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ABSTRACT

The horizontal-branch evolutionary tracks of Lee and Demarque (LD) are combined with the period/mean-density law of pulsation theory in an attempt to account for the observed periods and relative luminosities of 62 RR Lyrae stars in the extreme metal-poor globular cluster M15. These tests were performed for a number of assumed values for the mean luminosity of the M15 RR Lyrae sample and for different assumed boundaries for the pulsational instability strip. The results we obtained were contradictory in that no single set of models could account for all the RR Lyrae periods and luminosities while simultaneously reproducing the observed value for the ratio of blue to variable stars. The present detailed study thus agrees with the assertion by other authors that an RR Lyrae sample evolved near helium exhaustion along the LD tracks cannot explain the relatively large number of variable stars in M15. In addition, we find that this shortcoming remains even if the blue-variable ratio is assigned the generous value, $r_{obs} = 3.8$. For a main-sequence helium abundance $Y_{ms} = 0.23$, we show that the LD tracks corresponding to a largely unevolved RR Lyrae sample could reproduce the observed ratio, but still would not account for up to 15% of the stars. Furthermore, the mean mass of the RR Lyrae stars in this case, including that of the RR*d* stars, would be just under 0.90 $M_\odot$.

Subject headings: globular clusters: individual (M15) — stars: oscillations — stars: variables: other

1. INTRODUCTION

Globular clusters are traditional testing grounds of the theory of stellar evolution and thereby have implications as well for our knowledge of Galactic history and the age and distance scale of the universe. One of the best studied of these objects is the metal-poor Oosterhoff II cluster, M15. Because the horizontal branch (HB) in M15 is rich in RR Lyrae stars this cluster has played an important role in the discussion of theories of post–red giant evolution and also of stellar pulsation. The modeling of the HB in M15 has been particularly crucial in evaluating the Sandage (1982) period shift which has posed a challenge to standard models of HB evolution (see, e.g., the review by Renzini & Fusi-Pecci 1988).

Recently, Lee & Demarque (1990, hereafter LD) calculated new HB evolutionary tracks which attempt to reproduce, with standard assumptions, the Sandage period shift. The key to these tracks is the inclusion of the very late stages of core helium exhaustion. In fact, it is just the identification of these late evolutionary stages with the RR Lyrae stars in Oo II clusters like M15 which is claimed (Lee, Demarque, & Zinn 1990, hereafter LDZ) to yield the period shift and thus obviate the need for violating canonical assumptions in the treatment of HB evolution.

However, the LDZ claims have not been without their detractors. Renzini & Fusi-Pecci (1988) and Rood & Crocker (1989) have maintained that short-lived, late evolutionary stages cannot account for the M15 RR Lyraes which constitute at least 20% of the HB in that cluster. Simon (1990) has pointed out that the LD tracks seem to require an inverse correlation between mass and luminosity at given metallicity, a prediction which is not verified among the RR Lyrae stars in either ω Centauri or in the field. And, very recently, Simon & Cox (1991) have revisited and reiterated the old complaint that the HB evolutionary tracks yield masses which are in disagreement with those determined from studies of the double-mode RR Lyrae pulsators (RR*d* stars).

In the present investigation we shall make for the first time a detailed star-by-star comparison of pulsation and evolution theory, taking as our subject the RR Lyrae sample in M15. We shall find that the LD evolutionary tracks cannot account for the observed properties of the M15 RR Lyrae stars, particularly if these stars are taken to be evolved near the phase of helium exhaustion.

2. OBSERVATIONS AND THEORY

2.1. *Observational Data*

We choose for our observational sample the 62 M15 RR Lyrae stars whose data are given in Table 8 of Bingham et al. (1984, hereafter B84). From these data we employ for each star only two observed quantities—the period and mean V-magnitude. The magnitudes are then translated into luminosities as follows. We select from among the RR Lyrae stars an arbitrary fiducial star and assign it a luminosity $(\log L)_{fid}$. Then the luminosity of any other star in the sample will be given by

$$(\log L)_{obs} = (\log L)_{fid} + \frac{V_{fid} - V}{2.5}, \quad (1)$$

where V and V_{fid} are the observational magnitudes of the chosen and fiducial stars, respectively.

The use of equation (1) requires that we ignore any *differential* effects due to reddening or to bolometric corrections. The first of these is presumably very small and is always ignored. That the second effect is also small may be seen from the following argument. Table 1 of VandenBerg & Bell (1985) yields (for $\log g = 3$, $M/H = -2$) a gradient $\Delta(B.C.)/\Delta T_e \cong 8 \times 10^{-5}$. In § 3 below we shall determine temperatures for the M15 RR Lyrae stars with typical standard deviation $SD \lesssim 200$ K around a mean temperature for the sample. Thus we may estimate

$$\Delta(B.C.) \lesssim 8 \times 10^{-5}(200) \lesssim 0.016 ,$$

which translates via equation (1) into a differential error in the luminosity Δ log $L \lesssim 0.004$. That is, the average star cooler (hotter) than the sample mean will have its luminosity under- (over-) estimated by this amount. This error seems very small, and in fact we shall later demonstrate, using a specific case, that it is indeed negligible.

2.2. *Evolutionary Tracks*

We employ the recent calculations of LD who published HB tracks corresponding to various metallicities and to two values of the main-sequence helium abundance, $Y_{ms} = 0.20$ and 0.23. We choose the metallicity $Z = 0.0001$ to correspond to M15 and treat first the tracks with $Y_{ms} = 0.20$. For this case the LD calculations encompass masses between 0.64 and 0.92 $M_\odot$ with the tracks tabulated at intervals of 0.04 $M_\odot$. We shall be interested in the first three columns of the tables which give, respectively, the evolutionary time scale t (with $t = 0$ corresponding to the zero-age HB), the temperature log T_e, and the luminosity log L.

Let us now proceed as follows. We select one of the stars from the observational sample and calculate its observed luminosity, $(\log L)_{obs}$, from equation (1). We then consult the LD table corresponding to the smallest mass (i.e., 0.64 $M_\odot$) and interpolate in the log L and log T_e columns until we find the temperature $(\log T_e)_{evol}$ corresponding to $(\log L)_{obs}$. That is to say, if the M15 RR Lyrae star in question had a luminosity $(\log L)_{obs}$ and a mass 0.64 $M_\odot$, then the evolutionary tracks would predict its temperature to be $(\log T_e)_{evol}$. Now we go on to the next published LD mass, that is, 0.68 $M_\odot$ and perform the same operation as above, obtaining the value of $(\log T_e)_{evol}$ corresponding to $(\log L)_{obs}$ for 0.68 $M_\odot$. In this way we go through all eight of the LD tables, finding $(\log T_e)_{evol}$ for each of the masses 0.64, 0.68, 0.72, 0.76, 0.80, 0.84, 0.88, and 0.92 $M_\odot$.

What if the value of $(\log L)_{obs}$ from equation (1) falls completely outside the LD table for a given mass? In that case, either the evolutionary tracks cannot accommodate the mass and luminosity in question, or the required values have been "lost" between tables due to the rather coarse published mass grid. As an example of the latter instance, consider a luminosity $(\log L)_{obs} = 1.67\ L_\odot$. This value falls outside of the LD table for 0.84 $M_\odot$, but an examination of the tables for 0.80 and 0.84 $M_\odot$ clearly shows that this luminosity ought indeed to correspond to a ZAHB model with a mass in between these two masses. In all cases such as this we have interpolated between tables to find M_{evol} and $(\log T_e)_{evol}$ corresponding to $(\log L)_{obs}$. This has been done for both the ZAHB phase and for the phase of helium exhaustion.

Thus, to summarize: our use of the evolutionary tracks consists of finding in the LD tables eight pairs $(M, \log T_e)_{evol}$ corresponding to the observed value $(\log L)_{obs}$ of the luminosity for a given RR Lyrae star in M15.

We close this discussion with a brief word regarding the interpolations we have employed. They were performed in various ways. When the tabulated evolutionary data was thick with points and changed slowly (e.g., ZAHB and pre-helium-exhaustion phases for 0.80 $M_\odot$), simple linear interpolation was used. However, where changes in luminosity and temperature were rapid (e.g., helium exhaustion phase for 0.68 $M_\odot$), the data were fitted with polynomials. In these cases, considerable care was taken to ensure that the polynomial closely fit the tabulated points and that it behaved smoothly and sensibly in between.

2.3. *The Pulsation Equation*

For the fundamental mode we have employed a theoretical period/mean-density relation given by Cox (1988),

$$\log P_0 = 11.406 + 0.830 \log L - 0.659 \log M - 3.541 \log T_e\,, \tag{2}$$

and for the first overtone, the relation from Simon (1990),

$$\log P_1 = 10.933 + 0.8018 \log L - 0.6043 \log M - 3.3455 \log T_e\,. \tag{3}$$

These equations have been shown to be consistent to a very high degree (Simon & Cox 1991). Where the star to be treated is an RR*d* star, equation (3) is used since the published periods for all these objects are first-overtone periods.

Now, for a given RR Lyrae star, we use in equation (2) or (3) the observed values of log L and log P along with each of the masses, M_{evol}, obtained from the LD tables as described above. For each mass, equation (2) or (3) determines a temperature, $(\log T_e)_{pul}$, thus yielding eight pairs $[M_{evol}, (\log T_e)_{pul}]$.

To summarize here, for each of eight values of the mass we now have determined two temperatures, one of them, $(\log T_e)_{evol}$ from the evolutionary tracks and one, $(\log T_e)_{pul}$, from pulsation theory.

2.4. *Determination of Mass and Temperature*

Table 1 shows an example of results from the procedures described above. The star involved is number 31 as labeled in Table 8 of B84, with an (assumed) luminosity $(\log L)_{obs} = 1.753$. The three columns of Table 1 give, respectively, the assigned mass and the values of $(\log T_e)_{pul}$ and $(\log T_e)_{evol}$ associated with that mass. These two temperatures must be equal if a solution is to exist that satisfies both pulsational and evolutionary constraints. One notes that $(\log T_e)_{evol} > (\log T_e)_{pul}$ for 0.64 and 0.68 $M_\odot$, but this inequality has reversed by 0.72 $M_\odot$, This implies a solution corresponding to a mass in between 0.68 and 0.72 $M_\odot$. Indeed interpolating linearly between rows 2 and 3 of Table 1, we find $M = 0.718\ M_\odot$ and log $T_e = 3.831$.

We continue to refer to Table 1. For $M = 0.76$, 0.80, and 0.84 $M_\odot$ the condition $(\log T_e)_{pul} > (\log T_e)_{evol}$ continues. While this is also the case at $M = 0.88\ M_\odot$, one notices that the two temperatures have approached each other rather closely, while the gap has rewidened considerably at 0.92 $M_\odot$. Since the interpolations and fitting formulas we have used render the temperature values in Table 1 somewhat imprecise, we shall allow a generous offset within which we will say a solution has been obtained. Thus, here and elsewhere, whenever, for any mass, we have the condition $|(\log T_e)_{pul} - (\log T_e)_{evol}| \le 0.005$,

TABLE 1

EXAMPLE OF THE PROCEDURE FOR MASS AND TEMPERATURE DETERMINATION

$M/M_\odot$	$(\log T_e)_{pul}$	$(\log T_e)_{evol}$
0.64.........	3.840	3.957
0.68.........	3.835	3.876
0.72.........	3.830	3.828
0.76.........	3.826	3.806
0.80.........	3.822	3.797
0.84.........	3.818	3.801
0.88.........	3.815	3.813
0.92.........	3.811	3.788

that mass shall constitute a solution, with the temperature then obtained by averaging $(\log T_e)_{\rm pul}$ and $(\log T_e)_{\rm evol}$.

Thus, if the luminosity of star number 31 is indeed $\log L = 1.753$, then pulsation and evolution theory allow two consistent solutions: $(M, \log T_e) = (0.718, 3.831)$ and $(0.880, 3.814)$.

2.5. *The Blue-to-Variable Ratio*

For the purposes of this investigation we shall make the canonical assumption that for *any given mass*, the ratio of blue to variable stars is given by

$$\frac{n(\text{blue})}{n(\text{var})} \equiv r = \frac{t(\text{blue})}{t(\text{var})} ,$$

where t(var) and t(blue) are the respective times spent within the instability strip and blueward of the strip by models on the LD evolutionary track corresponding to the mass in question. Thus each RR Lyrae star of given mass implies the existence of r stars *of the same mass* blueward of the strip.

In that case, one notes that the two solutions given above for star number 31 have very different implications as regards the number of blue stars in M15. If the mass of this object is really 0.880 $M_\odot$, then no blue stars of the same mass are implied since the tracks for this mass do not penetrate blueward of the strip (see LD). On the other hand, if star number 31 has a mass $M = 0.718\ M_\odot$, then its existence implies that there ought to be about 6.1 stars with the same mass lying blueward of the strip. The latter number comes from an interpolation in the ratio r obtained by fitting a polynomial to the blue-variable ratios determined for the masses which appear in the appropriate LD tables (e.g., $r = 42.4$ at $M = 0.68$ and $r = 2.1$ at $M = 0.76$).

3. RESULTS FOR THE M15 SAMPLE

3.1. *Methodology*

The B84 results yield relative luminosities for their 62 star RR Lyrae sample. An absolute luminosity may then be assigned to each star, via equation (1), once the quantity $(\log L)_{\rm fid}$ is specified. This is, of course, equivalent to specifying a distance to M15. We shall try out many such specifications in order to effect an agreement between pulsation and evolution. It is clear that once a value is chosen for $(\log L)_{\rm fid}$, the average luminosity for the RR Lyrae sample, $\langle \log L \rangle$, is also fixed. In what follows we shall distinguish the different choices for M15's distance in terms of $\langle \log L \rangle$.

Before proceeding, let us once more clarify in the simplest terms how the star-by-star pulsation-evolution comparison is to be made. We choose a value of $(\log L)_{\rm fid}$ which then fixes the absolute luminosity $(\log L)_{\rm obs}$ for each star via equation (1). With $\log L$ and $\log P$ known, the period/mean-density law (eq. [2] or [3]) then gives rise to a relation

$$f_1(M, T_e) = 0 . \tag{4}$$

On the other hand, with given $(\log L)_{\rm obs}$, the evolutionary tracks yield a functional (though not analytic) relation

$$f_2(M, T_e) = 0 . \tag{5}$$

We then proceed to find pairs (M, T_e) for each star which simultaneously satisfy equations (4) and (5). While, in general, there will be multiple solutions (two, three, or more) for each star, we shall find many stars with only a single solution and others for which no solution can be determined at all.

Additionally, there will be some cases in which a formal solution to equations (4) and (5) exists but must be rejected because the temperature implied places the star outside the instability strip. In the present work we shall assign to the instability strip the rather generous width (e.g., Rood 1990) $\Delta \log T_e = 0.08$ and shall examine the effects of translating the strip boundary in temperature.

For those stars with multiple solutions it becomes necessary to select a single one in order that average properties may be determined for the sample as a whole. We shall calculate sample averages in two ways—namely (1) by selecting for each star the solution with lowest mass (Mass = Low); and (2) by selecting for each star the solution with highest mass (Mass = High). Once this has been done we will have determined a single mass and temperature for each star in the sample (except, of course, for those objects where no solution could be found). These values then determine, according to § 2.5, the number of blue stars that must exist corresponding to each of the observed RR Lyrae stars. Averaging for the entire sample we thus determine the blue-to-variable ratio predicted from the evolutionary tracks as *constrained by pulsation theory.*

3.2. $Y_{\rm ms} = 0.20$

In Table 2 we present our results ($Y_{\rm ms} = 0.20$) for five different instability strip boundaries and eight choices for the average M15 RR Lyrae luminosity, $\langle \log L \rangle$. The first column specifies $\log T_e$ at the red edge of the strip which then is assumed to extend blueward for a distance $\Delta \log T_e = 0.08$. The quantities tabulated are (1) the number of stars for which at least one solution could be found; (2) the blue-to-variable ratio predicted for the entire sample; and (3) the average mass of the sample. All of the rows in the table refer to the choice Mass = Low, except for the two footnoted rows which refer to Mass = High. For the sake of our initial discussion, we shall for the moment ignore these latter two rows.

TABLE 2

RESULTS FOR $Y_{\rm ms} = 0.20$, MASS = LOW

	$\langle \log L \rangle$							
$(\log T_e)_{\rm RE}$	1.73	1.74	1.75	1.76	1.77	1.78	1.79	1.80
	Number of Stars							
3.77.......	46	49	52	52	55	47	38	33
3.78.......	47	50	56	55	58	56	51	49
3.79.......	43	50	55	59	61	59	54	51
3.80.......	40	46	51	56	59	59	56	54
3.81.......	33	37	44	50	54	56	56	52
	B: Var							
3.77.......	2.2	4.5	9.0	14	22	32	39	57
3.78.......	2.5	5.2	8.7	15	24	36	46	64
3.79.......	2.7	5.2	9.9	16	25	36	50	66
3.79[a]......	1.2	3.8	7.7	14	23	35	49	66
3.80.......	2.9	5.7	11	17	26	39	53	66
3.81.......	3.2	6.5	12	18	28	40	53	68
	$\langle M \rangle$							
3.77.......	0.82	0.78	0.75	0.73	0.71	0.70	0.69	0.68
3.78.......	0.81	0.78	0.75	0.73	0.71	0.69	0.68	0.68
3.79.......	0.80	0.78	0.75	0.73	0.71	0.69	0.68	0.67
3.79[a]......	0.86	0.83	0.82	0.78	0.77	0.73	0.70	0.69
3.80.......	0.80	0.77	0.74	0.72	0.71	0.69	0.68	0.68
3.81.......	0.79	0.76	0.73	0.71	0.70	0.69	0.68	0.67

[a] Results in this row: Mass = High.

It is seen from Table 2 that none of the choices for strip boundary and average luminosity is able to accommodate all the stars in the sample, although the "best" choice, $(\log T_e)_{RE} = 3.79$, $\langle \log L \rangle = 1.77$, provides a solution for 61 of the 62 stars. (The recalcitrant object here is star number 102, which has the longest period in the sample.) One notes that a luminosity near $\langle \log L \rangle = 1.77$ is preferred for all values of $(\log T_e)_{RE}$ and that the number of stars that can be accommodated drops off as $\langle \log L \rangle$ is increased or decreased.

Turning to the blue-variable ratio, we see a weak dependence on the strip boundary but a strong dependence on $\langle \log L \rangle$. In particular, the ratio rises steadily as $\langle \log L \rangle$ is increased. The *observed* blue-to-variable ratio in M15 is given by LDZ as $r_{obs} = 3.8$. For $(\log T_e)_{RE} = 3.79$, this value is attained for $1.73 \leq \langle \log L \rangle \leq 1.74$.

The average mass is also seen to depend weakly on $(\log T_e)_{RE}$ and strongly on $\langle \log L \rangle$. One notes that the inferred masses fall as the luminosity rises. This effect is due to the nature of the evolutionary tracks: the lower mass stars are highly evolved and cross the instability strip at large luminosities.

Figure 1 summarizes the data of Table 2 for the case $(\log T_e)_{RE} = 3.79$, Mass = Low. All of the ordinates are plotted against $\langle \log L \rangle$. The lowest panel shows the number of "bad" stars, that is, the number of stars for which no solution could be found; this quantity is given by $NB = 62$ *minus* the relevant entry in the top segment of Table 2. The middle panel of Figure 1 displays the logarithm of the predicted blue-variable ratio obtained from the middle segment of Table 2. A dashed line indicates the observed value, $\log r = 0.58$.

In the top panel of Figure 1, we plot not the average mass of the entire sample, but rather that of the RR*d* stars. Among the 62 RR Lyraes are 11 objects considered to be double-mode pulsators (RR*d* stars) by Clement & Walker (1991). Once we have assigned masses to the stars in the sample, the average RR*d* mass may be calculated at once. We shall defer our discussion of the RR*d* masses until a later section.

It is clear from Figure 1 that the results we have obtained are contradictory. The number of bad stars reaches a minimum at $\langle \log L \rangle = 1.77$, which, however, corresponds to the totally unacceptable blue-variable ratio, $r = 25$. On the other hand, near $\langle \log L \rangle = 1.73$ or 1.74, where $\log r \approx \log r_{obs} = 0.58$, approximately 25% of the stars cannot be accounted for in a manner consistent with both pulsation and evolution.

Let us now turn briefly to the footnoted lines in Table 2. These correspond to Mass = High that is, to the choice of the highest mass in those cases where multiple solutions have been obtained. One notes only a small effect on the blue-variable ratio, with the condition $r = r_{obs}$ being pushed to slightly larger luminosity. These results can hardly be said to constitute an improvement over the choice Mass = Low.

There is one more test we can make with our data, this involving the temperatures. A similar exercise was suggested by Simon (1990) in a different context. Since the temperatures we have determined for the individual RR Lyrae stars are totally independent of the observed colors of the stars, a plot of temperature versus color constitutes an interesting consistency check. Figure 2 shows a plot of our derived temperatures versus observed $\langle B \rangle - \langle V \rangle$ colors (Table 8 of B84) for the case $\langle \log L \rangle = 1.77$, $(\log T_e)_{RE} = 3.79$, Mass = Low. The crosses, filled circles, and open circles denote RR*c*, RR*ab*, and RR*d* stars, respectively.

We note in Figure 2 a general fall of $\log T_e$ with increasing $\langle B \rangle - \langle V \rangle$. The RR*c* and RR*ab* stars are generally separated,

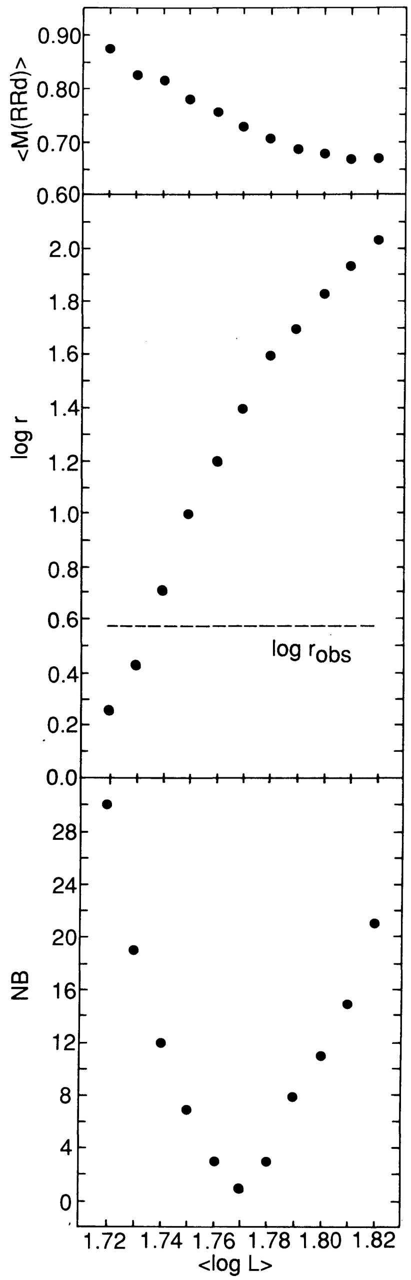

FIG. 1.—The number of "bad" stars (NB), logarithmic blue-to-variable ratio ($\log r$), and mean RR*d* mass, all vs. $\langle \log L \rangle$ for $Y_{ms} = 0.20$, $(\log T_e)_{RE} = 3.79$, Mass = Low. *Dashed line*: observed blue-to-variable ratio.

while the RR*d* stars lie in between these and, with the exception of one object, occupy a fairly narrow temperature range, $\Delta \log T_e \cong 0.015$. All this is as one would expect. A linear, least-squares fit to the points in Figure 2 yields $\log T_e = -0.32(\langle B \rangle - \langle V \rangle) + 3.95$, a relation whose slope is consistent, with that emerging from static model atmospheres (e.g., VandenBerg & Bell 1985). The average temperature of the Figure 2 sample is $\log T_e = 3.833$, with a standard deviation of about 0.01. Indeed, there is considerable scatter in Figure 2, with perhaps a dozen points lying far from their expected locations. Some of these seem attributable to errors in the color (e.g., the bluest RR*ab* star, which lies near the crosses), while others, in particular the far outliers from the mean, appear to be cases of

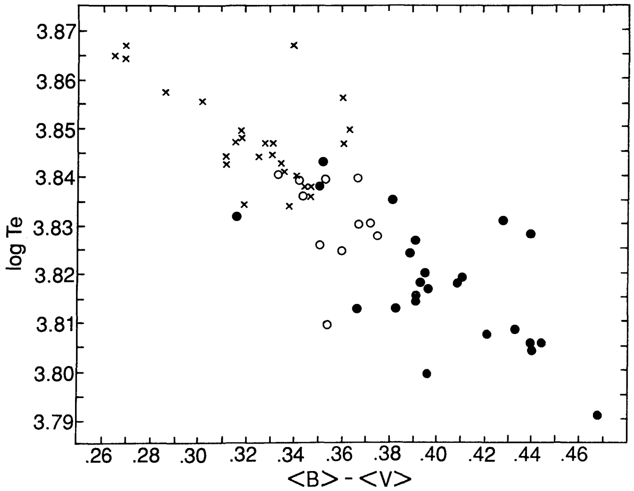

FIG. 2.—Inferred temperature vs. observed color for M15 RR Lyrae stars for the case $Y_{ms} = 0.20$, $\langle \log L \rangle = 1.77$, $(\log T_e)_{RE} = 3.79$, Mass = Low. *Filled circles*: RR*ab*; *crosses*: RR*c*; *open circles*: RR*d*.

uneasy solutions to the pulsational and evolutionary constraints. On balance, however, Figure 2 gives us little reason to doubt the results we have obtained: the derived temperatures and observed colors seem consistent. Furthermore, when the same analysis is performed for other cases in Table 2, the outcome is similar.

Before extending our treatment to the LD tracks for $Y_{ms} = 0.23$, let us return for a moment to the question of our neglecting differential bolometric corrections in equation (1). To test the effects of this approximation we again choose the case $\langle \log L \rangle = 1.77$, $(\log T_e)_{RE} = 3.79$, Mass = Low, and proceed as follows. We associate the mean bolometric correction with the mean temperature of the sample. Then, any star with a temperature below the mean will have a more negative bolometric correction, and any star hotter than the mean will have a less negative bolometric correction. The differential bolometric correction for each star is then given by $\Delta(B.C.) = 8 \times 10^{-5} \Delta T_e$ (see § 2), where $\Delta T_e = T(\text{star}) - T(\text{mean})$. Employing $\Delta(B.C.)$ appropriately in connection with equation (1) we obtain a new value of $(\log L)_{obs}$ for each star and redo the analysis as described in § 2. We find that the mean values of luminosity and temperature for the sample change by less than 0.001, while the entries in Table 2 for the number of stars, blue-variable ratio, and mean RR*d* mass become, respectively, 61, 24, and 0.72. These changes are totally negligible. The only alteration of substance is a peripheral one, namely a change in the linear fit to the log T_e versus $\langle B \rangle - \langle V \rangle$ plot, which becomes $\log T_e = -0.25(\langle B \rangle - \langle V \rangle) + 3.93$. Thus our estimate above of a negligible effect in neglecting differential bolometric corrections is vindicated.

3.3. $Y_{ms} = 0.23$

Table 3 displays our results for $Y_{ms} = 0.23$, Mass = Low (except for the footnoted lines which correspond to Mass = High). The notation is the same as in Table 2. We note the shift to higher luminosities and somewhat hotter temperatures than were obtained for $Y_{ms} = 0.20$. The largest number of stars, 59, is accommodated for $\langle \log L \rangle = 1.82$, $(\log T_e)_{RE} = 3.81$. The general behavior of the data for Mass = Low is similar to that of the corresponding case in Table 2. In particular, the same contradiction is present—namely, that the (high) values of $\langle \log L \rangle$ which are necessary to account for the largest number of stars yield blue-variable ratios far above that observed.

TABLE 3

RESULTS FOR $Y_{ms} = 0.23$, MASS = LOW

$(\log T_e)_{RE}$	$\langle \log L \rangle$: 1.77	1.78	1.79	1.80	1.81	1.82	1.83	1.84
				Number of Stars				
3.78	51	54	53	53	52	50	46	36
3.79	48	53	53	53	56	54	50	47
3.80	42	50	52	54	58	58	53	50
3.81	39	44	46	50	56	59	54	53
3.82	30	35	36	43	51	50	53	51
				B : Var				
3.78	1.8	4.8	9.3	15	31	45	35	50
3.79	2.5	5.8	9.3	18	32	49	56	73
3.79[a]	0.0	0.87	0.0	2.3	13	27	23	42
3.80	2.8	6.2	10	19	34	51	62	82
3.81	3.1	7.0	12	20	36	55	63	80
3.81[a]	1.1	3.0	4.1	6.6	21	37	39	60
3.82	3.6	7.9	13	23	39	63	64	82
				$\langle M \rangle$				
3.78	0.86	0.85	0.83	0.80	0.75	0.73	0.74	0.72
3.79	0.86	0.84	0.83	0.78	0.74	0.72	0.70	0.68
3.79[a]	0.87	0.88	0.88	0.87	0.85	0.84	0.86	0.81
3.80	0.85	0.84	0.82	0.78	0.74	0.72	0.69	0.68
3.81	0.84	0.83	0.81	0.77	0.74	0.71	0.69	0.67
3.82	0.84	0.83	0.80	0.76	0.73	0.70	0.69	0.67

[a] Results in this row: Mass = High.

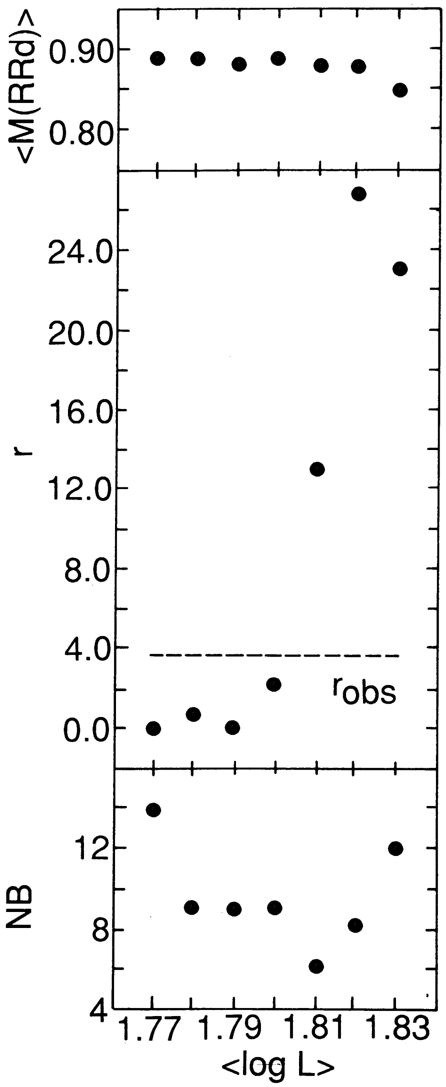

FIG. 3.—Same as Fig. 1 but for $Y_{ms} = 0.23$, $(\log T_e)_{RE} = 3.79$, Mass = High. The blue-to-variable ratio, r, is plotted directly here, rather than its logarithm.

The choice Mass = High is perhaps somewhat more promising. Figure 3 presents a plot of the $(\log T_e)_{RE} = 3.79$, Mass = High data versus $\langle \log L \rangle$. The notation is the same as in Figure 1, except that in the middle panel the blue-variable ratio is directly plotted, rather than its logarithm. The positive aspect of Figure 3 is that the minimum in NB (number of "bad" stars) and the condition $r = r_{obs}$ occur for approximately the same average luminosity, $1.80 \le \langle \log L \rangle \le 1.81$. This is, of course, a necessary condition for consistency in satisfying both the pulsational and evolutionary constraints. On the other hand, choosing $\langle \log L \rangle \cong 1.80$ still leaves up to 15% of the stars unaccounted for.

We shall undertake a discussion of these matters in the section which follows. Meanwhile we close the present section with Figure 4, a plot of inferred temperatures versus observed $\langle B \rangle - \langle V \rangle$ colors for $\langle \log L \rangle = 1.80$, $(\log T)_{RE} = 3.79$, Mass = High. The notation is that of Figure 2. A linear fit to the data yields $\log T_e = -0.27(\langle B \rangle - \langle V \rangle) + 3.92$. The mean temperature is $\langle \log T_e \rangle = 3.826$. The scatter in this diagram is less than that in Figure 2, but this is somewhat misleading since only 53 stars are included here versus 61 in Figure 2. Generally speaking, the same comments made with regard to Figure 2 also hold for Figure 4. That is, the temperature-color plot is satisfactory but does not seem capable of determining the appropriateness of different choices for Y_{ms}, $\langle \log L \rangle$ and $(\log T_e)_{RE}$, or for high or low masses in the case of multiple solutions.

4. DISCUSSION

4.1. "*Bad*" *Stars*

Let us begin this section with a question: namely, what is the extent of the uncertainty in the relative luminosities we have adopted for individual stars? We can get some handle on this by referring to the B84 study. Tables 5 and 6 of that study give values of mean V-magnitude obtained from two different data sets, the "Bologna" (B84) and "SKS" (Sandage, Katem, & Sandage 1981), respectively. These tables have 50 RR Lyrae stars in common, for which we have found the root mean square deviation of the V-magnitude ("Bologna" *minus* "SKS") to be 0.033, corresponding to an uncertainty $\Delta V = \pm 0.017$ mag. Since the values given for V-magnitude in Table 8 of B84 represent an average of Tables 5 and 6, weighted according to the quality of the light curves, the uncertainty may be actually smaller than this. On the other hand, our neglect in equation (1) of differential effects due to reddening and bolometric corrections will increase this uncertainty. Taking the latter effects to be somewhat larger than the former, we may crudely estimate the net uncertainty for individual stars to be $\Delta V \lesssim \pm 0.025$ or $\Delta \log L \lesssim 0.01$.

Let us now discuss the question of "bad" stars, using the data of Figure 3 as an example. Table 4 gives the specific identification numbers of the "bad" stars as a function of $\langle \log L \rangle$ for the case $Y_{ms} = 0.23$ and $(\log T_e)_{RE} = 3.79$ or 3.80 (footnote column). For $\langle \log L \rangle = 1.80$ (close to the point where NB = minimum and $r = r_{obs}$), there are nine "bad" stars. However, a change of their *individual* luminosities by amounts $\Delta \log L = \pm 0.01$ would suffice to remove six of these (star numbers 12, 20, 25, 28, 36, and 97). A seventh star (no. 57) could be accounted for by shifting the instability strip slightly blueward. This would leave only two stars without solutions—namely numbers 55 and 102, the longest period stars in the sample.

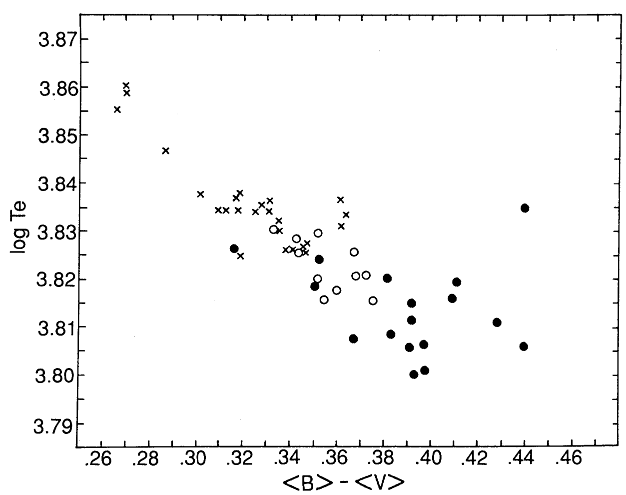

FIG. 4.—Same as Fig. 2 but for $Y_{ms} = 0.23$, $\langle \log L \rangle = 1.80$, $(\log T_e)_{RE} = 3.79$, Mass = High.

TABLE 4

IDENTIFICATION NUMBERS OF "BAD" STARS, $Y_{ms} = 0.23$, $(\log T_e)_{RE} = 3.79$

$\langle \log L \rangle$	Identification Number																										
1.78	...	...	6	...	...	...	...	20	...	...	...	...	...	...	...	46	...	...	52	55	...	...	65	67	...	97	102
1.79	...	...	6	8	...	...	...	20	...	...	23	...	...	...	...	...	...	...	...	55	57	...	65	...	...	97	102
1.80	...	...	...	...	...	...	12	20	...	...	...	25	28	...	36	...	...	...	...	55	57	...	...	...	...	97	102
1.80[a]	...	...	...	...	...	...	12	20	...	...	...	25	28	...	36	...	...	...	...	55	...	...	...	...	...	97	102
1.81	...	4	...	...	...	...	...	...	...	...	...	25	28	...	...	...	...	...	...	55	57	...	...	...	...	...	102
1.82	2	4	...	...	...	11	...	...	...	...	...	...	...	...	...	...	...	50	...	55	57	62	...	...	...	...	102
1.83	2	4	...	...	9	11	...	...	21	22	...	...	...	32	...	...	48	50	...	...	57	62	...	...	74	...	...

[a] $(\log T_e)_{RE} = 3.80$.

However, we will argue here that unless nature is playing some special trick, the "bad" stars cannot be attributed to errors in luminosity. The reason is that the same uncertainty invoked above ($\Delta \log L = \pm 0.01$) would be enough to move five stars *into* the bad column, namely, 4, 6, 8, 23, and 65. In fact, as Table 4 shows, apart from stars 55 and 102, the "cast of bad characters" changes so much with $\langle \log L \rangle$ that there are no other stars that one might reasonably suspect to be the victims of observational error. Thus it appears that the existence of "bad" stars represents a true deficiency of the theoretical framework we are testing and not merely an artifact caused by inaccuracies in the observational data. This argument will also apply to the data for $Y_{ms} = 0.20$, presented in Table 1 and Figure 3.

4.2. *The* RR*d Mass*

The two observed periods in RR*d* stars yield masses via pulsation theory, essentially by the application of equations (2) and (3). The "canonical" values of the RR*d* mass are 0.55 $M_\odot$ for an Oo I cluster and 0.65 $M_\odot$ for an Oo II cluster such as M15. However, these values are controversial (see Petersen 1990, 1991; Simon & Cox 1991) and could turn out to be significantly larger, particularly considering the potential effects of new opacities recently published by Iglesias & Rogers (1991).

It is clear from Figures 1 and 3 that the LDZ tracks are not compatible with the *canonical* Oo II RR*d* mass, $M = 0.65\ M_\odot$. At the relatively low values of $\langle \log L \rangle$ needed to match the observed blue-to-variable ratio in M15, the evolutionary tracks predict RR*d* masses which are considerably higher. This is particularly true in the case depicted in Figure 3 ($Y_{ms} = 0.23$, $(\log T_e)_{RE} = 3.79$, Mass = High), where the implied RR*d* mass is close to 0.90 $M_\odot$.

Could the latter result be changed by mixing the cases Mass = High and Mass = Low, such that when there are multiple solutions we choose the largest mass in all instances, except when an RR*d* star is involved, in which case we choose the smallest mass? For $\langle \log L \rangle = 1.80$, $(\log T_e)_{RE} = 3.79$, it turns out that there are three RR*d* stars with multiple solutions which yield low masses with values 0.71, 0.71, and 0.78 $M_\odot$, respectively. This reduces the average RR*d* mass to 0.83 $M_\odot$ and increases the blue-variable ratio to $r = 3.4$, very close to the observed value. However, it still leaves eight RR*d* stars with mass $\geq 0.88\ M_\odot$.

We conclude that the LDZ tracks and the RR*d* masses cannot be made consistent, unless the latter are raised substantially. This could perhaps be accomplished by opacity effects, as mentioned above.

5. FINAL REMARKS

We begin by noting that when the mean RR Lyrae mass is less than 0.80 $M_\odot$, the sample is dominated by highly evolved stars which have entered the instability strip from the blue. This is evident from the LD tables and is reflected in the high blue-variable ratios (see Tables 2 and 3) that correspond to these cases. These high ratios seem to rule out the LDZ contention that the M15 RR Lyrae stars are evolved, at least insofar as concerns the LD tracks that we have treated. However, as $\langle M \rangle$ rises above 0.80 $M_\odot$, less evolved stars begin to constitute more and more of the RR Lyrae sample, including objects which lie near the ZAHB. For the $Y_{ms} = 0.23$, Mass = High cases with small $\langle \log L \rangle$ (left side of Table 3, footnoted lines), the sample is dominated by such stars, most of which never leave the instability strip during HB evolution.

In previous sections we have taken the blue-variable ratio observed in M15 to be $r_{obs} = 3.8$ as given by LDZ. But this number has recently been criticized by Rood (1990) who points out that if one neglects the very blue clump of horizontal branch stars in M15, a much smaller blue-variable ratio is implied, namely $r_{obs} = 1.6$. According to Rood, if the smaller value is adopted, the LD tracks for M15 fail to demonstrate the Sandage period shift. However, in the present work we find a stronger result—namely, that the M15 RR Lyrae stars cannot be evolved near helium exhaustion along the LD tracks even if r_{obs} is taken to be 3.8. This result is implicit in Tables 2 and 3 (see also Fig. 1) and is *independent of any appeal to the masses of the* RR*d stars.*

If one is willing to accept a situation in which about 15% of the M15 RR Lyrae stars are not accounted for, then the remainder of the sample can be explained by the case $Y_{ms} = 0.23$, $\log L \lesssim 1.80$, Mass = High. In that event, the mean mass of the RR Lyrae stars in M15, including the RR*d* stars, would be just under 0.90 $M_\odot$ (see Table 3 and Fig. 3).

At this point let us recall that the pulsation input to the present study consists of the period/mean-density relation given in equations (2) and (3). To determine RR*d* masses an extremely accurate rendition of these equations is necessary, and, as indicated earlier, this has been a matter of some controversy and uncertainty. However, the arguments made in the present section turn upon the kind of coarse version of equations (2) and (3) which is used in practically every theoretical study of globular cluster RR Lyrae stars. If these equations are incorrect at this level, than all of these studies need drastic revision. We do not believe this to be the case.

On the other hand, if the LD tracks are at fault here, a potential remedy immediately comes to mind, namely that of oxygen enhancement. There is considerable evidence in the literature for such enhancement (see Abia & Rebolo 1989, and

references therein), and salutary effects on the problem of M15's horizontal branch have been suggested by Caputo, Castellani, & di Gregorio (1983). Recently, Dorman, Lee, & VandenBerg (1991) published oxygen-enhanced tracks for models appropriate to the study of M15. In addition to red loops (also present in LD), these tracks execute substantial blue loops, a circumstance which seems to add extra flexibility in meeting the pulsational constraints. Indeed, it is argued by Dorman et al. that the oxygen-enhanced models *are* able to reproduce the RR Lyrae population of M15.

While the Dorman et al. (1991) RR Lyrae sample taken from Sandage (1990) is virtually identical to the one we have treated, their method of comparing theory and observations requires the specification of both a reddening and a temperature-color relation. On the contrary, the procedure employed in the present work, while time-consuming, uses on the theoretical side only the overt, global properties of the models (i.e., M, L, T_e) and on the observational side the two best-determined quantities—namely, period and relative luminosity.

Although it is well known that oxygen enhancement can yield evolutionary models with masses close to the canonical RR*d* value (a circumstance which may or may not be desirable given the size of opacity augmentations found in recent calculations), it would be of great interest to test whether the Dorman et al. (1991) tracks can account for the entire M15 RR Lyrae sample while matching the observed ratio of blue-to-variable (and in this case perhaps, also, red-to-variable) stars. Such a test could surely be performed along the lines of the present investigation but for the moment must await publication of the Dorman et al. evolutionary calculation in full numerical form.

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