2014

Optimal Energy Scheduling for a Smart Entity

Amin Fakhrazari
University of Nebraska-Lincoln, amin@huskers.unl.edu

Hamid Vakilzadian
University of Nebraska-Lincoln, hvakilzadian@unl.edu

F. Fred Choobineh
University of Nebraska-Lincoln, fchoobineh@unl.edu

IEEE TRANSACTIONS ON SMART GRID, VOL. 5, NO. 6, NOVEMBER 2014

Pages: 2919 - 2928, DOI: 10.1109/TSG.2014.2319247

Optimal Energy Scheduling for a Smart Entity

Amin Fakhrazari, *Graduate Student Member, IEEE*, Hamid Vakilzadian, *Senior Member, IEEE*, and F. Fred Choobineh, *Member, IEEE*

***Abstract*—Real-time availability of electricity prices via a smart power grid has a potential bilateral benefit to electricity users and suppliers. The users can reduce their costs by consuming energy during low-price hours and balancing their energy usage during other hours. This in turn benefits energy utility companies by reducing their peak power demand. This article describes an optimal shrinking horizon model for electricity-consuming units based on user preferences. The proposed model optimizes the end user's electricity cost while meeting preferred comfort levels. The user can set preferences in the model using a tristate flexibility parameter for each electric-power-consuming unit. The electricity price model used in the optimization model is general and covers all pricing schemes in the electricity market today. The model derived can be described by a simple mixed integer linear program and solved by most optimization software in a short time. The most distinguishing characteristics of our proposed model are its simplicity, generality, comprehensibility, and ease of implementation. Simulation results are used to verify the model's performance in reducing consumer electricity costs and satisfying comfort preferences.**

***Index Terms*—Energy management, optimal scheduling, shrinking horizon scheduling, smart grids.**

NOMENCLATURE

Parameters:

$\mathcal{A}$	Set of electricity consuming units.
C_k	Energy price increase coefficient for the energy consumed between E_k and E_{k+1}.
E	Total amount of energy scheduled for an entity in planning horizon $\mathcal{H}$.
$E(t_h)$	The overall energy consumption by an entity in time interval t_h.
$E_i^{min/Max}$	The minimum/maximum energy use, in kWh for a scheduled unit i.
E_i^{Total}	The total energy in kWh needed by unit i in time horizon $\mathcal{H}$.
E_k	The energy level in kWh at which the base energy price factor C_k applies.
$\mathcal{H}$	Planning horizon in hours, indicating the number of hours in advance for which scheduling is assigned.
I	Total number of energy-consuming units.
K	Total number of discrete energy points ($K-1$ energy blocks).
m	Number of time intervals ahead of the current time that electricity prices are released.
$S(E_k)$	Energy cost for the consumption amount of E_k.
$p_i^{min/Max}$	The minimum/maximum operating power in kW that can be scheduled for unit i.
$S^{(j)}(\mathcal{H})$	The overall energy cost in cents for an entity in jth optimization step.
$S(t_h)$	The overall energy cost in cents for an entity in time interval t_h.
t_h	Time interval between $h-1$ and h.
$\eta_{i,h}$	Flexibility parameter for unit i in time interval t_h.
$\lambda_{k,h}$	Member of special ordered set of type 2 (SOS2) at t_h for $k \in \{1,\ldots,K\}$. In a special ordered set, only the adjacent variables can assume nonzero values.
τ_h	The price of electricity in cents per kWh in time interval t_h.

Decision Variable:

$x_{i,h}$	The energy consumption in kWh for unit i in time interval t_h.

The caret symbol (^) on any of the above parameters indicates the parameter's estimated value.

DEFINITION OF TERMS

Entity: An energy-consuming building with specific functionality for which energy planning is needed. Examples of entities are residential, industrial, or commercial buildings.
Unit: Machinery that consumes electric energy. Examples are electric motors, factory or commercial machinery, residential appliances, etc.
Planning Window (Planning Horizon): An interval during which the operation of a unit is scheduled.
Consumption: Energy usage by a unit based on the manufacturer's specification.
Price of Energy: The cost of energy delivered for an entity.

I. INTRODUCTION

WITH the debut of the smart power grid paradigm, the electric industry is being transformed from a centralized, producer-dependent network to a decentralized, consumer

Manuscript received October 24, 2013; revised February 17, 2014; accepted April 12, 2014. Date of publication July 15, 2014; date of current version October 17, 2014. Paper no. TSG-00804-2013.
The authors are with the Department of Electrical Engineering, University of Nebraska-Lincoln, Lincoln, NE 68588-0511 USA (e-mail: amin@huskers.unl.edu; hvakilzadian@unl.edu; fchoobineh@unl.edu).
Digital Object Identifier 10.1109/TSG.2014.2319247

interactive grid. Integrated communication and component connectivity are among the influential technologies considered for the smart grid [1]. These driving technologies will provide real-time communication for transmitting and/or receiving data. This will eventually empower consumers to obtain electricity prices in near real time via an advanced metering infrastructure (AMI) [2]. Such a pricing scheme will require consumers to have schedulers in order to analyze price data and plan their energy usage accordingly. From a supplier's point of view, the energy scheduler should accomplish the maximum achievable parsimony in order to reduce consumer loads. Consumers, however, are willing to reduce their energy cost only to such an extent that their comfort zones are not drastically impacted. Therefore, energy schedulers should provide consumers with options that take into consideration their cost benefits as well as their comfort.

A number of reports in the published literature focus on the design of energy usage schedulers for residential entities [3]–[8]. We briefly review these works from three basic perspectives that impact the effective performance of energy schedulers: how "*user preferences*" and "*electricity price*" are incorporated into the scheduler and how the "*optimization problem*" is performed.

- *User preferences*: In [3], the users define a time span for using each appliance, and the scheduler assigns energy to these appliances as soon as possible within the time spans defined. It is not feasible to define multiple preferred time spans for utilizing appliances. Therefore, using a "*waiting parameter*" to establish priorities between various preferred usage time ranges is not viable. User preferences are partially considered for thermostatically controlled household loads in [6]. In [4], [5], [7], and [8] user preferences are not considered which may impact consumer comfort zones.
- *Electricity price*: A real-time pricing scheme combined with a two-level inclining block rate is proposed for an electricity price model in [3]. A real-time pricing model is employed in [4]–[7] and [8] applies a time-of-use electricity pricing scheme. We believe, a general pricing model that can unify all available pricing strategies used by electricity suppliers (see Section II-C) is essential to the design of an ubiquitous energy usage scheduler.
- *Optimization problem*: In [3], the ultimate goal is to achieve a trade-off between minimizing electricity cost and reducing wait time for the operation of electric appliances. Implementation of a combined price scheme in [3] converts the linear energy optimization problem into a nonlinear model resulting in increase of complexity and solution time. The goal envisioned for optimization problems in [4], [5], [7], and [8] is minimization of energy costs. Reference [4] introduces a linear programming algorithm that is solved using CPLEX. Because of the optimization model's complexity in [5], the heuristic particle swarm optimization method has been utilized to find the optimal solution. This method is somewhat lacking in solid mathematical foundation for analysis and implementation. Reference [6] uses an algorithm to schedule thermostatically controlled loads in order to meet an optimization objective, such as minimum payment or maximum comfort. The computational cost of proposed stochastic dynamic programming algorithms for energy scheduling in [6] and [7] can be quite burdensome for long time horizons. The optimization model in [8] is formulated as a standard linear programming problem.

In this paper, we address the concerns and/or shortcomings of the above-mentioned approaches. This article proposes a general optimal energy scheduler for smart entities. The problem of energy management systems is addressed from the perspective of full user preference inclusion, general electricity price modeling, and development of a unified model that can be adapted for any smart entity. The proposed model provides a general framework for easy user involvement via definition of a flexibility parameter, incorporation of a general electricity price model, and design of an optimal energy scheduler in the form of an effective optimization problem that is easy to follow and straightforward to implement. To reiterate, our solution to the problem of energy scheduling includes:

- *Utilization of full user preferences*: This is achieved by introducing a tristate flexibility parameter for each electric unit that is fully controlled by the user. The advantage of this parameter is that it is very easy to set by just assigning three different values for each appliance in each time slot.
- *Utilization of general price modeling*: The model embeds proposed pricing schemes ranging from day-ahead pricing, time-of-use pricing, and real-time pricing into a tiered pricing structure. Our proposed tiered block rate price model has multiple nonflat rate segments instead of only two levels, as compared to [3].
- *Simple mixed integer linear programming optimization model*: The model is solved in a short time without the burden of computational cost as shown in Table IV.

The remainder of this article is organized as follows. Section II investigates the basic factors that influence the problem of energy scheduling. Section III describes the proposed optimization problem for optimal scheduling of electric energy consuming units based on user preferences. The linear programming optimization model and the problem of embedding price uncertainty into the model are also discussed in this section. Section IV provides important issues to be considered for using suitable data for simulation. The simulation results of several case studies are also presented and discussed in this section. Finally, Section V includes the advantages of the proposed model and the concluding remarks.

II. Foundations of Energy Planning

Proper planning for an entity's electricity consumption requires careful consideration of: 1) user preferences in operating the electricity-consuming units within the entity; 2) amount of electricity consumed by the units; and 3) the prevailing market price of electricity. These three items (preference, consumption, and price) form the foundation for proper energy planning for an entity. User preference is a function of the utility of an electricity-consuming unit and the importance of its use during a specific time period. Unit consumption is a function of the design characteristics and operational requirements of a unit and

the intensity of its use. Electricity price is a function of demand, supply, and regulation over a specific period of time.

Energy planning is generally done for a finite period of time and is referred to as the planning horizon. In order to make the energy planning model computationally tractable, we divide the planning horizon $\mathcal{H}$ into short intervals, namely t_h. We assume the user preference for using a unit, and the price of electricity remains constant during t_h of the planning horizon. In the next three subsections, we specify our modeling approach for preference, consumption, and price.

A. User Preference

In general, the preference for using a unit over an interval of time depends on the function of the unit and the satisfaction gained by using it. Satisfaction correlates positively to the importance and necessity of using the unit. The interplay between utility and reward constitutes the preference function of a user for a unit; in general, every unit has its own time-dependent preference function. For example, in a household setting, the user has a high preference for using the oven between 5 and 7 p.m.; because using the oven is necessary to cook food and most users eat dinner within this timeframe. On the other hand, although use of the dishwasher is necessary, users are generally flexible on the start time of this appliance.

In general, the user preference function is complex and indeterminate; and it varies for each unit among different users. The challenge for modeling purposes is to employ a preference function that realistically represents the user preference while simple enough to engage the user and is computationally tractable. We propose a three-value indicator parameter as the user preference function for using an electricity-consuming unit i during an interval t_h of the planning horizon. Clearly, the proposed function is a discrete approximation of a user's complex preference function; but for a short interval t_h, it captures the essence of the user preference. Equation (1) shows the proposed setting for the preference parameter:

$$\eta_{i,h} = \begin{cases} +1, & \text{unit } i \text{ may operate at time interval } t_h \\ 0, & \text{unit } i \text{ will operate at time interval } t_h \\ -1, & \text{unit } i \text{ will not operate at time interval } t_h \end{cases}. \tag{1}$$

In (1), a "$+1$" represents a user's flexibility in a unit's operation, which indicates either the user is indifferent to the use of the unit or the unit is in standby mode. A "0" represents required operation while a "-1" represents no operation, because either the unit is not needed or it is under repair during the period t_h. The proposed preference parameter is simple enough to engage the user and capture his/her preference easily while it is comprehensive enough to account for all important operational scenarios of the unit.

B. Unit Consumption

The amount of electricity consumed by a unit depends on its design and operational characteristics and the manner in which it is used i.e., user preference. The design and operational characteristics include a) whether the operation of the unit is interruptible or noninterruptible, b) the unit has single or multiple modes of operation, c) the unit uses electricity continuously or intermittently, and d) the unit has a periodic or an aperiodic operation cycle [9]–[12]. For example, the operation of a refrigerator is non-interruptible and periodic with varied operation cycles with a single mode of operation. The period of a refrigerator's operation depends on parameters such as the ambient temperature, the amount of its contents, its capacity, the number of times its door is opened, its age and the quality of its insulation. On the other hand, a television consumes electricity continuously while it is on.

Let $x_{i,h}$ be the amount of energy consumed by unit i in its operating mode during the interval t_h of the planning horizon. Clearly, $x_{i,h}$ is bounded by lower/upper bounds representing the minimum/maximum amount of electricity consumed when it is operated in time interval t_h. The energy consumed for any available mode of operation should be within these two bounds as shown in (2). It should be noted that when the unit is either in "OFF" or "STANDBY" mode, its energy consumption is either 0 or it can be determined from its standby energy consumption in a specific time interval:

$$E_i^{\min} \leq x_{i,h} \leq E_i^{\mathrm{Max}} \; \forall i \in I, h \in \{1, 2, \ldots, \mathcal{H}\}. \tag{2}$$

Equation (2) represents a constraint on the amount of energy consumed by a unit; and as such, it is a constraint in an energy planning model when the goal is to determine the optimal value of $x_{i,h}$. The bounds of (2) can be determined from operation specifications that are generally available from the manufacturer of the unit. Unit wattage specifications in *kW* should be multiplied by unit operation time in *hours* at time interval t_h to give energy bounds in *kWh* to be used in (2). Constraint (2) is not indicative of the different operating modes of a unit. For example, a battery charger for an electric vehicle can be operated in a range of continuous power levels where this power specifies the rate that the battery can be charged and there is a minimum and maximum energy level for the charger. On the other hand, for an electric range, the power consumption depends on the number of elements that are used simultaneously and their temperature settings. Similarly, for TV, the exact power consumption depends on the screen brightness, the volume level, and several other factors. Hence, because the power consumed by a unit with selectable operating modes depends largely on the consumer's usage habits, a user-dependent average power level can be assigned for these types of units.

For some units to be included in this framework prior data analysis is required to express user preferences and energy consumption bounds, as in (1) and (2), respectively. Thermostatically controllable units are good examples. A user can express his/her flexibility for an HVAC by assigning desirable lower and upper temperature limits. To translate this preference into a proposed flexibility parameter (1) and unit consumption constraint (2), one can use the dynamic power consumption model of the HVAC to obtain the corresponding flexibility parameters as well as allowable energy consumption bounds for every time interval in the planning horizon. Detailed load models for residential, commercial and industrial units can be found in [13].

C. Electricity Price

The price of electricity delivered to an entity is determined by the supplier. The supplier may follow a static or a dynamic

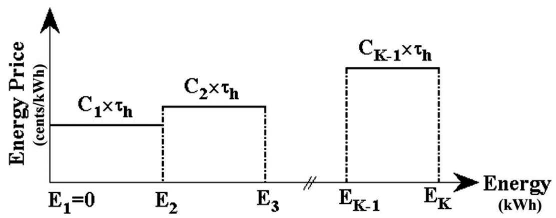


Fig. 1. Energy and time-dependent price model. Note that the price multiplier for the base consumption level is one, i.e., $C_1 = 1$.

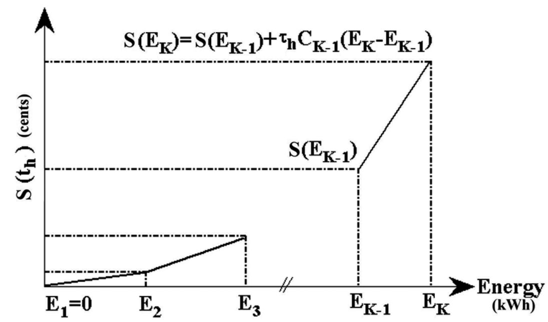


Fig. 2. Energy cost.

pricing scheme. Under a static pricing scheme, the price of electricity is known to the user and generally does not change within an energy planning horizon. However, under a dynamic pricing scheme, the price of electricity generally changes very frequently reflecting electricity market conditions. Therefore, energy planning for a specific horizon under dynamic pricing requires price forecasting. Examples of a static pricing scheme are *flat*, *tiered* and *time-of-use* pricing. Under a flat pricing scheme, the user pays a constant amount per *kWh* of electricity independent of the quantity of use. Under a tiered pricing scheme, either the user is penalized by an inclining block rate or incentivized by a declining block rate when the amount of electricity consumed exceeds a prespecified level. Finally, under the time-of-use pricing scheme, the price of electricity is tied to specific time periods, e.g., peak and off-peak seasons; and price changes occur a few times a year [14], [15]. Dynamic pricing has been pushed to the forefront of pricing schemes by deregulation of the electricity market and the advent of smart grid technologies. Examples of a dynamic pricing scheme are *real-time pricing*, *variable-peak pricing*, and *two-part real-time pricing*. Under a real-time pricing scheme, the price of electricity tracks the locational marginal price of electricity and changes at most every hour. The variable-peak pricing scheme is a combination of time-of-use and the real-time pricing schemes. Under this pricing scheme, time-of-use requiring price differentiation is defined in advance; and then the peak period prices for the next days are given based on the day-ahead forecast of wholesale market prices. Under the two-part real-time pricing scheme, real-time pricing takes effect when the electricity usage exceeds a historical baseline load [14]–[17].

To develop a general energy planning model, the model should be able to accommodate a hybrid static-dynamic pricing situation. Specifically for static pricing, we will use tiered inclining block pricing, since many other static pricing schemes are a simplified version of this pricing scheme. For dynamic pricing, we will use real-time prices if they are known for parts of the horizon. When real-time prices are not known, we will use their forecasted values. In the following two subsections, we describe how the static or dynamic prices for one *kWh* of electricity for an interval t_h of the planning horizon are determined.

1) Inclining Block With Base Real-Time Price: A generic inclining block pricing scheme is shown in Fig. 1. The price is τ_h for the base consumption level between $E_1 = 0$ and E_2 also known as a base real-time price. For other consumption levels, the price will be a multiple of the price for the base consumption level. The price multiplier for the consumption level between E_k and E_{k+1} is assumed to be C_k.

Let $E(t_h)$ be the total energy consumption during time interval t_h and assume it falls in the energy consumption range E_{k-1} and E_k ($k \in \{2, \ldots, K\}$). We can represent $E(t_h)$ as a linear combination of E_{k-1} and E_k as follows:

$$E(t_h) = \sum_{k=1}^{K} \lambda_{k,h} E_k \tag{3}$$

where

$$\sum_{k=1}^{K} \lambda_{k,h} = 1 \text{ and } \lambda_{k,h} \in [0, 1]. \tag{4}$$

Equation (4) is known as an SOS2 constraint, meaning that at most two of $\lambda_{k,h}$ variables can be nonzero; and these nonzero variables should be consecutive in their ordering [18].

The energy cost for total energy consumption of $E(t_h)$ at period t_h can be calculated by integrating the curve in Fig. 1 from $E_1 = 0$ up to $E(t_h)$. Fig. 2 is the integrated form of the inclining block graph of Fig. 1. Let $S(E_k)$ be the energy cost at E_k, as shown in Fig. 2. The total cost for energy consumption of $E(t_h)$, namely $S(t_h)$, can then be easily obtained in terms of the variables $S(E_k)$ and the values of $\lambda_{k,h}$, as in (5). Note that the values of $\lambda_{k,h}$ satisfy the constraint in (4). Equation (6) states that if the energy level of E_{k-1} is passed, the energy cost will be increased by a factor of C_{k-1}.

$$S(t_h) = \sum_{k=1}^{K} \lambda_{k,h} S(E_k) \tag{5}$$

where

$$S(E_k) = S(E_{k-1}) + \tau_h C_{k-1}(E_k - E_{k-1}) \quad \forall k \in \{2, \ldots, K\} \text{ and } S(E_1) = 0. \tag{6}$$

2) Dynamic Price Forecasting: Let us assume that the real-time energy price (τ_h) is announced by the electric utility company for m time intervals ahead of the current time. If m is less than the scheduling time horizon $\mathcal{H}$, then τ_h will be unknown for $h > m$. Hence, the energy prices for the aforementioned time intervals should be forecasted and their estimated values ($\hat{\tau}_h$) should be used.

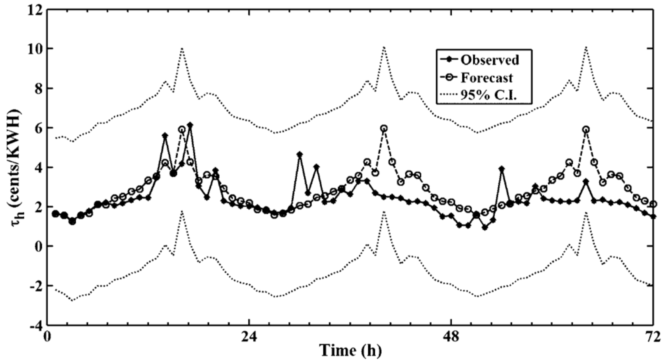


Fig. 3. Observed versus forecasted values of electricity prices with the autoregressive model. A 95% confidence interval is also shown for the forecasted prices.

The real-time electricity price depends heavily on the level of electricity demand, i.e., the specific time of day and specific day of the week [19]. In this paper, we adopt the recommended autoregressive model in [3] for price forecasting as it considers the existing hourly and daily correlations in price data for modeling and validation of price models. In [3], a price model is calculated for each day of the week. The electricity price for every hour in a specific day is the weighted average of the price at the same hour of the day before, same hour of two days before and the same hour of the same day last week. We use MATLAB to estimate the parameters of the electricity price models of [3] from the real-time price data over a one-year period, starting from September 2011 to August 2012 from Ameren Illinois [20]. Fig. 3 shows the forecasted values using these models and the observed values for a three-day forecast period (September 12–14, 2012). In this figure, the 95% confidence interval is also shown.

III. Optimization Problem

The objective of this research is to design an electric energy scheduler to optimize the energy use of electricity-consuming units within an entity for a predetermined time horizon such that the consumers can receive the most savings on their monthly electric bills, with a minimum impact on their usage preferences, by knowing the time-of-use electricity rates announced hours in advance.

A. Problem Formulation

Suppose that the scheduler is to determine the consumption vector, $X_i = [x_{i,1}, x_{i,2}, \ldots, x_{i,\mathcal{H}}]$ for each unit $i \in \mathcal{A}$ in the desired time horizon $\mathcal{H}$. In general, the dynamic electricity prices are announced every m time interval by a power utility company where m is less than $\mathcal{H}$. We assume there are M segments comprised of m time intervals in the planning horizon $\mathcal{H}$, i.e., $\mathcal{H} = M \times m$. This means that there will be M actual price announcements for m time intervals in each segment in the planning horizon $\mathcal{H}$. We believe that the "*Shrinking Horizon Scheduling*" approach fits well within this price release framework. In this approach, the scheduling is first performed over the whole planning horizon $\mathcal{H}$ based on actual prices available for the first segment; and for the remaining segments, the forecasted prices are used. Then, with the arrival of actual prices for the next segment, the scheduling is updated according to actual prices to account for the forecast error. Once the scheduling is updated, the scheduling horizon is shrunk one segment (m time intervals) for the next scheduling step. This mechanism is shown schematically in Fig. 4. Shrinking horizon optimization is used for fixed horizon problems in model predictive control methodology. In model predictive control, future events of a dynamical model are predicted; and then suitable control actions for the current time slot are taken and optimized. Model predictive control can be implemented in either receding (rolling) horizon or shrinking horizon form [21]–[25].

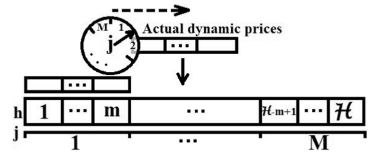


Fig. 4. Shrinking horizon optimization model.

Using the parameters in Sections II, we can formulate the shrinking horizon optimization problem as in (7) as follows:

for $j = 1$ to M

{Minimize $S^{(j)}(\mathcal{H})$ subject to constraints (8)–(11) where;

$$S^{(j)}(\mathcal{H}) = \sum_{h=jm-m+1}^{jm} S(t_h) + \sum_{h=jm+1}^{\mathcal{H}} \hat{S}(t_h)$$

$$= \sum_{h=jm-m+1}^{jm} \sum_{k=1}^{K} \lambda_{k,h} S(E_k) + \sum_{h=jm+1}^{\mathcal{H}} \sum_{k=1}^{K} \lambda_{k,h} \hat{S}(E_k)\} \qquad (7)$$

$$\sum_{h=jm-m+1}^{\mathcal{H}} \left[x_{i,h} \eta_{i,h} \left(\frac{1+\eta_{i,h}}{2} \right) \right] = E_i^{\mathrm{Total}}$$
$$- \sum_{h=1}^{\mathcal{H}} \left[\bar{x}_{i,h} (1 - \eta_{i,h}^2) \right] - \sum_{h=1}^{jm-m} \left[\tilde{x}_{i,h} \eta_{i,h} \left(\frac{1+\eta_{i,h}}{2} \right) \right]; \ \forall i \in \mathcal{A} \qquad (8)$$

$$\sum_{k=1}^{K} \lambda_{k,h} E_k = \sum_{i=1}^{I} [x_{i,h} \eta_{i,h} (\frac{1+\eta_{i,h}}{2})] + \sum_{i=1}^{I} [\bar{x}_{i,h} (1 - \eta_{i,h}^2)]; \quad h = \{1, \ldots, \mathcal{H}\} \qquad (9)$$

$$E_i^{\min} \le x_{i,h} \le E_i^{\mathrm{Max}}; \forall i \in \mathcal{A}, h = \{1, \ldots, \mathcal{H}\} \qquad (10)$$

$$\begin{aligned} &\lambda_{1,h} \le \alpha_{1,h} \\ &\lambda_{k,h} \le \alpha_{k-1,h} + \alpha_{k,h}; \ k = \{2, \ldots, K-1\} \\ &\lambda_{K,h} \le \alpha_{K-1,h} \\ &\sum_{k=1}^{K} \lambda_{k,h} = 1 \text{ and } \sum_{k=1}^{K-1} \alpha_{k,h} = 1 \\ &\alpha_{k,h} = 0 \text{ or } 1; k = \{1, 2, \ldots, K-1\} \\ &\lambda_{k,h} \ge 0; k = \{1, 2, \ldots, K\} \\ &h = \{1, \ldots, \mathcal{H}\}. \end{aligned} \qquad (11)$$

$S^{(j)}(\mathcal{H})$ in (7) refers to the total electricity cost in the jth optimization step. The first part on the first right-hand side (RHS) of (7) is the energy cost for m time intervals in the jth time segment based on actual electricity prices, and the second part is the estimated energy cost based on forecasted electricity prices for subsequent time intervals. The terms on second RHS of (7) are equivalent forms obtained from (5). It should be noted that at the

beginning of the jth time segment, the scheduler has assigned optimal values for previous time intervals in previous time segments of j, i.e., $j-1, j-2, \ldots, 1$ and these assigned energy costs do not need to be considered at this step.

To include user preferences in the optimization model, constraint (8) is constructed. The term $\eta_{i,h}((1+\eta_{i,h})/2)$ on the left-hand side (LHS) of (8) takes into account the time intervals (t_hs) where energy scheduling is performed for unit i. When unit i does not require energy scheduling at t_h because $x_{i,h}$ is already known (either because it should stay "OFF" for $\eta_{i,h}=-1$ or "ON" for $\eta_{i,h}=0$), this term will be 0. For $\eta_{i,h}=+1$ where energy scheduling is allowed for unit i, this term will take on the value of 1, bringing $x_{i,h}$ into consideration for scheduling. Let E_i^{Total} be the total amount of energy required by unit i in time horizon $\mathcal{H}$. Then, the total amount of energy in current optimization step [first term on the LHS of (8)] should be equal to E_i^{Total} minus the total amount of energy required for the unit on its non-flexible hours [second term on RHS of (8)] minus the total amount of assigned energies for unit i at previous optimization steps [third term on RHS of (8)]. Note that the term $(1-\eta_{i,h}^2)$ on RHS of (9) will only be nonzero for $\eta_{i,h}=0$ when unit i is "ON" at t_h and the amount of energy needed for its operation is known beforehand and noted here by $\bar{x}_{i,h}$. Also $\tilde{x}_{i,h}$ refers to the assigned energy for unit i in previous optimization steps.

Constraint (9) provides a link between $\lambda_{k,h}$ and $x_{i,h}$ variables. LHS of (9) is the total amount of scheduled energy for all units in time interval t_h, namely, $E(t_h)$, written in terms of a linear combination of $\lambda_{k,h}$ variables as in (3). This total amount is equal to the sum of scheduled energy for all units at time interval t_h if they are flexible [first term on RHS of (9)] plus the amount required if they are non-flexible [second term on RHS of (9)].

In addition, constraint (10) is the replica of constraint (2). As it was mentioned in Section II-C, (4) requires that at most two $\lambda_{k,h}$ variables can be nonzero; and these nonzero variables should be consecutive in their ordering. To guarantee this condition mathematically, $\alpha_{k,h}$ binary variables are introduced and constraints in (11) are added to the model. An exhaustive description on these constraints is given in [18], [26].

Equations (7)–(11) are linear combinations of $\lambda_{k,h}$, $\alpha_{k,h}$ and $x_{i,h}$. Together, they form a standard mixed integer linear optimization problem that can be easily solved using mixed integer linear programming techniques which are computationally efficient [26], [27]. In this research, we used the AIMMS suite to find the optimal solution to (7).

B. Remarks

The reason for choosing a fixed planning horizon along with a shrinking horizon scheduling framework is to alleviate problems due to dynamic price forecast errors and changing user preferences. It is well known that prediction confidence intervals widen as the forecast horizon increases due to the uncertainty on price levels and variation trends [28]. Indeed for this reason, we have fixed our planning horizon to lessen the effect of prediction errors. As to the second remedy, we have utilized a shrinking horizon scheduling framework for the proposed optimization problem such that with the release of actual dynamic prices, the optimal solution is updated to account for and reduce forecast errors.

The randomness of user preferences due to inherent stochasticity of human behavior is another characteristic that is expected specifically in residential entities. This issue has also been implicitly reflected and addressed in the definition of the flexibility parameter, selection of a fixed planning horizon, and recommendation of a shrinking horizon scheduling model for the optimization problem. Assume that the user changes his/her mind spontaneously about his/her previously defined preferences for a unit. The result would be to override the scheduled power consumption for that unit, and thereafter the corresponding unit would be regarded as a must-operate unit. Ultimately, a shrinking horizon scheduling framework will help capture spontaneous user preference changes and find suitable optimal solutions according to newly introduced preferences. The penalty for the user would be to pay more because of showing no or partial flexibility for that unit.

It is noteworthy to point out the generality of the optimization model developed and the sufficiency of its parameters in handling different scenarios that can be encountered in electric units. The proposed model is applicable to both interruptible and non-interruptible electric units by setting the proper values for the flexibility parameter in the planning horizon. For interruptible units that do not need to be operated in continuous time intervals, meaning that their operation can be stopped/started at any time and/or postponed to other suitable hours, there is no constraint on the flexibility parameter. For example, if a user is flexible on running a pausable clothes dryer that requires 2 hours to perform its task, at least two flexible time intervals need to be assigned for this unit; and these 2 hours do not necessarily need to be consecutive. For noninterruptible units, however, when the unit starts to operate, it should be given enough time to finish its operation without any interruption. For these units, the flexibility parameter for different time intervals should be set such that the length of the maximum continuous hours that the unit is set to be flexible is at least equal to the required time for the unit to finish its operation from its start time. For example, if a production line in an automobile manufacturing plant needs to operate 3 hours continuously to deliver its products to the assembly line, at least 3 flexible time intervals should be assigned for the production line, and these flexible hours should be consecutive in their ordering.

This model also considers the units with discrete energy consumption levels at different time intervals. This can be achieved by expanding constraint (2) such that for each time interval the minimum and maximum energy levels take on different values, and they are both equal to the discrete energy consumption level at that time interval.

Another important problem is that the overall energy usage of an entity should preferably be evenly distributed throughout the whole planning horizon. This helps for better "*Peak to Average Power Ratio*" (*PAPR*) values. *PAPR* is an important factor from the electric utility company's point of view for load balancing [29]. Because the general price model used in this study includes inclining block rate pricing and the price is increased when the overall electricity consumption in a time interval passes a threshold level, the concentration of scheduled units in a time

TABLE I
ELECTRIC APPLIANCE AVERAGE WATTAGE AND USAGE TIME PER DAY

Description	Average Wattage (kW)	Usage Time per Day (min.)	Total Energy per Day (kWh)
Clothes Dryer	5.5	45	4.125
Color TV	0.15	360	0.9
Range Top (Small Surface Unit)	1.6	20	0.53
Range Top (Large Surface Unit)	2.7	20	0.9
Dishwasher and Heater	1.2	32	0.64
PC	0.1	120	0.2
Oven	3.5	22	1.28
Microwave Oven	0.8	30	0.4
Washing Machine	0.665	45	0.50
Coffee Maker	0.35	60	0.35
Toaster Oven	1.5	10	0.25
Toaster	1.1	2	0.037
Vacuum Cleaner	0.74	30	0.37
Oven Electric Cleaning	3.5	24	1.4
Iron	1.1	30	0.55
PHEV[a] Charger[b]	1.4/3.3[c]	-	16

[a] Plug-in hybrid electric vehicle.
[b] The data for this appliance was obtained from [31] for a typical PHEV sedan for a daily driving range of 40 miles.
[c] These values indicate the minimum and maximum charging rate wattage respectively.

TABLE II
VALUES USED FOR INCREASE OF BASE PRICES (%)

Baseline Usage (%)	Energy Price Increase Coefficient (%)
101-130	114
131-200	247
201-300	280
Over 300	280

interval with minimum price during the scheduling horizon is avoided.

IV. CASE STUDY

A. Residential Entity

For the case study, we consider the scheduling of energy consuming units in a residential entity. The appliance data used for simulation in this paper has been obtained from a report by Duke Energy of Charlotte, NC, USA [30]. We chose 15 of the most commonly used electric appliances in U.S. households (Table I).

B. Electricity Price

We used real-time price data from Ameren Illinois [20] for the dynamic pricing part of the electricity price model. The required data for the inclining block static rate part of the model was obtained from Pacific Gas and Electric Company (PG&E) Electric Schedule E-1 that is applied for single-family dwellings in San Francisco, CA, USA[32]. Table II summarizes these values in terms of percent increase of base prices for different percentages of baseline usage, and they are used for the values of C_k in our model.

TABLE III
SIMULATION SCENARIOS

Scenario #	Pricing Scheme	$\mathcal{H}$	m[a]	Case
1	Real time	24	24	I:Full flexibility
2	Real time combined with tiered block rates	24	24	
3	Real time	24	2	II:Partial flexibility
4	Real time combined with tiered block rates	24	2	

[a] If $m = 2$, price is announced every 2 hours and price forecasting is required. For $m = 24$, no price forecasting is needed for the assumed planning horizon of 24 hours.

C. Simulation and Results

Four different scenarios were simulated arbitrarily for a 24-hour scheduling horizon, starting from 12 a.m. on September 14, 2012. These scenarios are summarized in Table III. Moreover, for each scenario, two different cases were considered. For the first case of each scenario, the family was assumed to be fully flexible, meaning that their ultimate goal was to save as much as possible on their electricity bill. The second case included the family's preferences for energy scheduling, and they specified the time intervals that they were flexible or not flexible for using the specific appliances. We assumed that the family wakes up at 6 a.m. and finishes breakfast by 8 a.m. Therefore, the coffee maker and toaster are only flexible during these hours. For lunch, the family needs the oven and the small and large plates on the cooktop from 9 a.m. to 12 p.m. Then, if needed, the oven can be self-cleaned in a suitable time from 12 p.m. to midnight. For dinner, the family will use the microwave and toaster oven from 3 p.m. to 6 p.m. The dishwasher will be used to wash the dirty lunch and dinner dishes from 7 p.m. to 12 a.m. The family also prefers to watch TV between noon and 10 p.m. and work with a PC from 8 a.m. to 6 p.m. They also find a suitable time for doing the laundry in the morning between 8 a.m. and 12 p.m. followed by drying the clothes and getting them ready by 9 p.m., and ironing them before 12 midnight. The house also needs to be vacuumed either from 9 a.m. to 11 a.m. or in the afternoon from 2 p.m. to 5 p.m. Finally, the electric car battery charger can operate in a time interval between 8 p.m. and 8 a.m. of the next day.

Fig. 5 shows the simulation results for all of the stated scenarios as well as observed and forecasted dynamic prices. As is seen in Fig. 5(a), for Case I of Scenarios I and III, the scheduled energy by the proposed optimal adaptive scheduler is biased towards the hours that the price of the energy is at its lowest amount. This is reasonably expected. On the other hand, in Case II where the user's preferences are included, the scheduled energy is shifted towards the hours when the user shows more flexibility. Thus, the user's preferences help to determine a somewhat more even distribution of energy consumption over the scheduling horizon that in turn will reduce the peak to average power consumption ratio. Moreover, it is apparent that the scheduled energy for these two scenarios under either Case I or II are almost the same. This indicates that the models used for price forecasting are valid and are able to forecast the real-time electricity prices and estimate the trend of price deviations satisfactorily.

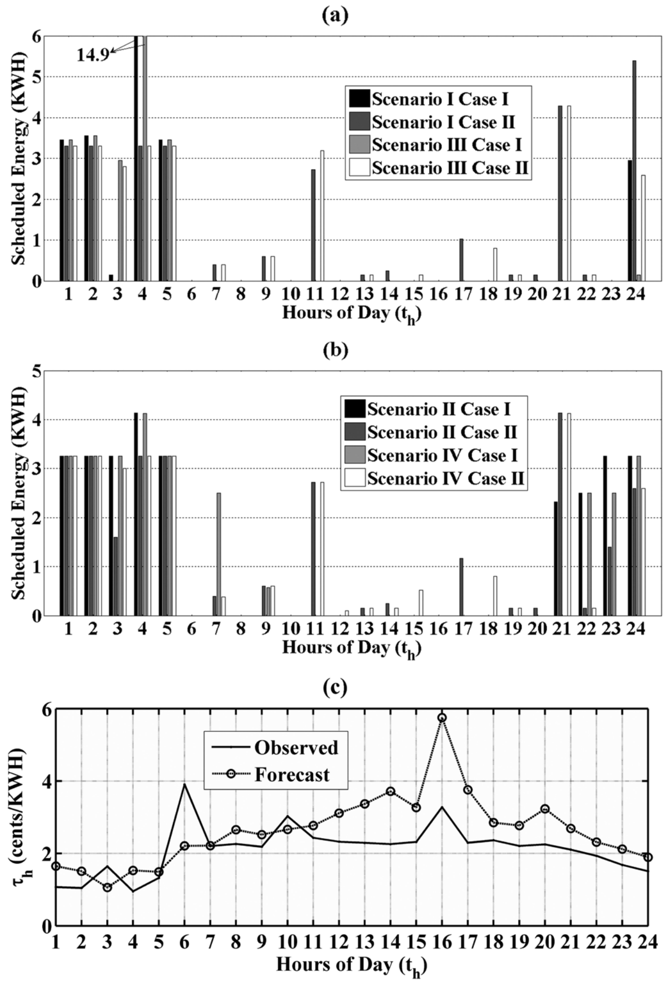


Fig. 5. Simulation results for electricity price model (a) without inclining block rates and (b) with inclining block rates. For Scenarios I and III, real-time prices are known for the scheduling horizon and for II and IV, real-time prices are announced every 2 hours and the forecasted price values are used for other time intervals. For Case I, the user is fully flexible during the scheduling horizon; while in Case II, the user's preferences are fully included. (c) shows observed and forecasted dynamic prices for simulation scenarios.

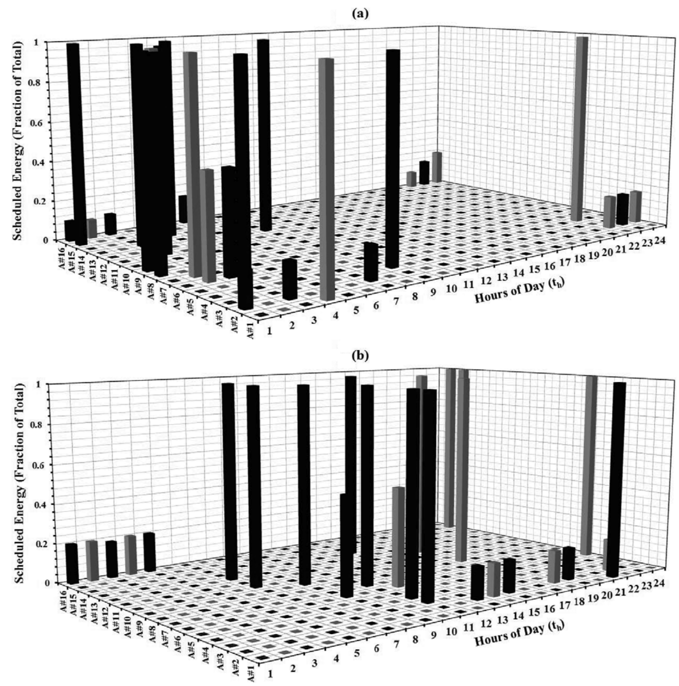


Fig. 6. Scheduled energies for appliances under simulation scenario IV (a) case I (b) case II. A# refers to the row number of the corresponding appliance in Table I. Scheduled energies are given in fraction of total required energy in the planning horizon for ease of representation.

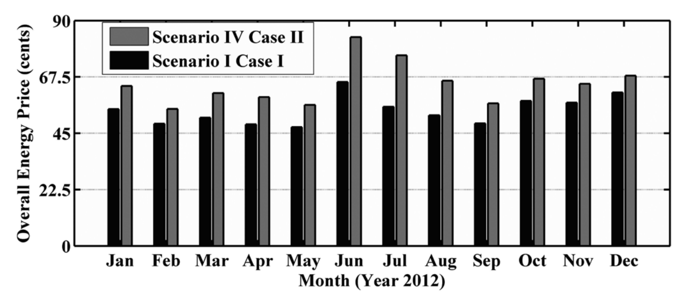


Fig. 7. Overall energy price for last days of months during year 2012 under simulation scenario I case I versus scenario IV case II.

TABLE IV
MINIMIZED OVERALL ELECTRICITY PRICE FOR THE SCHEDULING HORIZON AND COMPUTATIONAL TIMES UNDER SIMULATION SCENARIOS

Scenario No. Case No.	Overall Energy Prices (cents)	Computational Time (sec)
I-I	30.88	3.22
I-II	44.62	3.21
II-I	42.75	3.55
II-II	48.67	3.82
III-I	31.26	3.20
III-II	45.13	3.21
IV-I	43.20	3.69
IV-II	48.81	3.61

Fig. 5(b) shows the simulation results for the general case of electricity price model. Compared with Fig. 5(a), the scheduler assigns only a limited amount of energy in each time interval where the electricity prices are low within the planning horizon. This is because if the allocated energy for a time interval exceeds a certain level, the user will be penalized due to the inclining block rate price scheme. It is similarly seen that when the user is fully flexible, as in Case I, the consumption is again mostly distributed among the time intervals when the energy prices are at their lowest levels. Although during this time, the user shows flexibility over the whole scheduling horizon, compared to similar cases in Scenarios I and III, the energy is more evenly distributed. It is also seen that for the case where the user is partially flexible, the scheduling is done such that only a small portion of the consumption is tilted towards the high energy price hours to meet user preferences. Again, it is kept mostly towards the lowest energy price hours as far as the user preferences allow. It should be noted again that with price forecasting, energy scheduling is almost similar to the cases where the actual real-time prices are known. Fig. 6 shows how the scheduler has assigned required energy for the appliances in a 24 hour planning horizon for both cases of Scenario IV.

Table IV shows the overall minimized energy cost for each of the four simulated scenarios. It is apparent that the users will pay the lowest electricity price (30.88 cents) when they are fully flexible, and they can use as much energy as needed

without being penalized for passing consumption limits (baselines) in any time interval (Scenario I, Case I). For the similar case in the third scenario, where the forecasted prices are used instead, the minimized overall price is 31.26 cents. This is actually very close to 30.88 with 1.22% of error. This error is 1.13% for Case II of Scenarios I and III and 1.04% and 0.28% for Cases I and II of Scenarios II and IV. These very small error rates validate again that the autoregressive models for real-time electricity prices are suitable for price forecasting and optimizing electricity costs. Also it is seen that the user pays 30.80% and 30.73% more for applying their preferences in the electricity consumption for Scenarios I and III and 12.16% and 11.50% more for Scenarios II and IV, respectively. It is very interesting to see that the user's benefit on the electricity bill is less reduced when inclining block rate tariffs are applied, and at the same time, they prefer to be not fully flexible. It should also be pointed out that the user has to pay more in the most general case.

Computational times for each case of simulation scenarios are also given in Table IV. The model is solved using AIMMS 3.12.2 on a x64-based PC with eight processors clocking at 2.67 GHz and 8.00 GB of RAM. The average computational time required to achieve the optimal solution is 3.44 s.

To further investigate the efficiency of our proposed scheduler in optimizing electricity costs while satisfying user preferences, we have shown the overall energy prices that users have to pay under the best (scenario I case I) and worst (scenario IV case II) possible scenarios for the last days of months during the year 2012 in Fig. 7. According to these results, users have to pay 19.62% more on average if they choose to be partially flexible and inclining block rate tariffs are applied. For the month of December this percent increase is at its low, i.e., 10.88% where for the month of July this value reaches its high at 37.00%. Similar to what we concluded in the previous paragraph, the users have to pay more when they prefer to be less flexible under penalizing tiered block rate tariffs, while the scheduler tries to keep their energy costs at their lowest possible levels.

V. Conclusion

Easy user involvement in managing energy usage is among the ultimate goals of smart grid technology, benefiting both consumers and power utility companies. Our optimal energy scheduling model is based on simplifying the involvement of electricity subscribers in reducing their energy consumption costs compared to previous similar works. The users can specify their consumption preferences by simply setting a tristate flexibility parameter. Another novel characteristics of the model compared to similar works is that it considers the most general pricing scheme. In the meantime, the optimization model derived is a simple mixed integer linear programming problem that could be solved easily in a short time. The simulation data for electric units, real-time prices, and inclining block rate part were based on realistic available resources published by power companies throughout the U.S.

Simulation results show that the proposed model can minimize the electricity consumption costs while including the user preferences. It was also seen that the inclining block rate price model could drastically help in distributing the energy consumption more evenly throughout the scheduling horizon. The results also show that the model helps in significantly reducing the peak to average power consumption ratio.

References

[1] A. Chopra, V. Kundra, and P. Weiser, "A policy framework for the 21st century grid: Enabling our secure energy future," Executive Office of the President, National Science and Technology Council, 20502, Washington, DC, USA, Jun. 2011 [Online]. Available: http://www.whitehouse.gov/sites/default/files/microsites/ostp/nstc-smart-grid-june2011.pdf, Tech. Rep.

[2] FERC, "Assessment of demand response & advanced metering," Federal Energy Regulatory Commission, Dec. 2012 [Online]. Available: http://www.ferc.gov/legal/staff-reports/12-20-12-demand-response.pdf, Tech. Rep.

[3] A.-H. Mohsenian-Rad and A. Leon-Garcia, "Optimal residential load control with price prediction in real-time electricity pricing environments," *IEEE Trans. Smart Grid*, vol. 1, no. 2, pp. 120–133, Sep. 2010.

[4] A. J. Conejo, J. M. Morales, and L. Baringo, "Real-time demand response model," *IEEE Trans. Smart Grid*, vol. 1, no. 3, pp. 236–242, Dec. 2010.

[5] M. A. A. Pedrasa, T. D. Spooner, and I. F. MacGill, "Coordinated scheduling of residential distributed energy resources to optimize smart home energy services," *IEEE Trans. Smart Grid*, vol. 1, no. 2, pp. 134–143, Sep. 2010.

[6] P. Du and N. Lu, "Appliance commitment for household load scheduling," *IEEE Trans. Smart Grid*, vol. 2, no. 2, pp. 411–419, Jun. 2011.

[7] T. T. Kim and H. V. Poor, "Scheduling power consumption with price uncertainty," *IEEE Trans. Smart Grid*, vol. 2, no. 3, pp. 519–527, Sep. 2011.

[8] M. Erol-Kantarci and H. T. Mouftah, "Wireless sensor networks for cost-efficient residential energy management in the smart grid," *IEEE Trans. Smart Grid*, vol. 2, no. 2, pp. 314–325, Jun. 2011.

[9] R. Hendron and C. Engebrecht, "Building america research benchmark definition," National Renewable Energy Lab., Golden, CO, USA, Tech. Rep. NREL/TP-550-47246, Jan. 2010 [Online]. Available: http://www.nrel.gov/docs/fy10osti/47246.pdf

[10] T. P. Wenzel, J. Koomey, G. J. Rosenquist, M. Sanchez, and J. W. Hanford, *Energy Data Sourcebook for the US Residential Sector*. Berkeley, CA, USA: Univ. of California. Lawrence Berkeley Lab., 1997.

[11] R. Hendron and C. Engebrecht, "Building America house simulation protocols," National Renewable Energy Lab., 2010.

[12] M. M. Armstrong, M. C. Swinton, H. Ribberink, I. Beausoleil-Morrison, and J. Millette, "Synthetically derived profiles for representing occupant-driven electric loads in Canadian housing," *J. Building Perform. Simulat.*, vol. 2, no. 1, pp. 15–30, 2009.

[13] "Bibliography on load models for power flow and dynamic performance simulation," *IEEE Trans. Power Syst.*, vol. 10, no. 1, pp. 523–538, Feb. 1995.

[14] C. Goldman, Coordination of Energy Efficiency and Demand Response, 2010 [Online]. Available: http://www.epa.gov/cleanenergy/documents/suca/ee_and_dr.pdf

[15] J. W. Black and R. C. Larson, "Strategies to overcome network congestion in infrastructure systems," *J. Ind. Syst. Eng.*, vol. 1, no. 2, pp. 97–115, 2007.

[16] Time-Based Rate Programs [Online]. Available: http://www.smartgrid.gov/recovery_act/deployment_status/time_based_rate_programs

[17] "Rate design treatments in consumer behavior study designs," U.S. Dept. of Energy Smart Grid Investment Grant—Technical Advisory Group Guidance Document #4, Aug. 2010 [Online]. Available: http://www.smartgrid.gov/sites/default/files/pdfs/cbs_guidance_doc_4_ra te_design.pdf, Tech. Rep.

[18] E. Beale and J. Forrest, "Global optimization using special ordered sets," *Math. Program.*, vol. 10, no. 1, pp. 52–69, 1976.

[19] C. Nunes, A. Pacheco, and T. Silva, "Statistical models to predict electricity prices," in *Proc. 5th Int. Conf. Eur. Electricity Market (EEM'08)*, 2008, pp. 1–6.

[20] Real-time pricing for residential customers, [Online]. Available: https://www2.ameren.com/RetailEnergy/realtimeprices.aspx

[21] B. Joseph and F. W. Hanratty, "Predictive control of quality in a batch manufacturing process using artificial neural network models," *Ind. Eng. Chem. Res.*, vol. 32, no. 9, pp. 1951–1961, 1993.

[22] M. M. Thomas, J. Kardos, and B. Joseph, "Shrinking horizon model predictive control applied to autoclave curing of composite laminate materials," in *Proc . Amer. Control Conf.*, 1994, vol. 1, pp. 505–509.

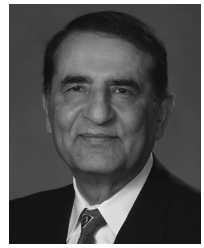
[23] Z. K. Nagy and R. D. Braatz, "Robust nonlinear model predictive control of batch processes," *AIChE J.*, vol. 49, no. 7, pp. 1776–1786, 2003.

[24] P. Mhaskar, N. H. El-Farra, and P. D. Christofides, "Predictive control of switched nonlinear systems with scheduled mode transitions," *IEEE Trans. Autom. Control*, vol. 50, no. 11, pp. 1670–1680, Nov. 2005.
[25] J. Skaf, S. Boyd, and A. Zeevi, "Shrinking-horizon dynamic programming," *Int. J. Robust Nonlinear Control*, vol. 20, no. 17, pp. 1993–2002, 2010.
[26] W. L. Winston and J. B. Goldberg, *Operations Research: Applications and Algorithms*. Belmont, CA, USA: Thomson/Brooks/Cole, 2004.
[27] M. W. Carter and C. C. Price, *Operations Research: A practical Introduction*. Boca Raton, FL, USA: CRC, 2001.
[28] G. E. Box, G. M. Jenkins, and G. C. Reinsel, *Time Series Analysis: Forecasting and Control*. New York, NY, USA: Wiley, 2013.
[29] B. Ramanathan and V. Vittal, "A framework for evaluation of advanced direct load control with minimum disruption," *IEEE Trans. Power Syst.*, vol. 23, no. 4, pp. 1681–1688, Nov. 2008.
[30] Electric Appliance Operating Cost List, [Online]. Available: http://www.duke-energy.com/pdfs/Appliance_Op-Cost_List_Duke_v8.06.pdf
[31] A. Ipakchi and F. Albuyeh, "Grid of the future," *IEEE Power Energy Mag.*, vol. 7, no. 2, pp. 52–62, Mar.-Apr. 2009.
[32] Residential Services: Electric Schedule e-1, [Online]. Available: http://www.pge.com/tariffs/tm2/pdf/ELEC_SCHEDS_E-1.pdf

Amin Fakhrazari (S'10) received the B.S. degree in electrical engineering from Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran, and the M.S. degree (with honors) from Sharif University of Technology, Tehran, Iran, in 2008. He is currently pursuing the Ph.D. degree in electrical engineering at the University of Nebraska-Lincoln, Lincoln, NE, USA.

His research interests include statistical modeling, digital signal processing, machine learning, intelligent/adaptive control and optimization and control of smart power systems.

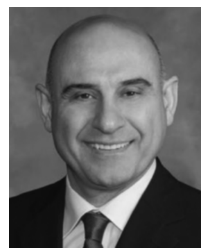
Mr. Fakhrazari is the recipient of University of Nebraska Engineering Fellowship. He is a lifetime member of the IEEE HKN Honor Society.



Hamid Vakilzadian (S'82–M'86–SM'91) received the B.S.E.E. degree from Sharif University, Tehran, Iran, and the M.S.E.E. and Ph.D. degrees from the Electrical and Computer Engineering Department, University of Arizona, Tucson, AZ, USA, in 1985.

After the B.S.E.E. degree, he was a Design Engineer, Quality Control Engineer, Field Engineer, and Consulting Engineer in the power industry. He is an Associate Professor of electrical engineering at the University of Nebraska-Lincoln, Lincoln, NE, USA, and serves as the Technical Committee Chair of IEEE Region 4.

Prof. Vakilzadian was the organizer and General Chair of the 2005 IEEE International Conference in electro information technology and served as a VP of conferences for the society for modeling and simulation (SCS). He has been the editor of the special issues of M&S in Education, and editor or co-editor of more than twelve volumes of SCS conference proceedings. His research interest includes modeling and simulation of continuous and discrete systems and their application in the design of digital systems.



F. Fred Choobineh (M'89) received the B.S.E.E., M.S.I.E., and Ph.D. degrees from Iowa State University Ames, IA, USA, in 1972, 1976, and 1979, respectively.

He is a Professor of electrical engineering and Blackman Distinguished Professor of Engineering at the University of Nebraska-Lincoln, Lincoln, NE, USA, where he also holds a courtesy appointment as a Professor of Management. His current research interests include electricity market and smart grid modeling and analysis. His research has been funded by the NSF and industry.

Prof. Choobineh is a recipient of the Nebraska Legislator Award for Distinguished Teaching in the College of Engineering and is a Fellow of the Institute of Industrial Engineers. He is a registered Professional Engineer in Nebraska where he also serves on the state Board of Engineers and Architects.