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THE RRd MASSES REVISITED

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ABSTRACT

We reexamine the RRd mass determination using both new and previously existing linear pulsation models. Our conclusion is that the weight of evidence remains strongly with the “canonical” values ($0.55 M_{\odot}$ for Oosterhoff I clusters and $0.65 M_{\odot}$ for Oosterhoff II clusters), unless the Population II metal opacities turn out to be very high.

Subject headings: stars: interiors — stars: pulsation — stars: RR Lyrae

1. INTRODUCTION

The question of the RR Lyrae masses is both important and controversial. Our knowledge of these masses has implications not only for our understanding of horizontal branch evolution, but also for the ages of globular clusters and the history of the Galaxy (see, e.g. Lee, Demarque, & Zinn 1990, hereafter LDZ; Sandage 1990).

In a pivotal investigation, Cox, Hodson, & Clancy (1983, hereafter CHC) determined RR Lyrae masses employing the RRd stars, which pulsate simultaneously in the fundamental and first overtone modes. Comparing the results from linear nonadiabatic (LNA) pulsation models with observed RRd periods via the Petersen diagram, (P_1/P_0 vs. P_0), CHC were able to infer masses of $M \sim 0.55 M_{\odot}$ and $M \sim 0.65 M_{\odot}$ for the RRd stars in Oosterhoff I (Oo I) and Oosterhoff II (Oo II) clusters, respectively.

While recent work on white dwarfs (Bergeron, Saffer, & Liebert 1990) has yielded results compatible with RR Lyrae masses as small as $0.55 M_{\odot}$, the CHC masses nonetheless disagree with those emerging from standard horizontal branch tracks. These tracks predict larger values ($\sim 0.75 M_{\odot}$) for the RR Lyrae stars in general and a smaller difference in mass between the Oo I and Oo II clusters (LDZ). In this connection, LDZ have made the suggestion that the CHC masses may be uncertain by significant amounts. Indeed when Kovács (1985) used the Petersen diagram along with a different set of linear pulsation models, he found RRd masses which exceeded those of CHC by as much as $0.1 M_{\odot}$.

Subsequently, Cox (1988) recalculated the RRd masses using a much wider grid of LNA models and examining the effects of changes in the helium abundance and convective mixing length and of using the Stellingwerf formula in place of tabular opacities. The new study produced results which were essentially the same as those of CHC. More recently, Simon (1990a) investigated the effects of enhanced metal opacities (e.g., Iglesias, Rogers, & Wilson 1990) on the RRd masses. He found that such effects should be small unless the opacity enhancement is greater than anticipated. This result supports the original CHC masses.

However, in another recent study Petersen (1990) concluded that a number of effects, especially those due to luminosity and metallicity, make the interpretation of the Petersen diagram

rather problematical, thus leading to significant uncertainties in the RRd masses. This topic continues to be highly controversial (see, e.g., the review of Simon 1990b, including the comments of other workers).

Our purpose in the present Paper is to try to clarify some of these issues. To do so we shall review results from a large number of linear pulsation models, calculated with two different pulsation codes and employing a variety of physical assumptions and parameters. Our conclusion shall be that there is no good reason to assume that the RRd masses are different from what CHC found them to be, unless the Population II opacities turn out to be very high.

2. EFFECTS OF NONADIABATICITY AND METALLICITY

In the study by Petersen (1990), adiabatic pulsation models were constructed with $M = 0.55 M_{\odot}$ and two luminosities (solar units), $\log L = 1.80$ and $\log L = 1.60$. The chemical composition was $X = 0.700$, $Z = 0.004$. The loci of the two luminosity sequences in the Petersen diagram are reproduced in Figure 1a, where the dot denotes a crude fiducial observed point for the RRd stars in Oo I clusters. The dashed line ($\log L = 1.80$) coincides with the original CHC locus (not shown here) and essentially passes through the observations. However, the solid line ($\log L = 1.60$) lies considerably below the observed point, implying that a mass larger than $0.55 M_{\odot}$ is necessary to match the observations. Since a difference in period ratio $|\Delta(P_1/P_0)| \cong 3 \times 10^{-4}$ corresponds to a mass difference $|\Delta M| \cong 1 \times 10^{-2} M_{\odot}$ (Petersen 1990), the lower line in Figure 1a points to a mass $M \cong 0.60 M_{\odot}$ for the Oo I RRd stars.

Figure 1b presents the results of new calculations for the same parameters as above, but employing an adiabatic version of the LNA pulsation code described by Aikawa & Simon (1983). Convection is neglected in this code and the Stellingwerf (1975a, b) opacity formula is used. The loci in Figure 1b agree crudely in slope with those in Figure 1a but lie somewhat lower in P_1/P_0 .

Figure 1c, the Aikawa code is again used for the same parameters but in its full nonadiabatic form. We note that while the $\log L = 1.80$ locus does not change much from Figure 1b, the $\log L = 1.60$ line is raised enough that the two loci alter their relation, with the lower luminosity now yielding

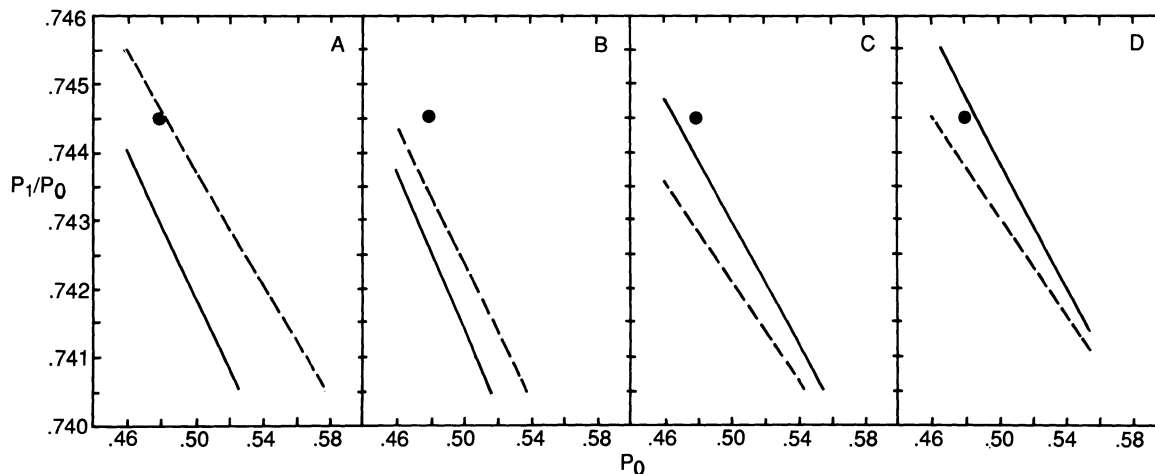


FIG. 1.—Petersen diagram for linear RR Lyrae models. The dot represents a fiducial observed point for Oo I RRd stars. Solid line: $\log L = 1.60$; dashed line: $\log L = 1.80$. (a) models from Petersen (1990), $Z = 0.004$; (b) present work, adiabatic, $Z = 0.004$; (c) present work, nonadiabatic, $Z = 0.004$; (d) present work, nonadiabatic, $Z = 0.001$.

higher values of P_1/P_0 . The reason for the larger effect at $\log L = 1.60$ is well known: namely, the increasing importance of nonadiabatic corrections as the temperature falls. For fixed mass and given period, the low-luminosity models must be cooler and thus suffer stronger effects due to nonadiabaticity.

Finally, in Figure 1d we again employ the nonadiabatic Aikawa code but change the chemical abundance parameters to $X = 0.700$, $Z = 0.001$, i.e., the same composition used by CHC. We note that the two loci both move up in period ratio and now closely span the observed domain, in essential agreement with the result of CHC.

In our opinion, the plots displayed in Figure 1 argue strongly against any code dependence or important luminosity effect in deriving RRd masses. In fact, there is every indication that, had Petersen (1990) included nonadiabatic effects (which certainly exist in stars) and employed a reasonable metallicity ($Z = 0.004$ is clearly much too large for either Oo I or Oo II clusters), he would have obtained a result very similar to that of CHC.

3. EFFECTS OF CONVECTION, HELIUM ABUNDANCE AND FORMULATION OF STANDARD OPACITIES

Cox (1988) calculated a large grid of models and published fitting formulas wherein the fundamental period, P_0 , and the period ratio, P_1/P_0 , are given as functions of L , M , and T_e for the following four cases: (1) Standard ($X = 0.700$, $Y = 0.299$; tabular opacities; mixing length convection);¹ (2) Same as standard except $Y = 0.199$; (3) Same as standard except Stellingwerf opacity formula is used; (4) Same as standard except convection is neglected.

We have used the Cox fitting functions to calculate periods and period ratios for a variety of models with parameters given in Table 1. The masses $M = 0.55$ and 0.60 are chosen to study the Oo I RRd stars while $M = 0.65$ and 0.70 are applied to the Oo II RRd stars. For each mass we employ three luminosities. The highest value of $\log L$ is chosen so a model with period corresponding to observed RRd periods ($0.48d$ for Oo I and

$0.55d$ for Oo II) will lie somewhat blueward of the F -mode blue edge. The lowest luminosity is selected so that a model of appropriate period for Oo I or Oo II will have a temperature $T_e = 6400$ K. These two cases clearly represent extreme limits on the luminosity, since (1) so far as we know, double-mode pulsation cannot occur for a linearly stable fundamental mode; and (2) the RRd stars cannot be as cool as 6400 K if standard color-temperature calibrations have even approximate accuracy. The third luminosity for each mass is selected simply to lie midway between the two extremes.

In Figure 2 we present an expanded Petersen diagram (see Clement et al. 1986) in which the observed domains of RRd pulsation have been indicated with boxes, the left box for Oo I, the right for Oo II. The symbols represent theoretical periods calculated from the fitting formulas. For each mass there are 12 such symbols corresponding to the four cases given above for each of the three luminosities listed in Table 1. One sees that the Oo I RRd mass emerging from these models is very close to $0.55 M_\odot$, irrespective of any effects due to “nonstandard” convection or helium abundance, or to use of the Stellingwerf formula. For the Oo II case the mass appears to be closer to $0.70 M_\odot$ than to $0.65 M_\odot$. However, the metallicity employed in the models was $Z = 0.001$, nearly 10 times larger than appropriate for an Oo II cluster like M15. We have done a few additional calculations which show that reducing the metal abundance to $Z = 0.0001$ raises the calculated period ratios by a nearly constant amount $\Delta(P_1/P_0) = 0.0003$. Applying this increase to Figure 2, one infers Oo II RRd masses roughly midway between 0.65 and $0.70 M_\odot$. These results depart very little from those of CHC.

TABLE 1
VALUES OF $\log L$ FOR LNA MODELS IN FIGURE 2

MASS (M_\odot)	TEMPERATURE (K)		
	7200	6800	6400
0.55.....	1.70	1.60	1.50
0.60.....	1.73	1.63	1.53
0.65.....	1.82	1.73	1.64
0.70.....	1.84	1.76	1.67

¹ Various values of mixing length were employed in these models. However, we have performed calculations which show only negligible changes in period ratio for mixing lengths between 1.0 and 2.5 times the pressure scale height.

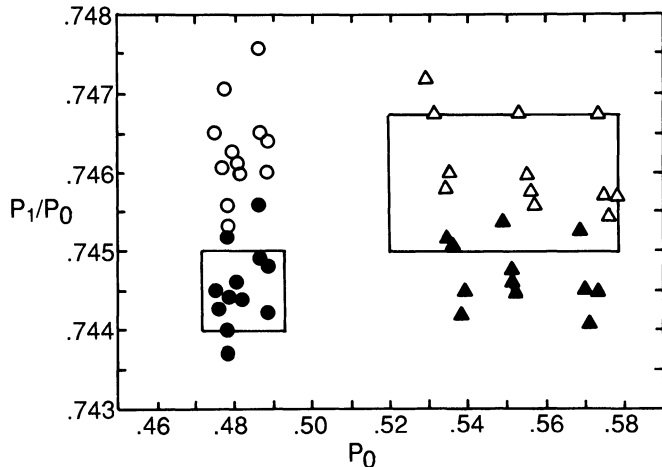


FIG. 2.—Petersen diagram, observed and theoretical. *Left box*: observed Oo I RRd stars; *right box*: observed Oo II RRd stars. Theoretical models are as follows. Dots: $0.55 M_{\odot}$; open circles: $0.60 M_{\odot}$; filled triangles: $0.65 M_{\odot}$; open triangles: $0.70 M_{\odot}$.

How do the periods and period ratios calculated by Cox (1988) compare with those obtained using the Aikawa code? We compare our current models (no convection, Stellingwerf formula) with the two most similar series from Cox (1988), namely Case 3 (standard convection, Stellingwerf formula) and Case 4 (no convection, tabular opacities). Figure 3 shows a plot of $(P_1/P_0)_A$ (Aikawa code) versus $(P_1/P_0)_C$ (Cox 1988) for models with the same parameters. The dots show the comparison of the current models with Case 3 of Cox (1988) and the open circles with Case 4. The solid line is the locus $(P_1/P_0)_A = (P_1/P_0)_C$. We note that in all cases the two codes give period ratios which agree to within 0.001. This agreement would be even closer had we compared codes with exactly the same physics and used the actual Cox (1988) models instead of fitting formulas which must necessarily possess some random error. We conclude that the two codes give nearly identical results.

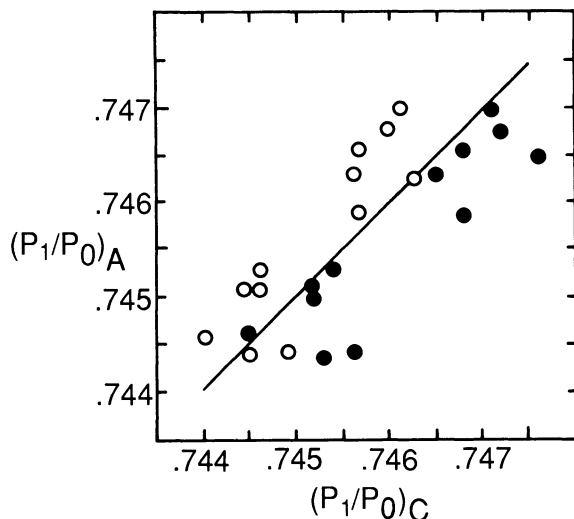


FIG. 3.—Period ratios from Aikawa code vs. Cox 1988 (dots: Case 3; open circles: Case 4). Solid line: $(P_1/P_0)_A = (P_1/P_0)_C$.

4. THE KOVÁCS CALCULATION

Kovács (1985) used an adiabatic pulsation code to calculate periods and period ratios for RRd models. He also investigated the importance of nonadiabatic corrections and encountered no systematic effect, although the adiabatic and nonadiabatic period ratios sometimes disagreed by amounts $0.001 \lesssim |\Delta(P_1/P_0)| \lesssim 0.002$. However, there was a strong systematic discrepancy vis-à-vis the CHC results in that Kovács found period ratios that were smaller by 0.002 to 0.003, which translates into inferred masses that are larger by 0.07 to $0.10 M_{\odot}$. Since CHC and Kovács (1985) both constructed models with $Z = 0.001$, the discrepancy found by Kovács has no contribution from metallicity.

In Figure 4 we further probe the influence of nonadiabaticity by plotting four loci from different calculations. In all cases, the mass is $0.65 M_{\odot}$ and the composition $X = 0.700$, $Z = 0.001$. The dot indicates a fiducial point for observed Oo II RRd stars. The upper and lower solid lines correspond, respectively, to the CHC and Kovács (1985) results, the former produced with nonadiabatic, and the latter with adiabatic, models. The two dashed lines represent our own adiabatic calculations (Aikawa code) at the two extreme luminosities, $\log L = 1.82$ and $\log L = 1.64$. Comparing the dashed loci to the CHC line, one observes, in disagreement with Kovács, a systematic effect wherein neglect of nonadiabatic terms lowers the period ratios by about 0.0015. This could account for as much as 40% of the discrepancy between the CHC and Kovács loci. However, this still leaves the Kovács period ratios smaller by about 0.002 and the inferred masses larger by approximately $0.07 M_{\odot}$. Such a difference is not negligible and requires further discussion. We shall return to it below.

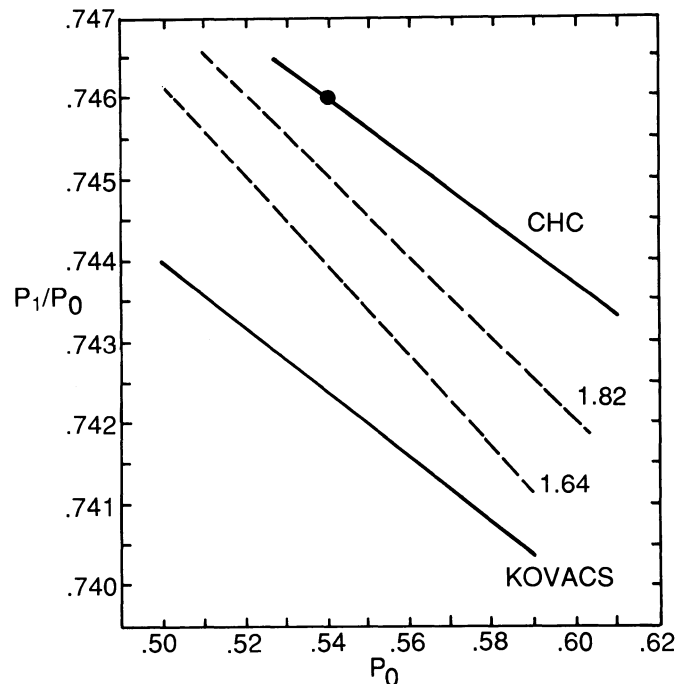


FIG. 4.—Petersen diagram loci for $M = 0.65 M_{\odot}$, $Z = 0.001$. Solid lines: CHC and Kovács (1985) as indicated. Dashed lines: present work, $\log L = 1.82$ and 1.64 as indicated. Dot: fiducial observed point for Oo II RRd stars.

5. EFFECT OF THE METAL OPACITY

It is now well-known that the problem of discrepant period ratios in Population I pulsators could be solved if the opacity due to heavy elements were increased over standard values by factors of 2 to 3 (Simon 1982; Andreasen 1988). Indeed, new opacities calculated by Iglesias et al. (1990) for a classical Cepheid model show an increase of about the required magnitude. Since the effect of augmented opacities is to lower the period ratio P_1/P_0 , the application of such opacities to RR Lyrae models could result in higher inferred RRd masses.

The size of such an effect has been estimated recently by two authors (Simon 1990a; Petersen 1990) who reached somewhat different conclusions. Whereas the two studies agreed that only a small effect should be present at the very low metallicities which characterize Oo II RR Lyraes, Petersen (1990) found indications that significant increases could be obtained in the masses of Oo I stars while Simon (1990a) dismissed such a possibility. However, this disagreement is more apparent than real since it was based upon the extrapolation to RR Lyrae models of two different ad hoc opacity formulas originally designed for the Population I Cepheid regime.

In fact, the Petersen & Simon studies both found that a significant effect would be present for $Z \gtrsim 0.001$ provided that the metal opacities were *high enough*. In that case, the inferred Oo I RRd masses could be raised by $0.05 M_\odot$ or even more. Thus the different conclusions reached by Petersen (1990) and Simon (1990a) were actually a function of different predictions as to the size of possible opacity increases in RR Lyrae stars. Since new opacities for Population II compositions have yet to be published, the resolution of this question must be left to the future.

6. DISCUSSION

In the present study, we have shown that the Los Alamos (Cox 1988; CHC) and Aikawa (Aikawa & Simon 1983) LNA pulsation codes agree very closely on RR Lyrae period ratios and thus on inferred RRd masses. These codes share the Castor (1971) algorithm but were otherwise written independently and contain different subroutines. We have further argued that any discrepancy between the results produced by these calculations

and those of Petersen (1990) are very likely explained by Petersen's (1990) use of an adiabatic approximation and in appropriate metallicity.

On the other hand, the Kovács (1985) calculation yielded systematically smaller period ratios and, consequently, RRd masses which are larger by as much as $0.1 M_\odot$. This result disagrees with those of all other published studies (assuming the Petersen calculation is suitably amended) and is not so easily explained. The Kovács models were constructed with the pulsation code described by Dziembowski (1977). This is a complex program, designed for nonradial pulsations with the radial modes constituting a limiting case obtained by setting certain constants equal to zero. We suggest that a possible cause for the discrepant Kovács period ratios might lie in the application of a nonradial code to radial modes. Be this how it may, to our knowledge the Kovács results have never been reproduced. Until they are, or until a detailed analysis is made of the Dziembowski (1977) code in the context of the RRd mass calculation, it does not seem prudent to assign the Kovács calculation a high weight.

Thus we are left essentially with the original CHC results: $0.55 M_\odot$ for Oo I RRd stars and $0.65 M_\odot$ (or slightly higher) for Oo II RRd stars. The only major uncertainty remaining here seems to be the question of the heavy element opacities. Should these turn out on the high side, the inferred masses of RRd stars in a relatively metal-rich Oo I cluster such as IC 4499 ($Z = 0.0008$) could be as large as $0.60 M_\odot$ or even somewhat larger. On the other hand, this would still leave the two RRd stars in M3 ($Z = 0.0004$) with smaller masses, say 0.55 to $0.60 M_\odot$. At the same time, the masses of the Oo II RRd stars would remain unchanged.

Thus while a large increase in metal opacity offers the prospect of somewhat better agreement with evolution theory by narrowing the gap between RR Lyraes in Oo I and Oo II clusters, the absolute RRd masses remain smaller than those obtained from standard horizontal branch models. When one adds to this the distinct possibility that the opacity increases may not turn out to be large enough to influence this problem at all, it would seem advisable that more attention be given to expedients such as oxygen enhancement which is known (LDZ) to reduce the evolutionary masses.

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