structure. It is only when they themselves are in firm possession of this logical structure, when they have constructed it for themselves according to the developmental laws which we shall discuss, that they succeed in understanding correctly the linguistic expression.

I come now to the fourth factor which is added to the three preceding ones but which seems to me to be the fundamental one. This is what I call the factor of equilibration. Since there are already three factors, they must somehow be equilibrated among themselves. That is one reason for bringing in the factor of equilibration. There is a second reason, however, which seems to me to be fundamental. It is that in the act of knowing, the subject is active, and consequently, faced with an external disturbance, he will react in order to compensate and consequently he will tend towards equilibrium. Equilibrium, defined by active compensation, leads to reversibility. Operational reversibility is a model of an equilibrated system where a transformation in one direction is compensated by a transformation in the other direction. Equilibration, as I understand it, is thus an active process. It is a process of self-regulation. I think that this self-regulation is a fundamental factor in development. I use this term in the sense in which it is used in cybernetics, that is, in the sense of processes with feedback and with feedforward, of processes which regulate themselves by a progressive compensation of systems. This process of equilibration takes the form of a succession of levels of equilibrium, of levels which have a certain probability which I shall call a sequential probability, that is, the probabilities are not established *a priori*. There is a sequence of levels. It is not possible to reach the second level unless equilibrium has been reached at the first level, and the equilibrium of the third level only becomes possible when the equilibrium of the second level has been reached, and so forth. That is, each level is determined as the most probable given that the preceding level has been reached. It is not the most probable at the beginning, but it is the most probable once the preceding level has been reached.

As an example, let us take the development of the idea of conservation in the transformation of the ball of plasticene into the sausage shape. Here you can discern four levels. The most probable at the beginning is for the child to think of only one dimension. Suppose that there is a probability of 0.8, for instance, that the child will focus on the length, and that the width has a probability of 0.2. This would mean that of ten children, eight will focus on the length alone without paying any attention to the width, and two will focus on the width without paying any attention to the length. They will focus only on one dimension or the other. Since the two dimensions are independent at this stage, focusing on both at once would have a probability of only 0.16. That is less than either one of the two. In other words, the most probable in the beginning is to focus only on one dimension and in fact the child will say, "It's longer, so there's more in the sausage." Once he has reached this first level, if you continue to elongate the sausage, there comes a moment when he will say, "No, now it's too thin, so there's less." Now he is thinking about the width, but he forgets the length, so you have come to a second level which becomes the most probable after the first level, but which is not the most probable at the point of departure. Once he has focused on the

width, he will come back sooner or later to focus on the length. Here you will have a third level where he will oscillate between width and length and where he will discover that the two are related. When you elongate you make it thinner, and when you make it shorter, you make it thicker. He discovers that the two are solidly related and in discovering this relationship, he will start to think in terms of transformation and not only in terms of the final configuration. Now he will say that when it gets longer it gets thinner, so it's the same thing. There is more of it in length but less of it in width. When you make it shorter it gets thicker; there's less in length and more in width, so there is compensation—compensation which defines equilibrium in the sense in which I defined it a moment ago. Consequently, you have operations and conservation. In other words, in the course of these developments you will always find a process of self-regulation which I call equilibration and which seems to me the fundamental factor in the acquisition of logical–mathematical knowledge.

I shall go on now to the second part of my lecture, that is, to deal with the topic of learning. Classically, learning is based on the stimulus–response schema. I think the stimulus–response schema, while I won't say it is false, is in any case entirely incapable of explaining cognitive learning. Why? Because when you think of a stimulus–response schema, you think usually that first of all there is a stimulus and then a response is set off by this stimulus. For my part, I am convinced that the response was there first, if I can express myself in this way. A stimulus is a stimulus only to the extent that it is significant, and it becomes significant only to the extent that there is a structure which permits its assimilation, a structure which can integrate this stimulus but which at the same time sets off the response. In other words, I would propose that the stimulus–response schema be written in the circular form—in the form of a schema or of a structure which is not simply one way. I would propose that above all, between the stimulus and the response, there is the organism, the organism and its structures. The stimulus is really a stimulus only when it is assimilated into a structure and it is this structure which sets off the response. Consequently, it is not an exaggeration to say that the response is there first, or if you wish at the beginning there is the structure. Of course we would want to understand how this structure comes to be. I tried to do this earlier by presenting a model of equilibration or self-regulation. Once there is a structure, the stimulus will set off a response, but only by the intermediary of this structure.

I should like to present some facts. We have facts in great number. I shall choose only one or two and I shall choose some facts which our colleague, Smedslund, has gathered. (Smedslund is currently at the Harvard Center for Cognitive Studies.) Smedslund arrived in Geneva a few years ago convinced (he had published this in one of his papers) that the development of the ideas of conservation could be indefinitely accelerated through learning of a stimulus–response type. I invited Smedslund to come to spend a year in Geneva to show us this, to show us that he could accelerate the development of operational conservation. I shall relate only one of his experiments.

During the year that he spent in Geneva he chose to work on the conservation of weight. The conservation of weight is, in fact, easy to study since there is a possible external reinforcement, that is, simply weighing the ball and the sausage on a balance. Then you can study the child's reactions to these external results. Smedslund studied the conservation of weight on the one hand, and on the other hand he studied the transitivity of weights, that is, the transitivity of equalities if A = B and B = C, then A = C, or the transitivity of the inequalities if A is less than B, and B is less than C, then A is less than C.

As far as conservation is concerned,

Smedslund succeeded very easily with five- and six-year-old children in getting them to generalize that weight is conserved when the ball is transformed into a different shape. The child sees the ball transformed into a sausage or into little pieces or into a pancake or into any other form, he weighs it, and he sees that it is always the same thing. He will affirm it will be the same thing, no matter what you do to it; it will come out to be the same weight. Thus Smedslund very easily achieved the conservation of weight by this sort of external reinforcement.

In contrast to this, however, the same method did not succeed in teaching transitivity. The children resisted the notion of transitivity. A child would predict correctly in certain cases but he would make his prediction as a possibility or a probability and not as a certainty. There was never this generalized certainty in the case of transitivity.

So there is the first example, which seems to me very instructive, because in this problem in the conservation of weight there are two aspects. There is the physical aspect and there is the logical–mathematical aspect. Note that Smedslund started his study by establishing that there was a correlation between conservation and transitivity. He began by making a statistical study on the relationships between the spontaneous responses to the questions about conservation and the spontaneous responses to the questions about transitivity, and he found a very significant correlation. But in the learning experiment, he obtained a learning of conservation and not of transitivity. Consequently, he successfully obtained a learning of what I called earlier physical experience (which is not surprising since it is simply a question of noting facts about objects), but he did not successfully obtain a learning in the construction of the logical structure. This doesn't surprise me either, since the logical structure is not the result of physical experience. It cannot be obtained by external reinforcement. The logical structure is reached only through internal equilibration, by self-regulation, and the external reinforcement of seeing that the balance did not suffice to establish this logical structure of transitivity.

I could give many other comparable examples, but it seems useless to me to insist upon these negative examples. Now I should like to show that learning is possible in the case of these logical–mathematical structures, but on one condition—that is, that the structure which you want to teach to the subjects can be supported by simpler, more elementary, logical–mathematical structures. I shall give you an example. It is the example of the conservation of number in the case of one-to-one correspondence. If you give a child seven blue tokens and ask him to put down as many red tokens, there is a preoperational stage where he will put one red one opposite each blue one. But when you spread out the red ones, making them into a longer row, he will say to you, "Now, there are more red ones than there are blue ones."

Now how can we accelerate, if you want to accelerate, the acquisition of this conservation of number? Well, you can imagine an analogous structure but in a simpler, more elementary situation. For example, with Mlle. Inhelder, we have been studying recently the notion of one-to-one correspondence by giving the child two glasses of the same shape and a big pile of beads. The child puts a bead into one glass with one hand and at the same time a bead into the other glass with the other hand. Time after time he repeats this action, a bead into one glass with one hand and at the same time a bead into the other glass with the other hand and he sees that there is always the same amount on each side. Then you hide one of the glasses. You cover it up. He no longer sees this glass but he continues to put one bead into it while at the same time putting one bead into the other glass which he can see. Then you ask him whether the equality has been conserved, whether there is still the same amount in one glass as in the other. Now you will find that very small children, about four years old, don't want

to make a prediction. They will say, "So far, it has been the same amount, but now I don't know. I can't see any more, so I don't know." They do not want to generalize. But the generalization is made from the age of about five and one-half years.

This is in contrast to the case of the red and blue tokens with one row spread out, where it isn't until seven or eight years of age that children will say there are the same number in the two rows. As one example of this generalization, I recall a little boy of five years and nine months who had been adding the beads to the glasses for a little while. Then we asked him whether, if he continued to do this all day and all night and all the next day, there would always be the same amount in the two glasses. The little boy gave this admirable reply. "Once you know, you know for always." In other words, this was recursive reasoning. So here the child does acquire the structure in this specific case. The number is a synthesis of class inclusion and ordering. This synthesis is being favored by the child's own actions. You have set up a situation where there is an iteration of one same action which continues and which is therefore ordered while at the same time being inclusive. You have, so to speak, a localized synthesis of inclusion and ordering which facilitates the construction of the idea of number in this specific case, and there you can find, in effect, an influence of this experience on the other experience. However, this influence is not immediate. We study the generalization from this recursive situation to the other situation where the tokens are laid on the table in rows, and it is not an immediate generalization but it is made possible through intermediaries. In other words, you can find some learning of this structure if you base the learning on simpler structures.

In this same area of the development of numerical structures, the psychologist Joachim Wohlwill, who spent a year at our Institute at Geneva, has also shown that this acquisition can be accelerated through introducing additive operations, which is what we introduced also in the experiment which I just described. Wohlwill introduced them in a different way but he too was able to obtain a certain learning effect. In other words, learning is possible if you base the more complex structure on simpler structures, that is, when there is a natural relationship and development of structures and not simply an external reinforcement.

Now I would like to take a few minutes to conclude what I was saying. My first conclusion is that learning of structures seems to obey the same laws as the natural development of these structures. In other words, learning is subordinated to development and not vice-versa as I said in the introduction. No doubt you will object that some investigators have succeeded in teaching operational structures. But, when I am faced with these facts, I always have three questions which I want to have answered before I am convinced.

The first question is: "Is this learning lasting? What remains two weeks or a month later?" If a structure develops spontaneously, once it has reached a state of equilibrium, it is lasting, it will continue throughout the child's entire life. When you achieve the learning by external reinforcement, is the result lasting or not and what are the conditions necessary for it to be lasting?

The second question is: "How much generalization is possible?" What makes learning interesting is the possibility of transfer of a generalization. When you have brought about some learning, you can always ask whether this is an isolated piece in the midst of the child's mental life, or if it is really a dynamic structure which can lead to generalizations.

Then there is the third question: "In the case of each learning experience what was the operational level of the subject before the experience and what more complex structures has this learning succeeded in achieving?" In other words, we must look at each specific learning experience from the point of view of the spontaneous operations

which were present at the outset and the operational level which has been achieved after the learning experience.

My second conclusion is that the fundamental relation involved in all development and all learning is not the relation of association. In the stimulus–response schema, the relation between the response and the stimulus is understood to be one of association. In contrast to this, I think that the fundamental relation is one of assimilation. Assimilation is not the same as association. I shall define assimilation as the integration of any sort of reality into a structure, and it is this assimilation which seems to me to be fundamental in learning, and which seems to me to be the fundamental relation from the point of view of pedagogical or didactic applications. All of my remarks today represent the child and the learning subject as active. An operation is an activity. Learning is possible only when there is active assimilation. It is this activity on the part of the subject which seems to me to be underplayed in the stimulus–response schema. The presentation which I propose puts the emphasis on the idea of self-regulation, on assimilation. All the emphasis is placed on the activity of the subject himself, and I think that without this activity there is no possible didactic or pedagogy which significantly transforms the subject.

Finally, and this will be my last concluding remark, I would like to comment on an excellent publication by the psychologist Berlyne. Berlyne spent a year with us in Geneva during which he intended to translate our results on the development of operations into stimulus–response language, specifically into Hull's learning theory. Berlyne published in our series of studies of genetic epistomology a very good article on this comparison between the results obtained in Geneva and Hull's theory. In the same volume, I published a commentary on Berlyne's results. The essence of Berlyne's results is this: Our findings can very well be translated into Hullian language, but only on condition that two modifications are introduced. Berlyne himself found these modifications quite considerable, but they seemed to him to concern more the conceptualization than the Hullian theory itself. I am not so sure about that. The two modifications are these. First of all, Berlyne wants to distinguish two sorts of response in the S-R schema: (*a*) responses in the ordinary, classical sense, which I shall call "copy responses;" (*b*) responses which Berlyne calls "transformation responses." Transformation responses consist of transforming one response of the first type into another response of the first type. These transformation responses are what I call operations, and you can see right away that this is a rather serious modification of Hull's conceptualization because here you are introducing an element of transformation and thus of assimilation and no longer the simple association of stimulus–response theory.

The second modification which Berlyne introduces into the stimulus–response language is the introduction of what he calls internal reinforcements. What are these internal reinforcements? They are what I call equilibration or self-regulation. The internal reinforcements are what enable the subject to eliminate contradictions, incompatibilities, and conflicts. All development is composed of momentary conflicts and incompatibilities which must be overcome to reach a higher level of equilibrium. Berlyne calls this elimination of incompatibilities internal reinforcements.

So you see that it is indeed a stimulus–response theory, if you will, but first you add operations and then you add equilibration. That's all we want!

Editor's note: A brief question and answer period followed Professor Piaget's presentation. The first question related to the fact that the eight-year-old child acquires conservation of weight and volume. The question asked if this didn't contradict the order of emergence of the pre-operational and operational stages. Piaget's response follows:

The conservation of weight and the conservation of volume are not due only to

experience. There is also involved a logical framework which is characterized by reversibility and the system of compensations. I am only saying that in the case of weight and volume, weight corresponds to a perception. There is an empirical contact. The same is true of volume. But in the case of substance, I don't see how there can be any perception of substance independent of weight or volume. The strange thing is that this notion of substance comes before the two other notions. Note that in the history of thought we have the same thing. The first Greek physicists, the pre-socratic philosophers, discovered conservation of substance independently of any experience. I do not believe this is contradictory to the theory of operations. This conservation of substance is simply the affirmation that something must be conserved. The children do not know specifically what is conserved. They know that since the sausage can become a ball again there must be something which is conserved, and saying "substance" is simply a way of translating this logical necessity for conservation. But this logical necessity results directly from the discovery of operations. I do not think that this is contradictory with the theory of development.

Editor's note: The second question was whether or not the development of stages in children's thinking could be accelerated by practice, training, and exercise in perception and memory. Piaget's response follows:

I am not very sure that exercise of perception and memory would be sufficient. I think that we must distinguish within the cognitive function two very different aspects which I shall call the figurative aspect and the operative aspect. The figurative aspect deals with static configurations. In physical reality there are states, and in addition to these there are transformations which lead from one state to another. In cognitive functioning one has the figurative aspects—for example, perception, imitation, mental imagery, etc.

The operative aspect includes operations and the actions which lead from one state to another. In children of the higher stages and in adults, the figurative aspects are subordinated to the operative aspects. Any given state is understood to be the result of some transformation and the point of departure for another transformation. But the pre-operational child does not understand transformations. He does not have the operations necessary to understand them so he puts all the emphasis on the static quality of the states. It is because of this, for example, that in the conservation experiments he simply compares the initial state and the final state without being concerned with the transformation.

In exercising perception and memory, I feel that you will reinforce the figurative aspect without touching the operative aspect. Consequently, I'm not sure that this will accelerate the development of cognitive structures. What needs to be reinforced is the operative aspect—not the analysis of states, but the understanding of transformations.

Physics Problems and the Process of Self-Regulation

Anton E. Lawson and Warren T. Wollman

In two previous articles[1,2] Jean Piaget's theory of intellectual development and its general implications for physics teaching were discussed. The purpose of this article is to examine more closely one aspect of that theory and discuss its implications for designing and using homework problems. We will briefly describe the process of self-regulation (the process Piaget hypothesizes governs all intellectual growth) and suggest a way in which homework problems can be used to provide students an opportunity for self-regulation. Further, we will discuss deficiencies of typical homework problems and provide a number of example problems which we believe can initiate self-regulation. Through the process of self-regulation initiated by thought-provoking problems, we believe students will not only be able to develop understandings of the concepts involved but will also progress from relatively concrete (or limited) to more abstract (or generalizable) modes of thinking.

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The process of self-regulation

The process by which Piaget hypothesizes that patterns of reasoning are refined, extended, or combined with other patterns of reasoning is called self-regulation. Initially, basic reasoning patterns serve to guide an individual's actions within his surroundings. As long as those actions promote satisfactory interaction, the basic patterns continue to guide behavior. However, owing to the individual's extended interaction with his environment he meets contradictions, that is, situations for which his initial patterns of reasoning do not serve as effective guides to behavior. These contradictions produce a state of disequilibrium. In other words, his patterns of reasoning are found wanting and must somehow be changed. If the disequilibrium is not too great, he will spontaneously begin to alter his patterns of reasoning in an attempt to assimilate the new situation. The process by which an individual actively seeks to reestablish equilibrium is termed self-regulation. The altered reasoning patterns which develop are then tried. If the patterns guide behavior successfully so that the person's efforts obtain positive feedback the patterns are reinforced. Continued positive feedback then produces an increasingly stable set of reasoning patterns. In this manner the person gradually builds new reasoning patterns and adapts to new situations.

Homework problems can initiate self-regulation

The gradual process of reestablishing equilibrium through self-regulation affords the possibility of initiating interactions between students and subject matter with the use of homework problems provided the following two factors are present: Problems must be chosen so that the student can partially but not completely understand

Typical homework problems seldom require a student to examine his own thinking.

them in terms of old ideas (i.e., a moderate state of disequilibrium must result from the problem); and sufficient time must be allowed for the student to grapple with the new situation, possibly with appropriate "hints" to direct his thinking, but allowing him to put the ideas together himself.

An important facet then in selecting problems which encourage self-regulation is to obtain a careful match between what the student knows and the kind of problem he is asked to work through. The ideal situation would seem to be one in which the problems are challenging but are felt to be solvable. The hypothesis is that a challenging but solvable problem will place a student into an initial state of disequilibrium. However, through his own efforts at bringing together what he has done in the laboratory, read in the textbook, heard in lectures, learned from other past experiences, and obtained from teacher or peer discussions he will gradually organize his thinking about this information and successfully solve the problem. This success will then establish a new and more stable equilibrium. The new state of equilibrium will be one with increased understanding of the subject matter and increased problem-solving capability. Before giving examples of the kind of problem we believe can initiate self-regulation a few comments will be made regarding deficiencies of standard homework problems.

What's wrong with typical homework problems?

Typical homework problems seldom require a student to examine his own thinking, make comparisons, and raise questions which, in fact, are crucial to scientific inquiry. These problems usually require students to apply an equation or sometimes two or three equations to obtain a solution. Students quickly come to realize that the name of this game is "Can you discover the correct equation?" This is a game of recognition—a sort of high order matching process involving little thought. Although this process can be an important one, we believe that little if any self-regulation takes place in this way. Typical homework problems do not require the student to think about:

1. *The data of the problem.* Usually there is just the right amount, no more nor less, whereas in real situations there is either a dearth or superfluity of information and the problem is to discover what is relevant.
2. *The approach to the problem.* Usually this is determined by the chapter heading. If, for example, a mechanics problem can be solved either by Lagrange's equations, Newton's laws, or energy conservation, the choice is dictated by irrelevant considerations, e.g., the problem comes from the chapter on Lagrange's equations. It is important for students to learn that many approaches may seem reasonable and the problem is to decide whether one is particularly appropriate.
3. *The tacit assumptions of a problem-solving strategy*, for example deciding between use of Boyle's law or the Van der Waals equation. This decision is usually made *for* the student, not by the student.
4. *The physical arguments involved in the problem as opposed to the mathematical ones.* Too often problems are only exercises in using mathematical tools (a necessary exercise) without ever demanding that the student try either to arrive at or qualitatively justify the mathematical result by physical (phenomenological) arguments utilizing both principles and order of magnitude calculations. Indeed, the physical or intuitive argument often precedes the mathematical in real research.
5. *The statement of a problem.* Problems are tailored to fit the text when, in fact, the real problem is doing the tailoring by conceptualizing a real situation in terms of a model. This involves all of the above points.

How to encourage self-regulation

A few points should be kept in mind when designing, discussing, using, and scoring problems to encourage self-regulation:

1. Open-ended problems (problems with no single solution) are often excellent tools to encourage thinking.
2. Problems which present an apparent paradox produce disequilibrium and can initiate self-regulation. Paradox problems by their nature are generally short and incisive. Leighton in his foreword to the exercise workbook written to accompany *The Feynman Lectures in Physics*[3] discussed the kinds of problems which appeared most suitable to him. He suggested that problems of a kind that are numerically or analytically simple, yet incisive and illuminating in content were particularly useful.
3. To encourage self-regulation it is often helpful to ask students to record and hand in all the various ideas they tried and found unsuccessful as well as the ones which were successful in arriving at the problem

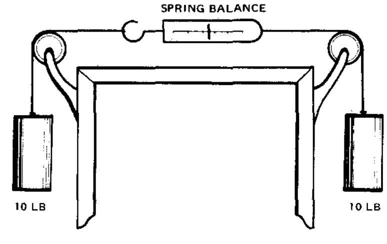
"Real" problems should, and indeed must, involve a certain amount of trial and error.

solution. Discussions of these steps in an atmosphere in which these ideas are recognized not only as worthwhile but as necessary, clue students into the fact that "real" problems *should* and indeed *must* involve a certain amount of trial and error, albeit informed trial and error.
albeit informed trial and error.

4. Have the students search for necessary data so they examine their conceptualization of the problem. Either give superfluous data or omit necessary data. To account for the latter, students should have to make plausible assumptions or introduce suitable symbols for quantities that are needed to solve the problem.
5. Require students to draw a diagram of the physical situation. To do this students have to think deeply about the spatial relationships of the interacting objects, and may find discrepancies as they compare their preconceptions with the diagram.
6. Provide for a "problem clinic" or tutorial service where students can get help with problems while they are solving them, and before they have to be turned in. Interaction with other persons can be very helpful and is often even necessary if students are to conceptualize, then critically analyze their own thinking.
7. For problems designed to engage a student over a period of, say, two weeks, the teacher should consult with the student several times in order to:
   A. Discuss with him his initial approach. If this approach is reasonable but known in advance to be inappropriate, the teacher should *not* intervene at this point, but rather let the student discover for himself why the approach will not work.
   B. Discuss with the student alternative approaches both when the initial approach is appropriate and when it is reasonable but not appropriate. In either case, let the student first discover which approach will work. *Then* discuss alternatives, *even* if the first approach worked. It may be that he will accept inappropriate alternatives as reasonable. He may then discover on his own why they are not.
   C. Discuss both semi-quantitative (order of magnitude) and qualitative arguments anticipating the outcome of more rigorous approaches. Limiting cases should be used as a check when solutions to simpler problems are already known.
   D. Discuss alternatives to an inappropriate and *time-consuming* approach. This is to *avoid* having the student spend too much time discovering the inadequacies of an approach. Overall, the student should get from the teacher a feeling for the general considerations appropriate to choosing and comparing strategies, i.e., a feeling for the process of inquiry.
8. Although solutions (numerical or algebraic) should be provided for all problems (not just the "odd-numbered" ones), students must understand that a premature glance at a solution will surely affect their conception of the problem and distort the problem solving procedure. Knowledge of the solution can provide stimulating feedback *after* the student has completed and carried through a formulation of a solution.

Examples of problems that can promote self-regulation

Problem 1 Since the net force on the spring scale shown in Fig. 1 is zero how can the scale register a non-zero reading? What does the scale register? Why isn't it 20 since it is pulled by 10 lbs at each end?



Comment: This example, which is especially useful when associated with a demonstration, illustrates how a little knowledge can go a wrong way. At first, concepts are only vaguely grasped and thus over-extended. Here we obviously have two forces whose sum is equal to zero and yet the scale does not read zero. Or, we might think that each force contributes 10 lbs of tension to the scale to give 20 lbs. These two approaches use unrestricted (over-extended) concepts which must be coordinated, via self-regulation, with other concepts, e.g., free-body diagrams and action-reaction, in order to resolve the discrepancy.

Fig. 1. Spring balance and suspended weights.

Problem 2 A capacitor and resistor are connected in a circuit as shown in Fig. 2. The values are $C = 250\ \mu\mu f$, $R = 10\ 000\ \Omega$, and $E = 400$ V. Initially the switch is closed and then it is opened suddenly. Use two methods to calculate the energy dissipated in the resistor after the switch is opened. Do both methods give the same result? Should they give the same result? If so, why? If not, why not?

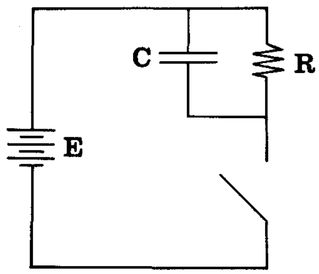


Fig. 2. Circuit diagram showing the capacitor, resistor, switch, and battery.

Comment: This problem calls for two quantitative analyses of the same situation. If the student is able to think of two methods of solution and obtain the same answer using both methods no disequilibration will result. However, if two different answers are obtained the student should check his own work. The discrepancy could be resolved quickly if the source of the difference was an error in calculation. If, however, the difference was due to difficulty in conceptualization, then the check will promote self-regulation.

Problem 3 The gas temperature at one level of the upper atmosphere is about 1000°K. The temperature at the surface of a burning match is about the same. Yet a person would be very cold in the upper atmosphere. How can that be?

Comment: This problem presents a paradox because 1000°K is a very high temperature and yet it is "cold up there." Resolution through self-regulation leads to a more scientific and less everyday notion of the relation between temperature and "cold" or "hot."

Problem 4 A glass is exactly full of water at 0°C and has a cube of ice floating in it. When the ice melts (still at 0°C) the water will not overflow, because the ice displaced a volume of water equal to the volume of the water into which the ice melted. OK. Let us look at some fine points. In what *direction* (slight overflow or the opposite) would each of the following affect the result? Give only the *direction.*

(a) The ice cube contained some grains of sand.

(b) The ice cube contained some air bubbles.

(c) The water (and the glass) were not at 0° to start with, but were at room temperature.

(d) The "water" is not water at all, but is a Martini which is close to 0° but, due to its alcoholic content, has density less than that of water.

Comment: This problem originally appeared in an article by Richard Crane.[4] It, as well as other problems in that article (for example, problems 8, 17, 18, 26-29), are excellent examples of problems which will promote self-regulation. Problems 34, 41, 42, and 48 which appeared in a second article by Crane[5] also are thought provoking and should encourage self-regulation.

Problem 5 If internal energy is partly molecular motion, what is the difference between a hot, stationary golf ball sitting on a tee and a cold golf ball rapidly moving off the tee.

Comment: Of course, the molecular motion part of internal energy refers to *random* motion. Thus, self-regulation refines or sharpens a global or relatively diffuse concept. It is typical of students that they only assimilate parts of a concept at first. By provoking them to discover or recover all the parts, the concept becomes more sharply defined.

Problem 6 When a cylinder, open at one end, is placed over a burning candle which is sitting in a container of water the candle flame goes out and water rises into the cylinder. Why does the flame go out and why does the water rise? Note: Not all observations are mentioned in the description. What other observations do you think you would make if the phenomenon was observed? Obtain the necessary materials and try the experiment yourself. Try the experiment varying the number of candles used, the amount of water in the container, the size and shape of the cylinder, the speed with which you place the cylinder over the candle, and anything else you can think of.

Comment: This problem is one which often yields a quick but erroneous solution. Most students will hypothesize that the candle goes out because it burned up all the oxygen in the cylinder and the water then came in to replace the oxygen. Selected items of information or questions could be supplied at this point to provoke students to abandon this idea and continue their search. For example: What is produced when a flame consumes oxygen? Two burning candles make more water rise than one. Small bubbles were observed escaping from the bottom of the cylinder. Why might this have occurred? These observations contradict the initial explanation and should provoke disequilibrium. Once other explanations are offered they can be analyzed to determine their suitability. They may lead

some students to try the experiment to collect further data. Explanations can then be evaluated in terms of their compatibility with the data and their compatibility with physical conceptions.

Problem 7 Everyone "knows" that to win a tug of war, a team has to pull harder than the other team. What everyone doesn't know is that, in fact, each team always pulls equally hard, even the winning team. Under these circumstances, how can one team ever win (short of the other team just letting go)?

Comment: Obviously one normally thinks that good teams pull harder than poor teams and this is why they win. This problem makes one apply the free-body diagram method and the action-reaction idea to resolve a problem already believed solved by common sense but now made to appear strange. This nonroutine use of physics concepts makes it more likely they will *not* be overlooked in the future.

Problem 8 Polishing surfaces reduces friction between them unless you polish them extremely well, then friction will increase. How can that be true?

Comment: One never expects polishing to increase friction. Resolution of this paradox leads to better understanding of the relation of macroscopic effects to microscopic phenomena, e.g., friction, to microscopic and molecular interaction.

Problem 9 (a) See Fig. 3a. The focal lengths of two identical, thin, convex lenses are the same and measured to be 20 cm each ($F_1 = 20$ cm, $F_2 = 20$ cm). The two lenses are placed next to each other as shown in Fig. 3b and taped together at their edges only. The focal length of this combination, F_c, is 10 cm. Write an equation that gives the focal length of a lens combination that consists of two lenses having identical focal lengths.

(b) One of the 20 cm focal length lenses is replaced by one having a focal length (F_3) of 5 cm. The focal length of the resulting combination is measured to be 4 cm. Write an equation that can be used to calculate the focal length of a lens combination that consists of two lenses of unequal focal lengths.

(c) Now check your two equations. Are they the same? Do you think they should be the same. If so, why? If not, why not? If you believe they should be the same but you have two different equations rethink the problem and try to reduce the two situations to one equation.

Comment: Students will generally solve parts (a) and (b) with little difficulty. However, they will seldom write an equation general enough to account for both situations. The suggestion in part (c) that the equations should be the same and the student's intuitive feeling that a general equation could be found, coupled with the original incompatible equations should produce disequilibrium and provoke the student to rethink the problem.

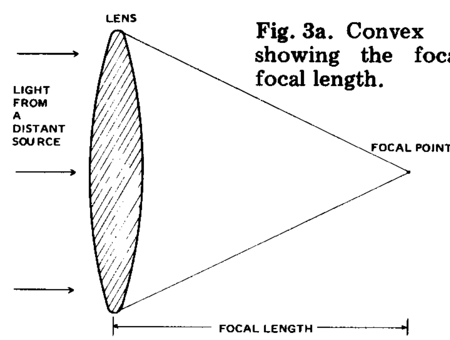


Fig. 3a. Convex lens diagram showing the focal point and focal length.

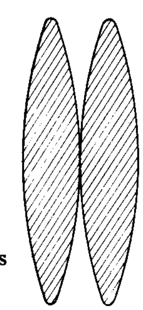


Fig. 3b. Two convex lenses fastened together.

Problem 10 A student measures his weight by climbing onto the large platform of a big spring scale. He takes a step to one side and notices that just as he started to do this, the scale registered less than his weight. Before he could puzzle this through, he noticed that just as he completed the step, the scale now registered more than his weight. If there is nothing wrong with the scale, then what was going on?

Comment: "Weight is weight is weight," a famous poet might have said. So how can a scale read less than one's weight? Worse, how can it also read more? Still worse, if it isn't the scale that must be fixed, then how am I, the student, to fix my ideas?

Problem 11 A brick is supported by a string A from the ceiling, and another string B is attached to the bottom of the brick. If you give a sudden jerk to B it will break, but if you pull on B steadily, A will break. Since the force is the same both ways how could this occur?

Comment: To be most effective this problem should be demonstrated. Anything actually seen makes a much greater and longer lasting impression than anything simply heard or read about. This comment of course applies to other problems as well. Since the student is used to thinking in a-temporal terms, he will think that

force is force and so equal forces have equal effects. So how can the string break in one instance and not in the other? Again, common sense is in conflict with observation and this use of physics to set the world straight is likely to be retained.

Acknowledgment

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projection pointers

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The Oersted effect on the overhead

It is well known that the effect on a magnetic compass needle of being deflected when placed near a current-carrying wire was discovered by Hans Oersted in 1820. An elementary demonstration of this effect is usually presented in any course dealing with electricity and magnetism, and it is a very convincing proof that moving electric charges produce magnetic fields. Several apparatus manufacturers* sell a simple device to demonstrate the Oersted effect to small classes. The apparatus consists of a metallic bar bent into a rectangular loop and mounted on an insulated base with a compass needle suspended at the middle of the loop. When a large current is sent through the loop the compass needle will deflect and line up perpendicular to the loop; i.e., tangent to the magnetic field line at that position. Reversing the current direction results in the needle reversing its direction, showing how the magnetic field direction is related to the current direction (right-hand rule).

In a large or auditorium-size lecture class it is difficult for all the students to see the effect demonstrated by this small apparatus. Since the overhead projector is used extensively in such situations it is natural to try to adapt this demonstration to the overhead. This is simply accomplished by replacing the opaque base with one made of Lucite and securing to it an inverted-U-shaped metal bar with screw terminals at each end for connection to a current source. The same compass needle that is used in the commercial apparatus is suspended under the bar by a needle point in the same manner as is found in the commercial device (see Fig. 1). When the apparatus is operated on the overhead the compass needle deflection is easily viewed by all. A small piece of paper can be taped to one end of the compass needle as a visible reference. A further modification (not shown in the figure) uses a smaller raised Lucite platform to place the compass needle above the metal bar for demonstrating the circular symmetry of the magnetic field.

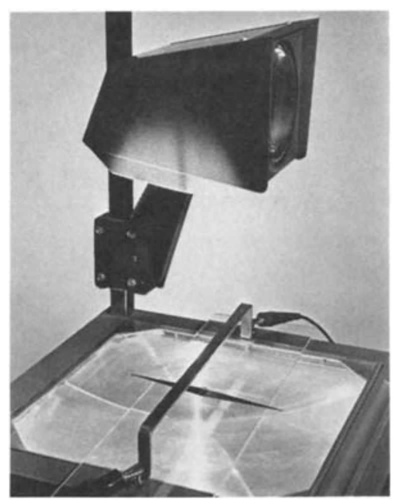

Fig. 1. The Oersted effect demonstrated in place on the overhead.

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*For instance, Oersted's Law Apparatus, manufactured by the Sargent-Welch Company, Skokie, Illinois.