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## On the Oscillations of AI Velorum

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**Summary.** Least-squares Fourier-Interaction fits are made to the observed light variations (Walraven, 1955) and radial velocity variations (Gratton and Lavagnino, 1953) of AI Velorum. The first-order amplitudes and phases emerging from these fits are then compared with the corresponding quantities from linear non-adiabatic pulsation models to attempt to determine the mass of AI Vel. The mass thus obtained is low ( $\sim 0.25$  to  $0.45 M_{\odot}$ ), but due to uncertainties in the method, this result is not considered definitive. The various uncertainties are analyzed in some detail and it is concluded that a new set of observed radial velocities will be needed in order to distinguish among different theoretical models for the oscillations of AI Vel.

**Key words:** stellar pulsations – AI Velorum stars

### 1. Introduction

AI Velorum is the prototype of the class of variable stars (also called Dwarf Cepheids) which bears its name. The mass and population of this star are still open to question. While certain investigations have assigned AI Vel a low mass ( $\approx 0.5 M_{\odot}$ ) and included it in Population II or old Population I (e.g., Bessell, 1969; Petersen and Jorgensen, 1972; Dziembowski and Kozłowski, 1974), it has been argued by others (Baglin et al., 1973; Breger, 1976, 1977) that the object may actually be a large-amplitude  $\delta$ -Scuti star in which case it is high mass ( $1\text{--}2 M_{\odot}$ ) and Population I. The implications of the conflicting hypotheses have been discussed by Percy (1975) and Breger (1976).

Because AI Vel has two stable, well-determined pulsation periods, it has been the object of considerable theoretical study. The strongest evidence for the low-mass hypothesis came from theoretical models which reproduced the observed period ratio ( $P_1/P_0 \approx 0.773$ ) with the fundamental and first overtone radial modes (Petersen and Jorgensen, 1972; Dziembowski and Kozłowski, 1974). Later investigations, however, have emphasized the difficulty of assigning masses and abundances on the basis of period ratios alone (Fitch and Szeidl, 1976; Petersen, 1976; Stellingwerf, 1976).

In the present work we attempt to use not only the observed periods, but also the structure of the light (Sect. 2) and radial velocity curves (Sect. 3) to deduce the mass of AI Velorum (Sects. 4 and 5). The value turns out to be in agreement with the low-mass hypothesis, but further investigation (Sect. 6) indicates that the result cannot be considered definitive at the present time. That being so, the uncertainties are analyzed in detail (Sect. 6) and suggestions made for improving the situation. A final discussion is undertaken in Sect. 7.

### 2. Decomposition of the Light Curves

The light of AI Vel was observed extensively in 1952 and 1953 by Walraven, who published (Walraven, 1955) well over 3000 individual observations. On the basis of this data he determined two strong periods with values (in days):

$$P_0 = 0.11157375; \quad P_1 = 0.08620767.$$

In addition, two other periodicities were proposed by Walraven, both with very small amplitudes.

When the attempt was made to fit the observed points by the superposition of two sine functions having periods  $P_0$  and  $P_1$ , respectively, substantial discrepancies were found, leading Walraven to propose ad-hoc but systematic distortions of both the phases ( $S$  distortion) and amplitudes ( $M$  distortion) of the fitted functions.

While it is clear that a fit with sine functions alone is bound to be inaccurate due to neglect of higher-order harmonics and interaction terms, it is only in recent years that nonlinear contributions have been included (Fitch, 1966, 1976; Faulkner, 1977a, b; Jerzykiewicz and Wenzel, 1977) and the success of the technique demonstrated. Implicit in this technique is the assumption that the nonlinear oscillations of a star which exhibits periodicities  $P_0, P_1, \dots, P_n$  can be described to within observational uncertainties by the set comprising a Fourier series for each period, plus all the interaction terms. Comparison of observed oscillations to theoretical models then seems to indicate that the periodicities in question are approximately those of the linear normal modes of the star.

To make the case of AI Vel amenable to computation it was decided to treat only the two major periods and to consider a subset of Walraven's data, consisting of 500 points. These points were selected in groups of 100, the points within each group being contiguous and covering a time which exceeds two fundamental periods, i.e., twice  $P_0$ . The selection was such that maxima and minima of both the strong and weak varieties (see Fig. 2 of Walraven, 1955) were covered at least once.

The analysis of the data – henceforth called, for convenience, a Fourier-Interaction (or  $F$ - $I$ ) decomposition – was done by least squares fitting of the observed points to an expression of the form

$$A_0 + A_j^i \cos((i\omega_0 + j\omega_1)(t - t_0) + \phi_j^i), \quad (1)$$

where the indices  $i$  and  $j$  run over all positive and negative integers, including zero, subject to the restrictions

$$0 < |i| + |j| \leq n$$

**Table 1.** Characteristics of  $F$ - $I$  fits

Name of Fit	$n$	$SD$	$SD/AMP$	$P_0$	$P_1$	Nature of Fit
W1	1	0.089	0.099	0.11157398	0.08620749	EM;(1)
W2	2	0.042	0.046	0.11157394	0.08620767	EM;(1)
W3	3	0.031	0.035	0.11157383	0.08620774	EM;(1)
W3'	3	0.030	0.035	0.11157383	0.08620774	EM;(1); zeroth-order term subtracted out.
W3(W3,2)	3	0.113	0.126	0.11157383	0.08620774	EM;(1); Periods from W3; $\phi_0^1$ , $\phi_1^0$ from A276 and G3(W3).
W3(W3,1)	3	0.041	0.045	0.11157383	0.08620774	EM;(1); Periods from W3; $\phi_0^1$ , $\phi_1^0$ from A174 and G3(W3).
W3m	3	0.028	0.034	0.11157378	0.08620775	OM;(1)
W4	4	0.029	0.032	0.11157384	0.08620778	EM;(1)
B3m	3	0.006	0.010	0.11154774	0.08616594	OM;(1)
B3m(W3m)	3	0.007	0.014	0.11157396	0.08620754	OM;(1); All phases from W3m
G1(W1)	1	8.48	0.134	0.11157398	0.08620749	TV;(2); Periods from W1
G2(W2)	2	7.72	0.122	0.11157394	0.08620767	TV;(2); Periods from W2
G3(W3)	3	7.65	0.121	0.11157383	0.08620774	TV;(2); Periods from W3
G3	3	7.63	0.120	0.11152604	0.08624179	TV;(2)
G3(W3,2)	3	8.07	0.127	0.11157383	0.08620774	TV;(2); Periods from W3; $\phi_0^1$ , $\phi_1^0$ from A276 and W3.
G3(W3,1)	3	7.99	0.126	0.11157383	0.08620774	TV;(2); Periods from W3; $\phi_0^1$ , $\phi_1^0$ from A174 and W3.
G3(W3,2')	3	7.75	0.122	0.11157383	0.08620774	TV;(2); Periods from W3; $A_0^1$ , $A_1^0$ from A276 and W3.
G3(W3,2'')	3	8.17	0.129	0.11157383	0.08620774	TV;(2); Periods from W3; $A_0^1$ , $A_1^0$ , $\phi_0^1$ , $\phi_1^0$ from A276 and W3.
G3(W3,.4)	3	7.63	0.120	0.11157383	0.08620774	TV;(2); Periods from W3; $A_0^1$ , $A_1^0$ , $\phi_0^1$ , $\phi_1^0$ from A.475 and W3.
G4(W4)	4	7.87	0.124	0.11157384	0.08620778	TV;(2); Periods from W4.

EM = Exponentiated magnitudes; OM = Observed magnitudes;

TV = Theoretical velocities; (1)=Fit to expression (1); (2) = Fit to expression (2).

$$i+j \geq 0$$

$$j > 0, \quad \text{when} \quad i+j=0.$$

Here  $n$  is the order of the fit,  $\omega_0 = 2\pi/P_0$ ,  $\omega_1 = 2\pi/P_1$  and the zero time is always taken to be

$$t_0 = 2433291.6281 \text{ J.D.}$$

To begin our investigation we attempted a third order fit (called W3m) of expression (1) to the 500 selected points<sup>1</sup>. In this mode the fitting routine has 27 variables – namely,  $A_0$ ; the 12 amplitudes  $A_j^i$ ; the 12 phases  $\phi_j^i$ ; and, finally, the two frequencies  $\omega_0$  and  $\omega_1$ , which were also left free to be determined from the fit. The results of this exercise are given in Table 1, in which the columns are, in order, the name of the fit, the order  $n$ , the standard deviation  $SD$  of the theoretical from the observed points, the dimensionless standard deviation  $SD/AMP$  (where  $AMP$  is the total amplitude of pulsation), the periods  $P_0$  and  $P_1$ , and finally remarks detailing the nature of the fitting procedure.

We see from Table 1 that the periods obtained from the fit are extremely close to those determined by Walraven, while the standard deviation is 0<sup>m</sup>028 ( $SD/AMP=0.034$ ), perhaps not too far from the expected error in the observations themselves. Thus this result gives us confidence that 1) the 500 points chosen are representative of the complete data; and 2) that the third order  $F$ - $I$  decomposition provides an excellent description of the observed points.

However, as pointed out by Stobie (1970), the comparison of observations and theory requires that the exponentiated magnitudes ( $EM$ ) be treated, rather than the observed magnitudes ( $OM$ ). With this in mind we next exponentiated each of the 500 observed magnitudes and again applied expression (1) with  $n=3$ . The result is fit W3, described in Table 1 and in Table 2, where the amplitudes and phases from (1) are given up to  $n=3$ . We note

that, as expected, the periods obtained from  $EM$  and those from  $OM$  are nearly identical.

We now inquire as to the effect of the order of fit on the amplitudes, phases, and periods which the fit determines. W1, W3, and W4 are fits of expression (1) to  $EM$  with orders 1, 2, and 4, respectively. We note from Table 1 that by the time third order is reached, the periods have essentially converged, and that there is very little improvement in  $SD$  on passing from  $n=3$  to  $n=4$ . With regard to the amplitudes and phases, we shall be especially interested later in the first-order quantities  $A_0^1$ ,  $\phi_0^1$ ,  $A_1^0$ ,  $\phi_1^0$ . While one sees from Table 2 that a greater accuracy might be obtained, particularly in  $\phi_1^0$ , by going to a higher order we have not done so for reasons which will become apparent in subsequent discussion.

This brings us to the important question of the stability of the periods  $P_0$  and  $P_1$  over time. To attack this problem we make use of a series of observations of AI Vel obtained by Breger (1977) in 1975, i.e., approximately 22 yr after the data of Walraven. Breger's observations consist of 102 points obtained with a  $y$  filter on two nights separated by about 10 days. Applying expression (1) to Breger's  $OM$  (fit B3m), we obtain the periods and  $SD$  indicated in Table 1. Comparing with W3m, one sees that substantial changes seem to have taken place in both periods. However, such a result can be misleading. In the first place, one cannot expect that the 102 points of Breger will yield as accurate periods as the 500 points of Walraven; and, secondly, the least squares fit can be made with periods differing in the fifth or sixth decimal place with little sacrifice in accuracy, as measured by  $SD$ .

It was thus decided to do the following. We imposed upon expression (1) all of the 12 phases  $\phi_j^i$  obtained from W3m, and then attempted to fit Breger's data with only the amplitudes and periods free. Table 1 shows the result [fit B3m(W3m)]. One notes that the periods which now emerge are very close to those of W3m, while  $SD$  (compared to that of B3m) has increased by less than 0<sup>m</sup>002. It thus seems permissible to assert the following: The data of Breger (1977) are consistent with the constancy of AI Vel's periods to a few parts in  $10^6$  over 22 yr. Any stronger statement would require new data that were more extensive.

<sup>1</sup> Note that one must multiply by minus unity the values given in Walraven's (1955) Table of Observations in order to obtain the observed magnitudes

**Table 2.** *F-I* Decompositions of observed data

Term	Name of Fit								
	W1	W2	W3	W3'	W4	G1(W1)	G2(W2)	G3(W3)	G4(W4)
$A_0^1$	0.199	0.195	0.196	0.190	0.196	12.0	12.0	11.9	12.0
$\phi_0^1$	4.90	4.87	4.80	4.80	4.81	3.15	3.15	3.13	3.14
$A_1^0$	0.140	0.134	0.133	0.128	0.133	11.8	11.9	12.0	12.0
$\phi_1^0$	1.16	1.30	1.36	1.36	1.39	5.64	5.63	5.63	5.64
$A_2^0$	-	0.056	0.055	0.053	0.057	-	2.98	3.01	2.97
$\phi_2^0$	-	4.12	4.00	4.00	4.01	-	2.63	2.68	2.72
$A_2^0$	-	0.035	0.033	0.032	0.032	-	1.40	1.43	1.61
$\phi_2^0$	-	3.76	3.90	3.90	3.96	-	1.33	1.40	1.55
$A_1^1$	-	0.068	0.067	0.065	0.067	-	4.33	4.33	4.49
$\phi_1^1$	-	0.693	0.708	0.708	0.712	-	5.98	5.99	6.02
$A_1^{-1}$	-	0.049	0.048	0.047	0.048	-	0.874	0.813	0.893
$\phi_1^{-1}$	-	2.84	2.94	2.94	2.97	-	0.495	0.767	0.819
$A_3^0$	-	-	0.014	0.013	0.012	-	-	1.50	1.28
$\phi_3^0$	-	-	3.17	3.17	3.36	-	-	2.49	2.33
$A_3^0$	-	-	0.009	0.008	0.008	-	-	1.12	1.36
$\phi_3^0$	-	-	6.16	6.16	6.25	-	-	4.01	3.98
$A_1^2$	-	-	0.023	0.022	0.023	-	-	2.05	2.12
$\phi_1^2$	-	-	6.17	6.17	6.20	-	-	5.50	5.48
$A_2^1$	-	-	0.013	0.013	0.013	-	-	1.34	1.38
$\phi_2^1$	-	-	3.17	3.17	3.20	-	-	2.03	1.99
$A_{-1}^2$	-	-	0.016	0.016	0.016	-	-	1.30	1.03
$\phi_{-1}^2$	-	-	2.76	2.76	2.84	-	-	3.46	3.62
$A_{-2}^1$	-	-	0.010	0.009	0.012	-	-	0.736	1.68
$\phi_{-2}^1$	-	-	4.75	4.75	4.84	-	-	3.13	3.38

### 3. Decomposition of the Radial Velocity Curves

Spectroscopic observations of AI Vel were made in 1950 at Bosque Alegre (Argentina) and radial velocities reported by Gratton and Lavagnino (1953). The plates considered of good quality by the authors contain 148 radial velocity points stretching over a period of about two weeks. Analysis of the data by Gratton (1953) showed periods close to those given by Walraven, and application to a double sine function of the same ad hoc phase distortions (*S* distortion) used by Walraven produced what Gratton considered to be a satisfactory fit to the observations. This was taken by both authors to be an indication of the physical reality of the *S*-distortions.

We continue the present investigation by applying an expression similar to (1), namely,

$$A_0 - A_j^i \sin((i\omega_0 + j\omega_1)(t - t_0) + \phi_j^i), \quad (2)$$

to the 148 radial velocity points, after multiplying each of them by the factor  $-24/17$  to obtain the "theoretical" velocities (*TV*). The form of expression (2) is dictated by convenience for subsequent comparison with theory.

The third order fit with all variables free is reported as G3 in Table 1. One notices immediately that this fit is far inferior to those obtained for the light curves, with a dimensionless  $SD/AMP = 12\%$ . This corresponds to  $SD = 7.63 \text{ km s}^{-1}$  for *TV*, or  $SD = 5.40 \text{ km s}^{-1}$  for the observed points. While Gratton (1953) estimates the observational error to be something less than  $3 \text{ km s}^{-1}$ , there are indications in Gratton and Lavagnino (1953) that the error could be much larger. In fact, the two observers, reading the

same plates, arrived at radial velocities which differed systematically by nearly  $5 \text{ km s}^{-1}$  per point. (This difference was split in half by the authors). Furthermore, the fit obtained by Gratton (1953) after applying the *S*-distortion was no more accurate than that reported here.

It may further be seen from Table 1 that the periods given by G3 differ substantially from those emerging from W3. Here, however, the remarks made in the previous section regarding Breger's data again apply. The entry G3(W3) in Tables 1 and 2 refers to a third order fit to the radial velocity points with the periods now fixed at the values given by W3. One notes that the *SD* of G3(W3) is essentially identical to that of G3.

Now, Walraven's data is separated from that of Gratton and Lavagnino by at most 3 yr, while the interval between Walraven's and Breger's observations is at least 22 yr. Thus, taking into account the results of the previous section, we will adopt the viewpoint that the periods were, for our purposes, constant between 1950 and 1953, and shall from now on impose the periods from the light data on all velocity fits. This assumption will lead to errors in subsequent comparison of the light and radial velocity curves only if the periods have indeed changed substantially (and anomalously) in the 3 yr in question.

G1(W1), G2(W2), and G4(W4) refer to fits of expression (2) to the radial velocities with orders 1, 2, and 4, respectively, with the periods fixed in each case at the values given by the corresponding fit of the light curve. We note that the fit improves almost not at all after second order, and is in fact worse at fourth order than at third (which we take to mean that we are already "overfitting"). In addition, the first-order quantities  $A_0^1$ ,  $\phi_0^1$ ,  $A_1^0$ ,

$\phi_1^0$  change very little from  $n=1$  to  $n=4$ . The fitting routine insists on these values regardless of the order of the fit.

#### 4. Comparison with Theory

Let us now write the luminosity and radius of AI Vel in the standard theoretical form

$$L = L_{\text{stat}} \left( 1 + \frac{\delta L}{L_{\text{stat}}} \right), \quad R = R_{\text{stat}} \left( 1 + \frac{\delta R}{R_{\text{stat}}} \right),$$

where the subscript "stat" indicates unperturbed values.

The observations then yield

$$\begin{aligned} \frac{\delta L}{L_{\text{stat}}} &= \exp \left( \frac{-\ln 10}{2.5} (m - m_{\text{stat}}) \right) - 1 \\ &= A_0(L) + A_0^1(L) \cos(\omega_0(t - t_0) + \phi_0^1(L)) \\ &\quad + A_1^0(L) \cos(\omega_1(t - t_0) + \phi_1^0(L)) \\ &\quad + \text{higher order terms} \end{aligned} \quad (3)$$

and

$$\begin{aligned} V &= \frac{-24}{17} \frac{dR}{dt} = A_0(V) - A_0^1(V) \sin(\omega_0(t - t_0) + \phi_0^1(V)) \\ &\quad - A_1^0(V) \sin(\omega_1(t - t_0) + \phi_1^0(V)) \\ &\quad + \text{higher order terms.} \end{aligned} \quad (4)$$

The right hand sides of Eqs. (3) and (4) represent the  $F-I$  decompositions of the light and velocity curves, respectively, according to expressions (1) and (2). Only the terms up to first order have been indicated explicitly. The quantities actually observed are the magnitude  $m$  in Eq. (3) and the velocity  $dR/dt$  in Eq. (4).

At the same time, the theoretical models give the perturbations of luminosity and radius as follows (e.g., Simon, 1977):

$$\begin{aligned} \frac{\delta L}{L_{\text{stat}}} &= \lambda_0 \alpha_L \cos(\omega_0 t + \phi_{L0}) + \lambda_1 \zeta_L \cos(\omega_1 t + \phi_{L1}) \\ &\quad + \text{higher order terms,} \end{aligned} \quad (5)$$

$$\begin{aligned} \frac{\delta R}{R_{\text{stat}}} &= \lambda_0 \alpha_R \cos(\omega_0 t + \phi_{R0}) + \lambda_1 \zeta_R \cos(\omega_1 t + \phi_{R1}) \\ &\quad + \text{higher order terms,} \end{aligned} \quad (6)$$

where  $\lambda_0$  and  $\lambda_1$  are arbitrary scale constants.

Let us define

$$\Delta\phi_0(\text{theor}) \equiv \phi_{L0} - \phi_{R0}, \quad \Delta\phi_1(\text{theor}) \equiv \phi_{L1} - \phi_{R1}$$

and

$$\Delta\phi_0(\text{obs}) \equiv \phi_0^1(L) - \phi_0^1(V), \quad \Delta\phi_1(\text{obs}) \equiv \phi_1^0(L) - \phi_1^0(V).$$

Then Eqs. (3)–(6) indicate the following comparisons between theory and observation:

$$\begin{aligned} \Delta\phi_0(\text{theor}) &\stackrel{?}{=} \Delta\phi_0(\text{obs}) \\ \Delta\phi_1(\text{theor}) &\stackrel{?}{=} \Delta\phi_1(\text{obs}) \end{aligned} \quad (7)$$

and

$$\lambda_0 \alpha_L \stackrel{?}{=} A_0^1(L), \quad \lambda_0 \omega_0 R \alpha_R \stackrel{?}{=} A_0^1(V) \quad (8)$$

$$\lambda_1 \zeta_L \stackrel{?}{=} A_1^0(L), \quad \lambda_1 \omega_1 R \zeta_R \stackrel{?}{=} A_1^0(V). \quad (9)$$

The comparison of phase shifts given by Eq. (7) is straightforward, but for the case of the amplitudes, one can proceed in a

number of ways. Given a specific model, the scale factors  $\lambda_0$  and  $\lambda_1$  may be fixed by one of Eqs. (8) and one of Eqs. (9), whereupon the two remaining equations can be checked for agreement or disagreement. In this way the fundamental and first-overtone amplitudes provide separate measures of the agreement between theory and observation. However, the danger in this method is that the "observed" light amplitudes  $A_0^1(L)$  and  $A_1^0(L)$  will in general vary with wavelength (e.g., Simon and Stothers 1970), a situation which makes the fixing of the scale constant highly uncertain. To get around this problem, we shall instead consider the quantity

$$r \equiv \frac{\alpha_R \zeta_L}{\zeta_R \alpha_L} \frac{A_0^1(V)}{A_1^0(V)} \cdot \frac{A_1^0(L)}{A_0^1(L)} \cdot \frac{\omega_1}{\omega_0}, \quad (10)$$

which contains the amplitude ratio  $A_0^1(L)/A_1^0(L)$ , a measure which ought to be independent of the wavelength at which it is observed.

The right hand sides of Eqs. (7) and (10) are now "observed" quantities, their values obtained through the mediation of the  $F-I$  decomposition. Because the velocity data does not justify a fit surpassing third order (see Sect. 3), we shall, for consistency, also consider the third-order fit for the light, even though the phase  $\phi_1^0(L)$  changes somewhat on passage from third to fourth order. Furthermore, consistent with Eq. (3), we shall use for our  $F-I$  decomposition of the light curve the fit W3' (see Tables 1 and 2) which is obtained as follows: The zeroth-order quantity  $A_0(L)$  from W3 m is taken to be the unperturbed value  $m_{\text{stat}}$  appearing in Eq. (3). This quantity is subtracted from each of the observed magnitudes which are then exponentiated according to Eq. (3) and fit according to expression (1). We note from Tables 1 and 2 that, as expected, W3 and W3' yield nearly identical values of  $P_0$ ,  $P_1$ ,  $\phi_0^1(L)$ ,  $\phi_1^0(L)$ , and  $A_1^0/A_0^1$ .

Thus, finally, we have from the decompositions G3(W3) and W3' the "observed" quantities

$$\begin{aligned} \Delta\phi_0(\text{obs}) &= 95.8^\circ, \quad \Delta\phi_1(\text{obs}) = 115^\circ \\ r_{\text{obs}} &= 0.864 \end{aligned} \quad (11)$$

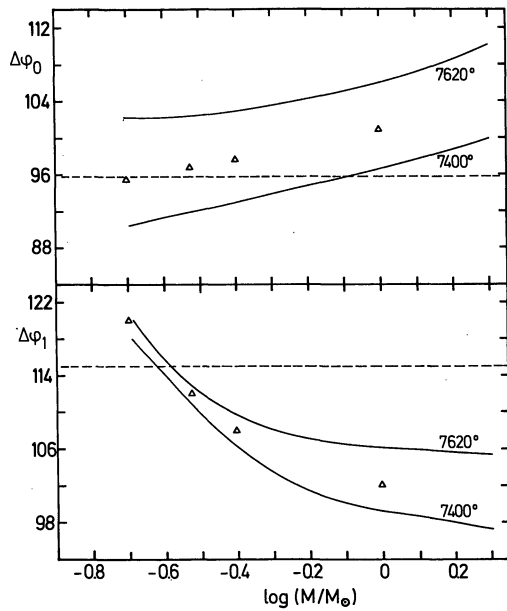
#### 5. The Theoretical Models

Static envelopes were generated as in Simon and Schmidt (1976) and Simon (1977), except that the opacity was described by the analytic formula of Stellingwerf (1975a, b). Convection was neglected. Models were integrated over a range of masses and for temperatures of 7620 K (Breger, 1977) and 7400 K (Bessel, 1969). In addition a few models at 7500 K were included. The composition was usually  $X=0.695$ ,  $Z=0.005$ , although some different abundances were employed occasionally.

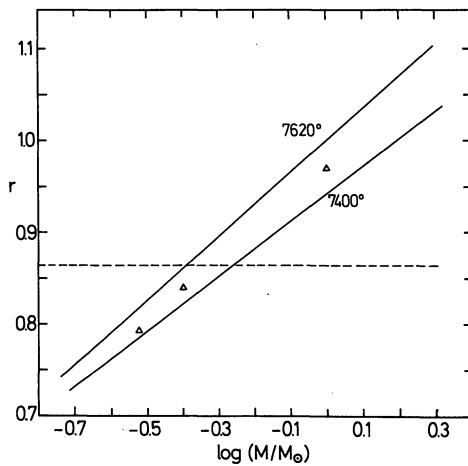
The linear nonadiabatic pulsation equations were solved as in Simon (1977). Model characteristics and pulsation results are shown in Table 3. No attempt was made to precisely match the observed periods of AI Vel, since the theoretical amplitudes and phases are insensitive to small changes in the models.

In Fig. 1 the theoretical phase shifts  $\Delta\phi_0(\text{theor})$  and  $\Delta\phi_1(\text{theor})$  are plotted against mass for the various effective temperatures. The "observed" phase shifts (Eq. 11) are indicated as dotted horizontal lines. It is clear from Fig. 1, that the high mass models cannot simultaneously reproduce both  $\Delta\phi_0(\text{obs})$  and  $\Delta\phi_1(\text{obs})$ , this being true at either temperature. Furthermore, this circumstance is defined rather well since lowering (raising) the temperature brings the high mass models into better (worse) agreement with  $\Delta\phi_0(\text{obs})$  while yielding worse (better) agreement with  $\Delta\phi_1(\text{obs})$ .





**Fig. 1.** Phase shifts VS. mass for theoretical models with suffix *A* in Table 3. Upper diagram: phase shift for fundamental mode; lower diagram: first overtone. Solid lines: models with 7620 K and 7400 K, as indicated; triangles: models with 7500 K. Dotted lines give “observed” phase shifts from *F-I* decompositions



**Fig. 2.** Amplitude ratio *r* VS. mass for theoretical models with suffix *A* in Table 3. Solid lines: models with 7620 K and 7400 K, as indicated; triangles: models with 7500 K. Dotted line gives “observed” amplitude ratio from *F-I* decompositions

The low mass models show much better agreement with the “observed” phase shifts, although no single model agrees completely at either 7620 K or 7400 K. However, because of the trends indicated in Fig. 1 a few additional models were calculated at 7500 K. These are indicated by triangles in Fig. 1. One sees that the theoretical and “observed” phase shifts will agree almost precisely for  $M \approx 0.25 M_{\odot}$ ,  $T_e = 7500$  K. Thus the comparison of linear phase shifts seems to determine not only the mass of AI Vel, but also its temperature!

Figure 2 shows the theoretical amplitude ratio *r* (Eq. 10) plotted against mass for various temperatures. The “observed”

**Table 3.** Characteristics of theoretical models

Model No.	$\frac{M}{M_{\odot}}$	$\frac{L}{L_{\odot}}$	$T_e$	$\chi$	$z$	$P_0$	$P_1$	$\Delta\phi_0$	$\Delta\phi_1$	$r$
A276	2.00	24.5	7620	0.695	0.005	0.110	0.0864	110	106	1.09
B276	2.00	24.5	7620	0.680	0.020	0.110	0.0852	112	107	1.11
C276	2.00	22.5	7620	0.530	0.020	0.108	0.0847	89.8	81.7	1.03
A176	1.00	16.0	7620	0.695	0.005	0.111	0.0861	106	106	1.00
A.576	0.50	10.0	7620	0.695	0.005	0.112	0.0871	104	108	0.904
A.276	0.20	5.00	7620	0.695	0.005	0.112	0.0842	102	121	0.738
A.176	0.10	2.90	7620	0.695	0.005	0.112	0.0831	101	139	0.636
A175	1.00	15.1	7500	0.695	0.005	0.111	0.0866	101	102	0.971
A.475	0.40	8.00	7500	0.695	0.005	0.113	0.0869	97.6	108	0.840
A.375	0.30	6.40	7500	0.695	0.005	0.112	0.0855	96.8	112	0.793
B.375	0.30	6.40	7500	0.699	0.001	0.111	0.0852	96.5	112	0.792
C.375	0.30	6.40	7500	0.680	0.020	0.114	0.0861	97.9	112	0.799
A.275	0.20	4.70	7500	0.695	0.005	0.112	0.0845	95.6	120	0.731
B.275	0.20	4.70	7500	0.599	0.001	0.111	0.0839	75.5	87.9	0.502
A274	2.00	21.5	7400	0.690	0.010	0.110	0.0855	100	97.8	1.04
B274	2.00	21.5	7400	0.790	0.010	0.109	0.0839	113	110	1.13
A174	1.00	14.3	7400	0.695	0.005	0.111	0.0865	96.5	99.4	0.941
A.474	0.40	7.45	7400	0.695	0.005	0.111	0.0857	93.0	106	0.822
A.274	0.20	4.30	7400	0.697	0.003	0.109	0.0821	90.5	119	0.734

value of *r* (Eq. 11) is indicated as a dotted line. Once more one sees that agreement comes at a low mass, somewhat dependent upon temperature, the value for  $T_e = 7500$  K being  $M \approx 0.45 M_{\odot}$ . Thus, both the linear amplitudes and the linear phases emerging from *F-I* decomposition of the observed data indicate a low mass for AI Velorum.

## 6. Sources of Uncertainty

It is now necessary to inquire as to the reliability of the rather striking result obtained in the previous section. The sources of uncertainty may be conveniently divided into three categories. First there is the uncertainty in the observations themselves vis à vis the *F-I* decompositions. Second, there is the question of the error introduced by comparing the linear quantities (phase shifts and amplitude ratios) from observation and theory. And, finally, there are the uncertainties connected with physical assumptions adopted in constructing the theoretical models. We shall attempt to consider each of these in order.

### a) The Observations

We ask the question: how accurate are the “observed” phase shifts and amplitude ratios obtained from the least-squares fitting? From the discussion of previous sections, it is clear that if the observations have a weak point, it will lie in the radial velocity data rather than in the light. Let us thus focus on the radial velocities and perform the following experiment. We shall impose on the radial velocity data the phase shifts  $\Delta\phi_0$  and  $\Delta\phi_1$  corresponding to the high mass theoretical models A 276 and A 174. The fits in which this is done are listed in Table 1 as G3(W3,2) and G3(W3,1), respectively. In each case the fitting routine has 23 free parameters to work with, corresponding to a third-order fit with the two periods fixed at the values given by

W3, and the two phases  $\phi_0^1(V)$  and  $\phi_1^0(V)$  fixed vis à vis Walraven's data at the values necessary to match the theoretical models A 276 and A 174, respectively. Thus, e.g.,

$$\phi_0^1(G3(W3,2)) \equiv \phi_0^1(W3) - \Delta\phi_0(A276), \text{ etc.}$$

From Table 1, one sees that the *SD* of G3(W3,2) and G3(W3,1) are only very slightly greater than those of G3(W3). Put another way: while the least-squares fit to the velocity data prefers values of  $\phi_0^1$  and  $\phi_1^0$  corresponding to low-mass models, it will accept those corresponding to high-mass models with very little worsening of what was already a very poor fit.

We now try a similar experiment with the amplitudes, imposing on the radial velocity data the amplitude ratio corresponding, vis à vis W3, to the model A 276. This fit is called G3(W3,2'). Thus

$$\frac{A_0^1(G3(W3,2'))}{A_1^0(G3(W3,2'))} = \frac{A_0^1(W3)}{A_1^0(W3)} \frac{\omega_0}{\omega_1} r(A276).$$

One sees in Table 1 the same disappointing result, namely an *SD* very little different from that of G3(W3).

Let us finally attempt to distinguish high and low mass models by imposing on the velocity data phases and amplitudes simultaneously. We thus construct the fits G3(W3,2'') and G3(W3,0.4). In each case the fitting routine now has 21 variables to work with, the linear amplitudes and phases all having been fixed at the values corresponding to A 276 and A 0.475, respectively. Comparing G3(W3,2'') and G3(W3,0.4) in Table 1 we find a difference of only  $1/2 \text{ km s}^{-1}$  between the respective *SD*'s. While the fact that the fitting routine prefers a low mass for AI Vel in both the amplitude and the phase comparisons is suggestive that perhaps the result is real, we nonetheless are forced to conclude that the radial velocity observations are not demonstrably accurate or extensive enough to allow the *F-I* decompositions to decide definitively between high and low mass models.

Given this situation it becomes reasonable to inquire as to what sort of observational accuracy and coverage will suffice to distinguish the models of high and low mass. A convenient way to put the question is in terms of the data of Walraven. Let us then pretend for a moment that the radial velocity phases  $\phi_0^1(G3(W3))$  and  $\phi_1^0(G3(W3))$  are correct and examine the effect of forcing on the light data phases corresponding to the high-mass models. We are thus asking how the luminosity fit will react to imposed phases that differ from the preferred ones by amounts equal to those recently forced on the velocity data.

We have called the fits in question W3(W3,2) and W3(W3,1), representing forced phase shifts corresponding to the models A 276 and A 174, respectively. Thus, for example,

$$\phi_0^1(W3(W3,1)) \equiv \phi_0^1(G3(W3)) + \Delta\phi_0(A174), \text{ etc.}$$

We note that in these cases, Table 1 tells a different story. Compared with that of W3, the *SD* of W3(W3,2) is totally unacceptable, while that of W3(W3,1) is arguably inferior (e.g., *SD* has climbed to the value yielded by the second-order fit W2). Furthermore, if we were to force on the light data the amplitude ratio from the high-mass models in addition to the phase shifts, the fits would clearly become still worse. We thus conclude that the first-order quantities ( $A_0^1, A_1^0, \phi_0^1, \phi_1^0$ ) emerging from the *F-I* decomposition of Walraven's data are true structural components of the oscillations of AI Vel and not merely artifices produced by the combination of observational errors and the vagaries of the fitting routine. It therefore follows that if we had radial velocity data of accuracy and coverage similar to that of the 500 points selected from Walraven's measurements, the observational phase

shifts  $\Delta\phi_0, \Delta\phi_1$ , and amplitude ratio  $r$  would provide meaningful physical standards for distinguishing among different theoretical models for the oscillations of AI Vel.

Before moving to the next point it is well to mention that observational-theoretical comparisons are also possible using only the luminosity data of Walraven. Such comparisons involve the second-order amplitudes  $A_0^2, A_2^0$  and phases  $\phi_0^2, \phi_2^0$ , which on the theoretical side can be calculated from the iterative theory of Simon (1977). To look at this possibility we employed the same sort of forced fits described above, and ascertained that the *F-I* decomposition does not give values accurate enough to allow the second-order comparison under present conditions. However, it is not precluded that future improvements in the iterative calculations or for that matter in fully nonlinear techniques may someday make the second-order comparison feasible.

### b) Neglect of Nonlinear Corrections

We now turn to an important problem, about which, unfortunately, not too much can presently be said. The matter in question turns upon the fact that the amplitudes and phases given by linear pulsation theory do not correspond identically to the first-order quantities emerging from *F-I* decomposition of the observed oscillations. To see this we can think heuristically of building up nonlinear oscillations through a hierarchy of terms of increasing order. Thus the linear frequencies ( $\omega_0, \omega_1$ ) produce by their interaction the second order frequencies ( $2\omega_0, 2\omega_1, \omega_1 + \omega_0, \omega_1 - \omega_0$ ), which in turn themselves interact with the linear terms to produce third-order frequencies, among them  $2\omega_0 - \omega_0 = \omega_0, 2\omega_1 - \omega_1 = \omega_1, (\omega_1 - \omega_0) + \omega_0 = \omega_1, (\omega_1 + \omega_0) - \omega_1 = \omega_0$ , etc. In this way one notes that the terms coming from the linear theory will be corrected in third order as nonlinear effects are taken into account. (These arguments appear in quantitative form in Simon, 1972). The actual size of these corrections can thus only be determined by nonlinear calculations which extend to at least third order.

Lacking these calculations, one can only make the crudest sort of estimates from the *F-I* decomposition of the observations themselves. These estimates turn upon the size of the third-order *F-I* terms, which are found to run about 10% of the first-order contributions (see, e.g., fit W3, Table 2). We shall now imagine that the third-order corrections to linear pulsation theory also have a relative size of 10%, and shall in addition take the worst possible case – namely, that in which these corrections produce maximum changes in the theoretical quantities  $\Delta\phi_0, \Delta\phi_1$  and  $r$ . Under those circumstances, one can calculate that the theoretical phase shifts would be altered by as much as  $15^\circ$  and the amplitude ratio by a factor as great as 1.5. While changes of this magnitude are large enough to obliterate the distinctions between the high and low mass models, the true corrections are likely to be much less. One reason for believing so is the fact that nonlinear corrections to linear periods (also a third-order effect; see Simon, 1972) have been found to be very small (e.g., Stellingwerf, 1975b). Another reason is simply that one doesn't expect the corrections to actually be such as to maximize the changes in phase shift and amplitude ratio. At any rate, the size and form of these corrections can actually be calculated, although this is a task for the future.

### c) Physical Assumptions

The most important sources of uncertainty in the physical structure of the models would seem to be the boundary conditions,

the treatment (or omission) of convection, and the opacity. The effects of all of these on RR Lyrae pulsation models were studied by Iben (1971). Unfortunately, the results given by Iben do not seem applicable in the present context, except for the question of convection, where Iben reported that first-overtone phase shifts showed very little sensitivity to treatment of convection in models of high temperature. On the other hand Castor (1971) found phase shifts for classical Cepheid models which differed substantially from these obtained by Baker and Kippenhahn (1965). Castor attributed these differences to the inclusion of convection in the latter models, but the two calculations also employed different boundary conditions. In general, one expects that the alteration of any of the physical assumptions will affect both phase shifts and amplitude ratios. Whether these changes will have a narrow range for models of AI Vel or whether it is in effect possible to get any values one likes with different combinations of reasonable physics is a question that remains to be studied.

One possible insight into the question of opacity changes may be obtained by studying the effects of altering the chemical composition. Comparison of models A 276 with B 276 and A 0.375 with B 0.375 and C 0.375 (Table 3) shows that reasonable changes in  $Z$  tend to make little difference in either the amplitude ratios or phase shifts. On the other hand, large (but perhaps not entirely unreasonable) changes in  $Y$  affect the phase shifts rather drastically, but have a relatively modest effect on the amplitude ratios (see B 276 vs. C 276 and A 274 vs. B 274). However one sees that the effects of changing the helium abundance are contradictory in that lowering  $Y$  can bring the high mass models into better agreement with  $\Delta\phi_0(\text{obs})$  only at the expense of a worse discrepancy with  $\Delta\phi_1(\text{obs})$ , while raising  $Y$  has the opposite effect. We conclude that simple manipulation of the chemical composition cannot make the  $2 M_\odot$  models agree simultaneously with  $\Delta\phi_0(\text{obs})$  and  $\Delta\phi_1(\text{obs})$ , nor can it duplicate  $r_{\text{obs}}$  at high mass.

We close this section by noting that both the phase shifts and amplitude ratios of our theoretical models were highly insensitive to the place in the atmosphere at which they were calculated. For the phase shifts, the differences over the entire atmosphere typically did not exceed  $1^\circ$ , while the amplitude ratios were constant to within about 1% over the range  $0.1 < \tau < 0.667$ . We note the contrary result of Iben (1971) who found much larger variations, particularly in the case of the amplitudes. The causes of this discrepancy probably include the following: 1) differences over the atmosphere are softened in the amplitude ratio  $r$  which involves two modes, as opposed to simply  $v/l$  (Iben, 1971) which involves only one; 2) Iben's results for  $v/l$  are quoted over the whole atmosphere, including the very outer layers; and 3) the RR Lyrae models of Iben have a higher ratio of luminosity to mass, which tends to increase variations over the atmosphere.

## 7. Discussion

In the present investigation we have attempted to use  $F-I$  decompositions of the observed oscillations of AI Velorum to estimate its mass. Employing the observational data and the theoretical models described in previous sections, we found the mass of AI Vel to be low ( $0.25$  to  $0.45 M_\odot$ ), but subsequent scrutiny of the radial velocity data indicated that the phases and amplitudes determined by  $F-I$  decomposition were not accurate enough to distinguish definitively between high and low mass models. On the other hand, the  $F-I$  decompositions of Walraven's light measurements did seem to have the requisite accuracy.

Thus the attempt to determine the mass of AI Vel in this manner must await a more extensive and accurate set of observed radial velocities. Furthermore, if such observations are too long in coming, an additional set of luminosities will also be necessary in order to avoid the difficulties connected with slow period changes in the star. The ideal situation would be to have a dual set of light and radial velocity measurements obtained over the same months. While such observations, particularly the radial velocities, might pose serious problems in terms of telescope time, the rewards of obtaining them could be substantial. The availability of such data ought to stimulate much theoretical work, both linear and nonlinear, looking toward the day when fully nonlinear pulsation models themselves give results accurate enough for  $F-I$  decomposition and thus direct term by term comparison with the observations. In the meantime, given the observed data, linear calculations could attempt to match the phase shifts and amplitude ratios and to quantify the effects on the models of differing physical assumptions. It is clear that much work remains to be done before the mass of AI Velorum can finally be ascertained.

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