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The History and Applications of Fibonacci Numbers

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THE HISTORY AND APPLICATIONS OF FIBONACCI NUMBERS

Cashous Bortner with adviser Dr. Allan Peterson

1. Origins

Leonardo Bonacci, (seen in figure 1) known by most as Fibonacci, was arguably one of the most influential mathematicians in Europe in the 13th century. The book for which he is now famous, *Liber Abaci* (1202 A.D.) not only brought the Hindu-Arabic numeral system to the Western World, but also brought many interesting problems that had not been considered at the time.

Figure 1. Leonardo Bonacci

Iteration	Pairs of Rabbits
1	1
2	1
3	2
4	3
5	5
6	8
7	13
8	21
9	34
⋮	⋮

Table 1.

Fibonacci's most recognized contribution to mathematics came in the form of one of these problems, which is now generally referred to as **The Rabbit Problem**. The problem is read as follows:

- Begin with one pair of [juvenile] rabbits.
- The rabbits must wait one iteration after birth to [mature and] begin to give birth.
- Every iteration (after their first iteration of life), each pair of rabbits gives birth to one pair of rabbits that will also eventually reproduce.
- The rabbits live and reproduce indefinitely.

So then, as can be seen in the table 1, a pattern forms for the total number of rabbits in a particular iteration. To find the current number of rabbits, one can take the sum of the previous two iterations' number of rabbits!

This is the origin of the now famous Fibonacci Sequence, with the first two numbers in the sequence being one (or equivalently zero and one depending on the source).

2. Fibonacci Numbers in Nature

Fibonacci numbers are found in numerous areas of nature including, the spiral bracts of a pinecone as seen in figure 2 (and pineapple), and many other "perfect" specimens of vegetation such as branches on trees and bushes. More often found are examples of equiangular spirals, which can be created using the Fibonacci numbers, in Nautilus shells as seen in figure 3, a spiral galaxy's arms, a hurricane, an ocean wave, and some growing ferns.

The Fibonacci numbers can also be seen in the genealogy of a drone (male) bee as depicted in figure 4. A drone is born of only a mother, and does not need a father (i.e. does not result from a fertilized egg), while a female is born from a fertilized egg.

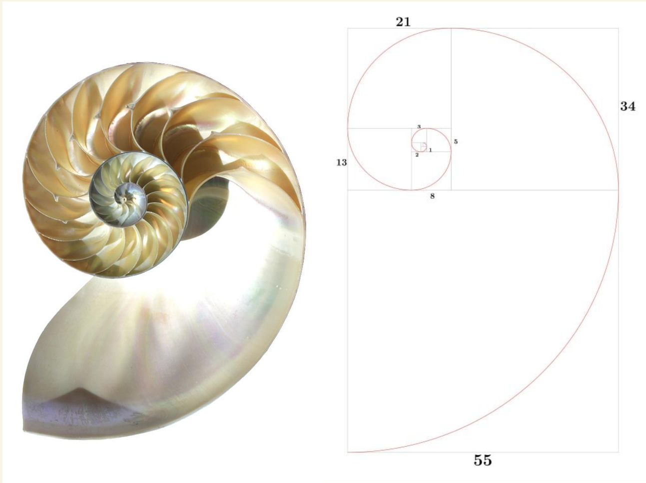

Figure 3. Nautilus shell and Equiangular Spiral

Retrieved from http://deography.com/nautilus-shell-isolated/

Figure 2. Pinecone

Retrieved from http://curiosamathematica.tumblr.com/post/114294953296/golden-ratio-in-phyllotaxis

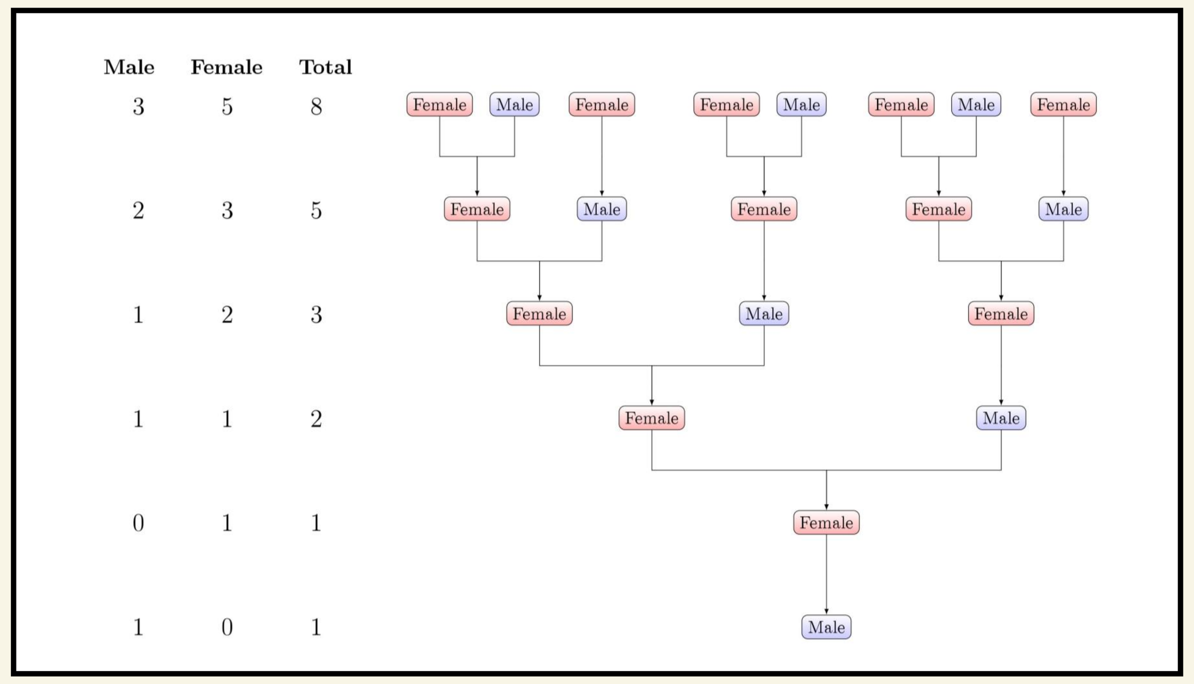

Figure 4. Male Bee Genealogy

3. Mathematical Applications

As previously mentions, one can compute a Fibonacci number by using the previous two Fibonacci numbers as reference:

$$F_n = F_{n-1} + F_{n-2}.$$

However, in 1843 Binet derived a new formula for calculating the nth Fibonacci number:

$$F_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}.$$

Fibonacci numbers have been applied in areas of combinatorics including a search algorithm that uses the Fibonacci numbers to find an element in a sorted array much like a binary search.

Fibonacci numbers also have many more applications as it can be applied to areas such as Pythagorean triples, or integers solutions to $a^2 + b^2 = c^2$, Pascal's triangle and many more.

The Fibonacci numbers also share a special bond with the golden ratio which is approximately $\phi \approx 1.618$ as the limit to infinity of the quotient of consecutive Fibonacci numbers converges to the golden ratio:

$$\lim_{n\to\infty} \frac{F_n}{F_{n-1}} = \phi.$$

4. Application to Trading

One of the main applications of Fibonacci numbers outside of the realm of mathematics is in the area of stock market analysis. Many investors use what is called the Fibonacci Retracement Technique to estimate the action that the price of a particular stock will take, based on certain ratios found within the Fibonacci numbers.

Figure 5. Fibonacci Retracement

Retrieved from http://dailypriceaction.com/forex-beginners/fibonacci-retracement-levels

As seen in figure 5, the retracement uses lines across the 0, 23.6, 38.2, 50, 61.8, and 100 percentiles of the chosen high and low values. A trader would then use these estimates to buy stock when the value decreases to one of these percentages, and sell stock as it peaks at another of the percentages.

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