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Lameck O. Odhiambo
University of Nebraska-Lincoln, lodhiambo2@unl.edu

R. E. Yoder
University of Tennessee, Knoxville, ryoder2@unl.edu

D. C. Yoder
University of Tennessee, Knoxville

J. W. Hines
University of Tennessee, Knoxville

OPTIMIZATION OF FUZZY EVAPOTRANSPIRATION MODEL THROUGH NEURAL TRAINING WITH INPUT–OUTPUT EXAMPLES

L. O. Odhiambo, R. E. Yoder, D. C. Yoder, J. W. Hines

ABSTRACT. *In a previous study, we demonstrated that fuzzy evapotranspiration (ET) models can achieve accurate estimation of daily ET comparable to the FAO Penman–Monteith equation, and showed the advantages of the fuzzy approach over other methods. The estimation accuracy of the fuzzy models, however, depended on the shape of the membership functions and the control rules built by trial–and–error methods. This paper shows how the trial and error drawback is eliminated with the application of a fuzzy–neural system, which combines the advantages of fuzzy logic (FL) and artificial neural networks (ANN). The strategy consisted of fusing the FL and ANN on a conceptual and structural basis. The neural component provided supervised learning capabilities for optimizing the membership functions and extracting fuzzy rules from a set of input–output examples selected to cover the data hyperspace of the sites evaluated. The model input parameters were solar irradiance, relative humidity, wind speed, and air temperature difference. The optimized model was applied to estimate reference ET using independent climatic data from the sites, and the estimates were compared with direct ET measurements from grass–covered lysimeters and estimations with the FAO Penman–Monteith equation. The model–estimated ET vs. lysimeter–measured ET gave a coefficient of determination (r^2) value of 0.88 and a standard error of the estimate (S_{yx}) of 0.48 mm d^{-1}. For the same set of independent data, the FAO Penman–Monteith–estimated ET vs. lysimeter–measured ET gave an r^2 value of 0.85 and an S_{yx} value of 0.56 mm d^{-1}. These results show that the optimized fuzzy–neural–model is reasonably accurate, and is comparable to the FAO Penman–Monteith equation. This approach can provide an easy and efficient means of tuning fuzzy ET models.*

Keywords. *Evapotranspiration estimation, Fuzzy logic, Fuzzy–neural–model, Neural network.*

Evapotranspiration (ET) estimation models are used to estimate ET from weather parameters owing to the difficulty of obtaining accurate field measurements. The FAO Penman–Monteith equation is recommended as the standard method for computation of daily reference ET (Allen et al., 1998). More recently, there have been some attempts to model ET and/or evaporation using fuzzy logic and neural network approaches. Fuzzy systems acquire knowledge from domain experts, and this is encoded within the algorithm in terms of the set of IF–THEN rules. Fuzzy systems employ this rule–based approach and interpolative reasoning to respond to new inputs (Kaufmann and Gupta, 1991; Eberhart et al., 1996; Tsoukalas and Uhrig, 1997). Some of the pioneering studies on daily ET estimation using fuzzy logic include work by Clyma and Martin (1996), who developed a method based on fuzzy principles to forecast reference crop ET from forecasted weather information, and Ribeiro and Yoder (1997), who used fuzzy logic concepts to develop a fuzzy ET estimator for an automated irrigation control system. In a previous study (Odhiambo et al., 2001), we examined the suitability of fuzzy logic for estimating daily ET under different types of climatic conditions. Two fuzzy ET Models, one using two input weather parameters (daily solar irradiance, RS, and daily average relative humidity, RH), and the other using three input weather parameters (RS, RH, and daytime wind speed, U_d), were developed and applied to estimate grass ET. Independent weather parameters and measured ET from sites representing arid and humid climates were used to test the models. Comparison of the fuzzy–estimated ET values with direct ET measurements from grass–covered weighing lysimeters gave values for the standard error of the estimates (S_{yx}) in the range of 0.22 to 0.97 mm d^{-1}, and coefficients of determination (r^2) in the range of 0.72 to 0.90. The ET values estimated using the fuzzy model with three input weather parameters were comparable to the ET values estimated with the FAO Penman–Monteith equation at all sites evaluated. The fuzzy ET models performed better than the Hargreaves–Samani equation (Hargreaves and Samani, 1985). The results showed that fuzzy ET models could yield accurate estimation of ET with simpler and fewer input parameters. However, at low temperatures, both models tended to overestimate ET. Another drawback of the fuzzy approach is that the estimation accuracy depended on the shape of the membership functions and the control rules built by trial–and–error methods.

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The authors are **Lameck O. Odhiambo**, Post–Doctoral Research Associate, **Ronald E. Yoder**, *ASAE Member Engineer*, Professor and Head, and **Daniel C. Yoder**, *ASAE Member Engineer*, Professor; Biosystems Engineering and Environmental Science Department; and **J.Wesley Hines**, Associate Professor, Nuclear Engineering Department, The University of Tennessee, Knoxville, Tennessee. **Corresponding author**: Ronald E. Yoder, Biosystems Engineering and Environmental Science Department, 2506 E. J. Chapman Drive, Knoxville, TN 37996–4531; phone: 865–974–7266; fax: 865–974–4514; e–mail: ryoder@utk.edu.

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This study is an extension of our previous work on estimation of reference crop evapotranspiration using fuzzy state models (Odhiambo et al., 2001). Here we present a method of eliminating the reliance on the trial and error method to design the membership functions and control rules. Air temperature difference was included as an additional input parameter to resolve the problem of ET overestimation during winter months. The strategy consisted of fusing the FL and ANN on a conceptual and structural basis. The neural component provided supervised learning capabilities for optimizing the membership functions and extracting control rules from a set of input–output examples. The optimized model was applied to estimate grass ET using climatic data from different types of climates, and the results were compared with direct ET measurements from grass–covered lysimeters and estimations with the FAO Penman–Monteith equation.

ESTIMATING ET USING NEURAL NETWORKS

In contrast to fuzzy logic, neural networks offer a highly structured architecture, with learning and generalization capabilities that attempt to mimic the neurological mechanisms of the brain. A neural network stores knowledge in a distributed manner within the connection weights between elements, which are determined by training (learning) with known input–output examples. The generalization ability for new inputs is based on the inherent algebraic structure of the neural network (Rumelhart et al., 1986). A few studies that use artificial neural networks to estimate ET and/or evaporation have been reported in literature. Han and Felker (1997) developed a neural network model to estimate daily soil water evaporation from average air relative humidity (RH), air temperature (T), wind speed (U), and soil water content. The model achieved a good agreement between predicted and measured values. The average absolute percentage error and the root mean squared error were 21.0% and 0.17 mm d^{-1}, compared to 30.1% and 0.28 mm d^{-1} for a multiple linear regression model. The neural network model appeared to perform better than the multiple linear regression technique in estimating soil evaporation. Similarly, Tahir (1998) developed a neural network model to forecast monthly potential evapotranspiration (ET). The model used relative humidity (RH), solar irradiance (RS), temperature (T), and wind speed (U) as input parameters. The results showed that the neural network model was superior to the conventional methods of estimation of potential evapotranspiration.

Bruton et al. (1998) developed an artificial neural network (ANN) model of pan evaporation. The development was based on various combinations of the following daily weather data: rainfall, occurrence of rainfall, maximum temperature, minimum temperature, average temperature, maximum relative humidity, average relative humidity, total solar radiation, average wind speed, and calculated values for day length and clear sky solar radiation. Elevation and latitude of the location were also included in the data set. They found that an ANN pan evaporation model with all the variables was the most accurate, and gave a correlation coefficient of 0.72 and root mean square error of 1.11 mm d^{-1} on the evaluation data set. They also found that the ANN models of pan evaporation were slightly more accurate than multiple linear regression estimates of pan evaporation.

FUSION OF FUZZY AND NEURAL SYSTEMS

The foregoing review indicates that individual applications of FL and ANN were successful in modeling ET and/or evaporation. Although both FL and ANN approaches possess remarkable properties when employed individually, there are great advantages to using them in combination. For example, combining FL and ANN endows the fuzzy system with neuronal learning capabilities for the purpose of making them more adaptive. At the same time, the FL also improves the overall expressiveness and flexibility of the neural network. Thus the aim of combining FL and ANN is to exploit their complementary nature to develop a powerful approximate reasoning framework, which has learning and generalization capabilities.

Several examples of successful fusion of fuzzy and neural systems have been reported in the literature over the past few years. Jang and Sun (1995) developed an adaptive network–based fuzzy inference system (ANFIS) that identifies a set of parameters through a hybrid learning rule combining the backpropagation gradient descent and least square method. Takagi and Hayashi (1991) proposed a neural–network driven fuzzy reasoning algorithm. This algorithm is capable of automatic determination of inference rules and adjustment according to the time–variant reasoning environment. Nie and Linkens (1992) demonstrated that a backpropagation neural network that is based on the fuzzy set theory could implement approximate reasoning. Horikawa et al. (1992) presented a fuzzy modeling method, which uses a fuzzy system fused into a neural network with backpropagation algorithm. The method can identify the fuzzy model of a nonlinear system automatically. Mitra and Pal (1994) developed a fuzzy layered neural network for classification and rule generation. This model is a logical version of the feedforward multilayer perceptron using the concept of fuzzy set at various stages. The model can handle uncertainty and/or impreciseness in the input and output representations. Simpson and Jahns (1993) proposed a fuzzy min–max neural network for function approximation. This network is realized by fusion of fuzzy sets and neural networks in a unified framework. In general, all these methods interpret a fuzzy system in terms of a neural network such that each step in the process is equivalent to at least one layer in the network.

THEORETICAL CONSIDERATIONS

Evapotranspiration (ET) is a combination of two separate water–transfer processes whereby water is transferred from the soil, water, and wet plant surfaces to the atmosphere by evaporation, and also through the crop by transpiration. In this section we present the theoretical concepts and the assumptions considered in the development of the ET estimation model by looking at the controlling forces of ET and how these affect the rate of water transfer by a cropped surface to the atmosphere. A more complete review of the physics and relationships useful in analyzing the phenomenon of evaporation are found in Monteith (1973) and Brutsaert (1982). Briefly, ET is controlled by two conditions: the amount of energy available for use in the vaporization process (ε), and an atmospheric factor (C) representing the capacity of the atmosphere to absorb water vapor. Solar irradiance (RS) is the main source of latent heat of vaporization. In hot arid cli–

ates, sensible heat from the air may contribute energy for the vaporization process. At low temperatures, part of the solar energy is generally converted to sensible heat to raise the air temperature (T) and is unavailable for the vaporization process. The amount of incoming solar irradiance converted to sensible heat is best reflected by the difference between daily maximum and minimum air temperatures, i.e., temperature difference, TD (Hargreaves and Samani, 1985). In our trial runs, the TD appeared to best represent the air temperature factor involved in the evapotranspiration process when used in combination with RS Therefore, ε was taken as a function of RS and TD.

When the energy available for use in the vaporization process (ε) is low, the amount of water transferred by ET is low. Evapotranspiration increases with increase in ε and is only limited by C, when soil water is not limiting. The atmospheric factor (C) depends on the relative humidity (RH) of the air. Relative humidity is dependent on air temperature and varies considerably throughout the day such that when the temperature rises, relative humidity falls and vice versa. This behavior occurs irrespective of the actual changes in atmospheric moisture levels. Wind speed (U_d) influences the capacity of the atmosphere to absorb moisture by replacing the nearly saturated air layers near the crop surface with unsaturated air from outside the crop canopy. The contribution of U_d to C depends on RH. When RH is high, the contribution of U_d to C is low, and when RH is low, the contribution of U_d to C is high. Thus C is a function of RH and U_d. The plant canopy resistance (r_c) provides an important link between the plant canopy and the atmosphere. Hence, ET is a function of ε, C, and r_c, where r_c is plant canopy resistance to vapor exchange between the plant canopy and the atmosphere. The plant canopy resistance depends on the leaf area index and stomatal resistance of the leaves.

Conceptual and Structural Basis of the Model

The conceptual model in figure 1 can represent the processes involved in ET. The relation between inputs and output inside boxes 1, 2, and 3 are not fully understood and/or defined. In trying to model these processes, we are faced with two idealized extremes, where either (1), we know exactly how the system should be working but have no example of its input–output behavior, or (2), we know its input–output behavior but know nothing of the system's internal working (black box). In the first case, it is convenient to use fuzzy logic (FL) reasoning to describe the system behavior. In the second case, it is convenient to use the available input–output examples to train artificial neural networks (ANN) to model the internal workings of the system.

In the real world system, we have partial knowledge of what is inside the boxes, and some examples of the system's input–output behavior. Hence, we may use a combination of FL and ANN tools to model the ET process. The main issues considered in integrating FL and ANN systems for the ET model are the strategy to be adopted in combining the two together, and how to facilitate cognitive learning and knowledge representation. Many methods for combining FL and ANN systems have been suggested (Khosla and Dillon, 1997; Takagi, 1997; Tsoukalas and Uhrig, 1997). In this study, we fused the FL and ANN together on a conceptual and structural basis to give the resultant model the ability to learn and deal with new situations.

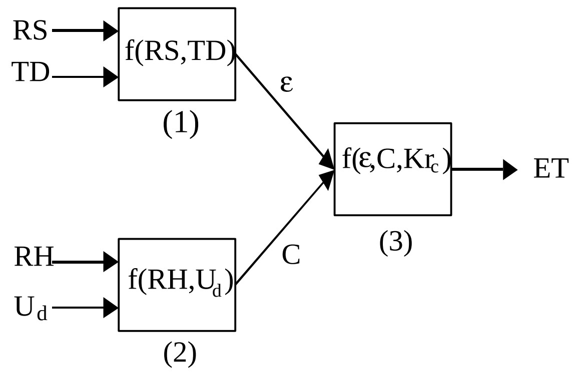

Figure 1. A physically based conceptual model of the evapotranspiration process where weather parameters (RS, TD, RH, and U_d) get mapped to intermediate ε and C, and then to ET.

In the fused fuzzy–neural system, the information–processing features of the FL are fused into the representation structure of ANN. The resultant structure consists of a six–layer feed–forward neural network (fig. 2), which corresponds to the conceptual model in figure 1. The model generates output by implementing a fuzzy inference scheme through the various layers of the network. Each layer of the network performs one stage of the fuzzy inference process. The stages of the fuzzy inference process are: (1) transformation of real numerical input data into fuzzy sets (a process known as fuzzification); (2) fuzzy reasoning based on the control rules (rule base); and (3) transformation of the fuzzy output of a fuzzy inference into real numerical numbers (a process known as defuzzification). A detailed description of the fuzzy inference process can be found in several references, including Kaufmann and Gupta (1991), Jang and Sun (1995), and Tsoukalas and Uhrig (1997).

Data Space and Membership Functions

The input and output data spaces used for model development are shown in table 1. The input and output spaces were selected to include a wide variety of climates between latitudes 60° N and 60° S. The model inputs include measured daily solar irradiance (RS) in MJ m^{-2} d^{-1}, percent relative humidity (RH) computed as the average of maximum and minimum daily relative humidity, average daytime wind speed (U_d) in m s^{-1}, and air temperature difference (TD) computed as the difference between maximum and minimum daily air temperature in °C. The model output was ET in mm d^{-1}. The intermediate parameters were available energy for vaporization (ε) expressed in equivalent water evaporation, and an atmospheric factor (C) representing the capacity of the atmosphere to absorb water vapor. The input, intermediate, and output data spaces were categorized into five fuzzy sets. These were VERY LOW (VL), LOW (LO), MEDIUM (ME), HIGH (HI), and VERY HIGH (VH). Any arbitrary curve whose shape is suitable from the point of view of simplicity, convenience, speed, and efficiency can be used as a neuron activation function. In this case, Gaussian distribution membership functions were used to determine the degree of membership of data points to the respective fuzzy sets (fig. 3).

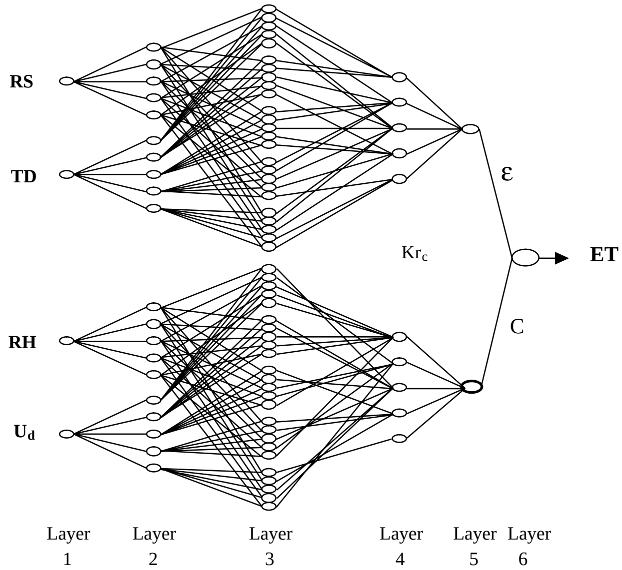

Figure 2. A fuzzy–neuron system involving a fuzzy system fused into a neural network where weather parameters (RS, TD, RH, and U_d) get mapped to intermediate ε and C, and then to ET. Layer 3 represents all possible combinations of fuzzy sets between RS and TD, and RH and U_d.

Table 1. Input and output data space used in the fuzzy–neural ET model.

Input / output parameters	Minimum	Maximum	Units
Solar radiation (RS)	2	37	MJ m^{-2} d^{-1}
Temperature difference (TD)	0	25	°C
Relative humidity (RH)	20	100	%
Wind speed (U_d)	0	10	m/s
Evapotranspiration (ET)	1	12	mm/d

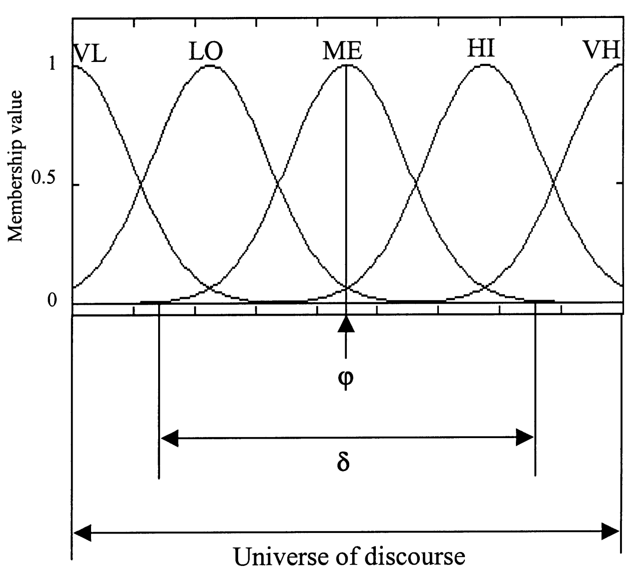

Figure 3. Illustration of the input /output data space divided into five fuzzy sets (VL, LO, ME, HI, and VH) and showing the definition of center (φ) and spread (δ) for the set ME.

Implementation of the Fuzzy Algorithm

The fuzzy algorithm was implemented through the various layers of the neural network (fig. 2). The nodes in layer 1 are fan–in neurons. They receive the input weather parameters, i.e., RS, TD, RH, and U_d, and distribute them to the neurons in layer 2 without doing any computation. The neurons in layer 2 represent the fuzzy sets used in the antecedent parts of the control rules, i.e., VL, LO, ME, HI, and VH. Each neuron consists of an activation function used to compute membership functions for the input variable it receives from layer 1. Each neuron has only one input variable and one output membership function. For example, if an input variable RS is passed through the neuron representing the fuzzy set LOW, then the output is μ_{LOW}(RS). This neuron feeds its output to all the rules using the clause ‘if RS is LOW’ in the ‘IF’ part of the rules. The Gaussian distribution curve was used as the neuron activation function and is given by:

$$\mu_{LOW}(x) = \exp\left(-(x-\varphi)^2 / 2\delta^2\right) \qquad (1)$$

where $\mu_{LOW}(x)$ is the membership value for the fuzzy set LOW, x is the input variable, φ describes the ‘center’ of the membership function, and δ is the spread of the membership function (fig. 3). The shape and position of the membership function will change if either φ or δ is changed. The center and spread are considered as weights on the input links to this layer, analogous to the approach taken with radial–basis–function units in neural networks (Moody and Darken, 1989). Jang and Sun (1993) showed that under some minor restrictions, the functional behavior of radial basis function networks and fuzzy inference systems are equivalent.

A neuron in layer 3 corresponds to a rule in the FL rule base. Its inputs come from the neurons in layer 2 which participate in the ‘IF’ part of that rule. The number of neurons in this layer is equal to the number of control rules that are used to capture the relationship between the input parameters and desired output. For example, we have 25 rules (5 × 5 combination) drawn to capture the relationship between RS, TD, and ε in box 1 (table 2). Similarly, there is also another set of 25 rules defining the relationship between RH, U_d, and C in box 2 (table 2). Thus layer 3 has a total of 50 neurons representing 50 rules. Each neuron in this layer performs the fuzzy ‘AND’ operation. The choice of the implication operator is based on the interpretation of the connective ‘AND’. In trial runs, we tried three commonly used implication operators, namely, Zadeh minimum implication operator, the arithmetic implication operator based in multi–valued logic, and the Larsen product implication operator. The *Larsen product* (ϕ_p) implication operator (Larsen, 1980) gave the best results and was used in the model to evaluate the conjunction ‘AND’ in the rules. For example, if μ_{HIGH}(RS) = 0.88 and μ_{MEDIUM}(TD) = 0.59 in a rule, then the firing strength of the rule (ω_1) can be expressed as follows:

$$\omega_1 = \phi_p(\mu_{HIGH}\text{RS}), \mu_{MEDIUM}(\text{TD}))$$

$$= \phi_p(0.88, 0.59) = 0.52 \qquad (2)$$

Table 2. Initial fuzzy rules before optimization. The antecedent fuzzy sets are defined by their centers and spreads, and the consequent fuzzy sets are defined by their centers only.

Box 1					
TD \ RS	VL $\varphi = 2.0$ $\delta = 3.72$	LO $\varphi = 10.75$ $\delta = 3.72$	ME $\varphi = 19.5$ $\delta = 3.72$	HI $\varphi = 28.25$ $\delta = 3.72$	VH $\varphi = 37.0$ $\delta = 3.72$
VL $\varphi = 0.0$ $\delta = 2.65$	0	0	0	0	0
LO $\varphi = 6.25$ $\delta = 2.65$	0	0	0	0	0
ME $\varphi = 12.5$ $\delta = 2.65$	0	0	0	0	0
HI $\varphi = 18.75$ $\delta = 2.65$	0	0	0	0	0
VH $\varphi = 25.0$ $\delta = 2.65$	0	0	0	0	0
Box 2					
U_d \ RH	VL $\varphi = 20.0$ $\delta = 8.5$	LO $\varphi = 40.0$ $\delta = 8.5$	ME $\varphi = 60.0$ $\delta = 8.5$	HI $\varphi = 80.0$ $\delta = 8.5$	VH $\varphi = 100.0$ $\delta = 8.5$
VL $\varphi = 0.0$ $\delta = 1.06$	0	0	0	0	0
LO $\varphi = 2.5$ $\delta = 1.06$	0	0	0	0	0
ME $\varphi = 5.0$ $\delta = 1.06$	0	0	0	0	0
HI $\varphi = 7.5$ $\delta = 1.06$	0	0	0	0	0
VH $\varphi = 10.0$ $\delta = 1.06$	0	0	0	0	0

The neurons in layer 4 evaluate the consequent 'THEN' part of the rules. A neuron in this layer corresponds to a consequent label (i.e., VL, LO, ME, HI, and VH). Input to this layer comes from all the rules in layer 3, which use this particular consequent label. Layer 4 neurons aggregate the consequents of all the rules that feed them and each computes the output by using the center of gravity method (Tsoukalas and Uhrig, 1997). Layer 5 has two neurons. The first neuron in this layer combines the recommendations from all the fuzzy control rules in the rule base governing the relations between inputs RS, TD, and ε. The second neuron combines the recommendations from all the fuzzy control rules in the rule base governing the relations between inputs RH ,U_d, and C. The output of each neuron is expressed as follows:

$$Y_m = \frac{\sum_{i=1}^{m}\sum_{j=1}^{k}\omega_{r(i,j)}\varphi_{cr(i,j)}}{\sum_{i=1}^{m}\sum_{j=1}^{k}\omega_{r(i,j)}} \tag{3}$$

where Y_m is the output of the neuron in layer 5 (i.e., ε or C), ω_r is the activation strength of rule r, φ_{cr} is the center of the consequent fuzzy set the rule r, m and k are the total number of fuzzy sets in column 1 and row 1 in the fuzzy rule table (see table 1), i is the row number, and j is the column number. The rule number is calculated as r = (i–1)k + j. The neuron in layer 6 computes the estimated ET based on ε, C and r_c. The atmospheric factor (C) acts as an adjustment on ε, and hence ET is computed as ET = f(ε,C,r_c). It uses the algebraic product T–norm operation (i.e., ET = ε × C × Kr_c). The parameter r_c is represented by a constant Kr_c assumed to be 1 for the reference crop for which the model is optimized.

Optimization of the Model

Optimization of the model was achieved by adjusting the center (φ_a) and spread (δ_a) of all the antecedent membership functions, and the center (φ_c) of all the consequent membership functions. Centers of the membership functions of individual fuzzy sets (VL, LO, ME, HI, and VH) define the input vectors causing maximal activation of these sets, and the spreads of the membership function of individual fuzzy sets determine the radii of the areas of the input space around the centers where activations of the fuzzy sets are maximum (fig. 3). The parameters φ_a and δ_a are represented as input connection weights to neurons of layers 2, and φ_c as input connection weights to neurons of layer 4. The process of adjusting the connection weights between layers of neurons is called training, and consists of presenting the network with examples of input–output pairs, and adjusting the connection weights until the objective function is minimized. The objective function to be minimized is defined as the mean sum squared error (MSE), which is expressed as MSE = $(Y_t - Y_m)^2/d$, where Y_t is the target output, Y_m is the model output, and d is the number of training data points.

Training Procedure

The training process consisted of two separate stages. During the first stage, parameters φ_a and δ_a for individual input fuzzy sets in layers 2 and 4 were set such that the five fuzzy sets were distributed uniformly over the data space (see fig. 3 and table 2). The centers for individual output fuzzy sets were set to zero. Training pairs were selected consisting of typical samples and patterns from the available lysimeter and weather data range. In a forward pass, the input data (RS, RH, U_d, TD, and corresponding lysimeter ET) were propagated from the input to the output. Calculation of the output was carried out, layer by layer, in the forward direction. The second stage consisted of a reverse pass. The connections between neurons were adjusted starting with the input connections into layer 4, and moving in reverse to the weight of the input connections into layer 2. The other weights were fixed at unity. This means that the weight adjustment works on only two layers of weights, rather than all six. A batch–training mode where the weights are only adjusted after all of the inputs have been presented was used.

Training Algorithm

Several algorithms for updating the parameters φ_a, δ_a, and φ_c have been developed (Moody and Darken, 1989; Berenji and Khedkar, 1992; Wang and Mendel, 1992; Mizumoto and Shi, 1997). The gradient descent algorithm developed by Mi-

zumoto and Shi (1997) was adopted for training because of its simplicity, efficiency, and capability to optimize the fuzzy control rules and membership functions without changing the form in which the rules are presented in the rule tables. For example, the expressions for adjusting the centers (φ) and spreads (δ) of the input fuzzy sets in column 1 of a fuzzy rule table were as follows:

$$\varphi_i(t+1)=\varphi_i(t)+\frac{\eta_1(E)(x-\varphi_i(t))\sum_{j=1}^{k}\omega_{r,j}(\varphi_{cr,j}-Y_{m,j})}{\delta_i^2\sum_{i=1}^{m}\sum_{j=1}^{k}\omega_{r(i,j)}} \quad (4)$$

$$\delta_i(t+1)=\delta_i(t)+\frac{\eta_2(E)(x-\varphi_{1i}(t))^2\sum_{j=1}^{k}\omega_{r,j}(\varphi_{cr,j}-Y_{m,j})}{\delta_{1i}^3\sum_{i=1}^{m}\sum_{j=1}^{k}\omega_{r(i,j)}} \quad (5)$$

where i is the index of fuzzy sets in column 1, and j is the index of fuzzy sets in row 1 (see tables 2 and 3), x is the input parameter, φ_i is the center of membership functions for fuzzy set i; δ_i is the spread of membership functions for fuzzy set i, η_1 and η_2 are the learning rates (both = 0.0005); t is the learning iteration; m and k are the total numbers of fuzzy sets in column 1 and row 1 in the fuzzy rules table. The same expressions were used to adjust the fuzzy sets in row 1 of the rule table, but the summation in the numerator of equations 1 and 2 run from i = 1 to m instead of j = 1 to k. The expressions for adjusting the centers of the consequent fuzzy sets was expressed as follows:

$$\varphi_{cr}(t+1)=\varphi_{cr}(t)+\frac{\eta_3(E)\omega_r}{\sum_{i=1}^{m}\sum_{j=1}^{k}\omega_{r(i,j)}} \quad (6)$$

where η_3 is the learning rate (η_3 = 0.0065). The final parameters (centers and spreads) obtained for the training data set used are presented in table 3.

Table 3. Final fuzzy rules after optimization. The antecedent fuzzy sets are defined by their centers and spreads, and the consequent fuzzy sets are defined by their centers only.

Box 1					
RS / TD	VL φ = 1.973 δ = 3.665	LO φ = 11.067 δ = 4.214	ME φ = 20.397 δ = 4.974	HI φ = 27.059 δ = 4.985	VH φ = 37.096 δ = 3.645
VL φ = 0.0601 δ = 2.8180	0.071	0.584	0.190	0.105	0.001
LO φ = 7.0121 δ = 3.6890	0.292	1.277	2.826	3.440	0.040
ME φ =12.4076 δ = 4.3578	0.155	1.819	1.477	3.636	0.402
HI φ =17.2962 δ = 3.7805	0.022	1.383	2.175	4.101	0.978
VH φ =25.4441 δ = 2.0232	0.000	0.004	0.598	2.059	0.166
Box 2					
RH / Ud	VL φ = 19.994 δ = 8.483	LO φ = 39.964 δ = 8.496	ME φ = 60.657 δ = 9.225	HI φ = 79.783 δ = 9.283	VH φ = 99.722 δ = 8.998
VL φ = 1.1518 δ = 2.618	0.002	0.163	0.290	1.089	0.416
LO φ = 2.983 δ = 2.362	0.001	0.849	0.892	2.581	1.737
ME φ = 4.150 δ = 2.026	0.026	2.443	3.520	2.932	1.440
HI φ = 5.468 δ = 2.087	0.146	4.857	2.884	0.267	0.509
VH φ = 10.418 δ = 0.243	0.013	0.347	0.385	0.018	0.000

Simulation and Results

Daily evapotranspiration (ET) data for well watered grass along with supporting climatic data from three sites (Crossville, Tennessee; Bushland, Texas; and Paraipaba, Ceara, Brazil) representing different climates ware used in the study. A description of the sites and climates used are presented in table 4, where RS, RH, U_d, and T are the average solar irradiance, average daily relative humidity, average daytime wind speed, and average air temperature respectively for the periods considered. The climatic parameters were measured from automatic weather stations, and ET was directly measured from well–watered, grass–covered weighing lysimeters. A summary of the characteristics of the lysimeter facilities at each site is presented in table 5. The integrity of the climatic data was assessed based on guidelines by Allen

Table 4. Description of location and climates of lysimeter sites evaluated.

Site (and date)	No. of data	Latitude	Altitude (m)	RS (MJ m^{-2} d^{-1})	RH (%)	U_d (m/s)	T (°C)
Crossville, Tennessee (July–Sept. 1997)	29	35°55’N	573	19.9	79.5	1.0	21.0
Crossville, Tennessee (May–June 1994)	50	35°55’N	573	22.3	78.1	1.1	17.5
Crossville, Tennessee (Jan.–Feb. 1997)	60	35°55’N	573	7.0	84.7	1.7	5.2
Paraipaba, Ceara, Brazil (March–May 1998)	60	3°29’S	30	19.0	84.8	3.2	27.6
Bushland, Texas (May–Sept. 1998/99)	37	35°11’ N	1170	23.5	59.4	4.2	22.2

Table 5. Summary of the characteristics of the lysimeter facilities at the evaluated sites.

Characteristic	Crossville, Tennessee	Paraipaba, Ceara Brazil	Bushland, Texas
Type of lysimeter	Weighing	Weighing	Weighing
Type of scale system	Lever load cell[a]	Floor stand scale	Lever load cell[a]
Soil profile	Monolith	Reconstructed	Monolith
Wall material	Steel	Steel	Steel
Surface area (m^2)	4.0	2.25	9.0
Soil depth (m)	1.8	1.0	2.3
Drainage type	Free drainage	Periodic drainage	Free drainage
Sensitivity (ET mm)	0.05	0.18	0.05

[a] Counterbalance lever load cell.

(1996) and found to be of good quality. The records of lysimeter condition and maintenance were used to select days with good measured ET data. A total of 118 days ET data were selected from the three sites and divided into two sets, the training data set and the test data set. The two sets were of equal size (n = 59). The training data were selected to cover the available data hyperspace reasonably well, and especially to include the data close to the decision boundaries of the hyperspace. This set was used to train the model as described in the training procedure.

Training was completed within MSE $\leq$ 0.25 mm^2, and the model parameters frozen at the prevailing values. The model was then used in a simulation mode with the test data to obtain ET estimates. The standard errors of the estimate (S_{yx}) were calculated based on the ET estimates that have not been adjusted by regression. A plot of the estimated ET by the fuzzy–neural–model using training data versus lysimeter measured ET (fig. 4a) shows that the model was able to capture the relationship between the presented input weather parameters and the output ET with a good fit. The coefficient of determination (r^2) was 0.87, and the standard error of the estimate (S_{yx}) was 0.66 mm d^{-1}. The FAO Penman–Monteith equation gave an r^2 value of 0.93 with the same data. When the training completion level was set at MSE $\leq$ 0.15 mm^2, the model realized an r^2 value of 0.97, but then it became over–trained and less general. The MSE level of 0.25 for training completion was selected as a compromise between accuracy and better model generalization. The model does not need to be trained at every location so long as the predictor inputs are within the hyperspace of the training data.

The performance of the model was evaluated using the independent test data. A plot of model estimated ET versus lysimeter measured ET using the test data (fig. 4b) shows a good fit between the two, with r^2 = 0.88 and S_{yx} = 0.48 mm d^{-1}. Figure 4c shows a comparative plot of the estimated ET by the FAO Penman–Monteith equation using the test data versus lysimeter measured ET. The r^2 value in this case was 0.85 and the S_{yx} value was 0.56 mm d^{-1}. In order to evaluate the performance of the fuzzy–neural–model during winter months for which lysimeter measured data were not available, the model was trained with the FAO Penman–Monteith equation estimated ET for a winter month (January 1997, Crossville). The trained model was then used to estimate ET for an independent winter month (February 1997, Crossville). The model outputs were plotted versus the FAO Penman–Monteith equation estimated ET. The results with training data (fig 5a) show an almost perfect fit, with an r^2 =

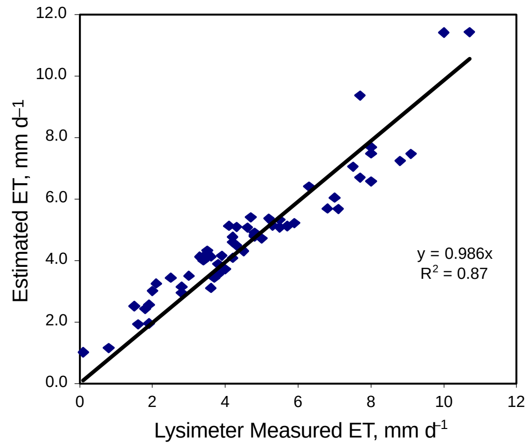

Figure 4a. Estimates of ET by fuzzy–neural model versus daily lysimeter ET using training data.

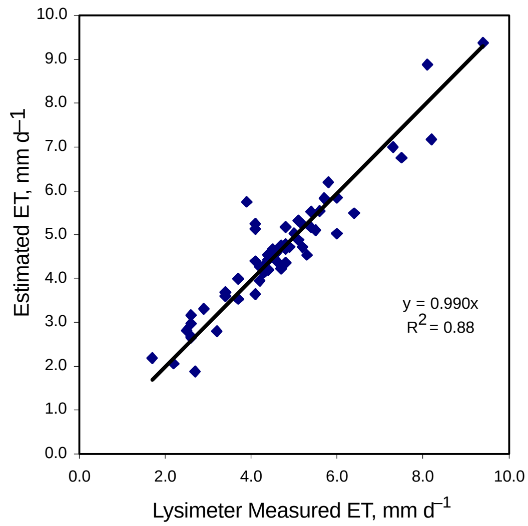

Figure 4b. Estimates of ET by fuzzy–neural model versus daily lysimeter ET using test data.

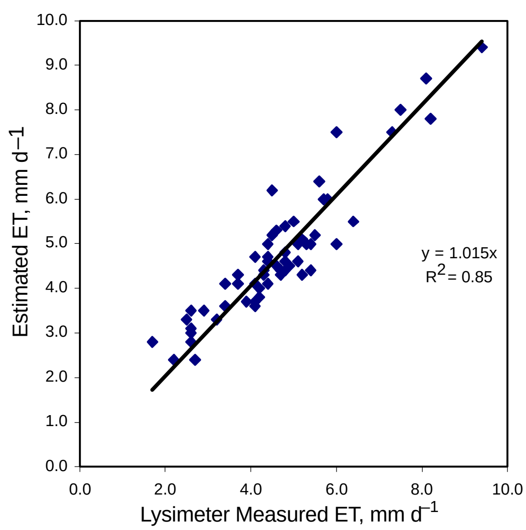

Figure 4c. Estimates of ET by FAOP Penman–Monteith equation versus daily lysimeter ET using test data.

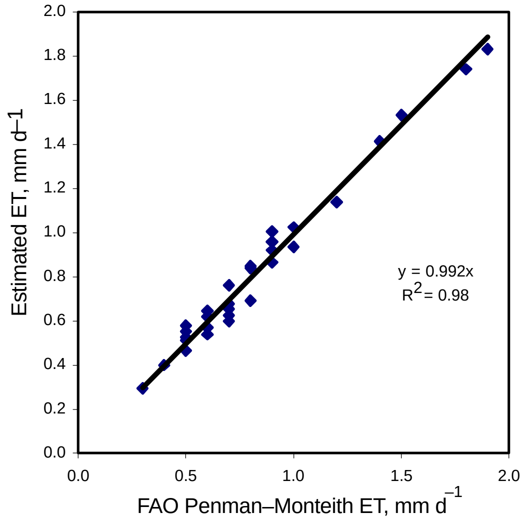

Figure 5a. Estimates of ET by fuzzy–neural model versus estimates of ET by the FAO Penman–Monteith equation using training data.

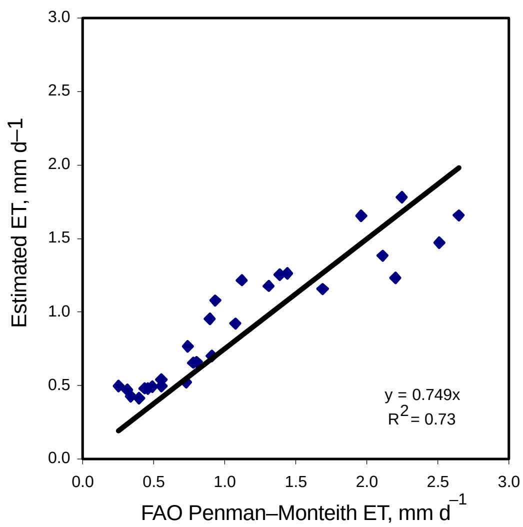

Figure 5b. Estimates of ET by fuzzy–neural model versus estimates of ET by the FAO Penman–Monteith equation using test data

0.98 and a S_{yx} = 0.06 mm d^{-1} (n = 31). The plot of ET estimated with independent winter test data is shown in figure 5b. The r^2 value was 0.73 and the S_{yx} was 0.16 mm d^{-1} (n = 29). These results indicate that the fuzzy–neural–model is reasonably accurate and is comparable to the FAO Penman–Monteith equation in different climates. The inclusion of daily air temperature as a temperature difference appears to successfully address the issue of ET overestimation during low temperatures observed in our previous three input parameter fuzzy ET model (Odhiambo et al., 2001).

Summary and Conclusions

The study presented a method of eliminating the reliance on the trial and error method to design the membership functions and control rules in fuzzy ET models. The strategy consisted of fusing the FL and ANN on a conceptual and structural basis. The structure of the fuzzy–neural–model provided a systematic and easy way of optimizing the membership functions, and extracting the fuzzy rules from input–output examples. The results show that the optimized fuzzy–neural–model is reasonably accurate, and is comparable to the FAO Penman–Monteith equation. Thus, optimization of fuzzy ET models through neural training with input–output examples can provide an easy and effective method of tuning fuzzy ET models to new sets of climatic conditions. Fuzzy ET models can yield accurate estimation of ET with simpler and fewer input parameters.

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